A Robust Iterative Kalman Filter Based On Implicit Measurement Equations

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Abstract

In the field of robotics and computer vision recursive estimation of time dependent processes is one of the key tasks. Usually Kalman filter based techniques are used, which rely on explicit model functions, that directly and explicitly describe the effect of the parameters on the observations. However, some problems naturally result in implicit constraints between the observations and the parameters, for instance all those resulting in homogeneous equation systems. By implicit we mean, that the constraints are given by equations, that are not easily solvable for the observation vector.

We derive an iterative extended Kalman filter framework based on implicit measurement equations. In a wide field of applications the possibility to use implicit constraints simplifies the process of specifying suitable measurement equations. As an extension we introduce a robustification technique similar to [17] and [8], which allows the presented estimation scheme to cope with outliers.

Furthermore we will present results for the application of the proposed framework to the structure-from-motion task in the case of an image sequence acquired by an airborne vehicle.

1 Introduction

Recursive estimation or Kalman filtering is a classical technique [10] and has been widely used in robotics and computer vision [19]. Some examples are ego-motion estimation and structure-from-motion, object tracking or calibration tasks. All those recursive estimation schemes assume a functional model, where the observations are explained by an explicit function in the unknown parameters.

However, many problems encountered in computer vision naturally result in implicit constraints between the observations and the parameters [5, 6, 7, 13].

Although it is always possible to reduce the solution of an implicit problem to the solution of an explicit problem [11, p.231ff], it is often much easier and straightforward to specify the measurement equations as implicit functions relating the state vector with the observation vector [15]. A first order approximation to use implicit constraints in the classical Kalman filter without any iteration was introduced by [14].

The main goal of this paper is to provide a recursive estimation scheme, that can be applied to such problems comprising of implicit constraints in a black-box manner thereby simplifying the task of recursive estimation from the modeling point of view. The scope is not to present a run-time optimized estimation scheme tailored specifically for the task of structure-from-motion, as the proposed method is a framework, which is applicable in a much broader context.

The Kalman filter consists of two parts, namely a time update and a measurement update. The scope of our work is not the time update but the measurement update, for which we will present a solution based on implicit constraints.

Recently the Kalman filter based on the unscented transformation [9] has obtained a lot of attention, which aims at improving the stochastic properties of the filter. Our work on the other hand aims at simplifying the specification of measurement equations, which are often much easier and straightforward to derive as implicit functions.

We will demonstrate the applicability of our approach for the task of on-line structure-from-motion from image sequences acquired by an unmanned aerial vehicle (UAV), which may be modeled using explicit functions

[1] as well as using implicit functions (see section 3.2). The results from the recursive estimation will be compared to a global optimized solution obtained by an overall bundle adjustment.

We are aware that a lot of highly optimized non-linear methods for the task of on-line structure-from-motion from image sequences are available [2, 4, 18], which exploit the specific structure of the normal equation matrix. However, this is not the scope of our paper as the presented methods are applicable to a variety of problems beyond structure-from-motion, which can be specified using implicit functions. The structure-from-motion problem is only used to demonstrate the applicability of the proposed method, as it is well-known to many researchers and test-sequences are readily available.

This work is structured as follows: first we will derive the prerequisites for the recursive estimation algorithm based on implicit measurement functions in section 2.1. Then we will show, how outliers can be detected and the algorithm can be made more robust in section 2.2. The final algorithm will be given in a black-box manner in section 2.3. Finally we will present some results for the task of recovering the flight path of an unmaned aerial vehicle in section 3.

2 Recursive Estimation using Implicit Functions

We will now derive a robust estimation scheme for implicit constraints. First we will derive the basic equations followed by a section on robustification. The final algorithm will then be summarized in section 2.3.

2.1 Recursive estimation

We will now derive a recursive estimation scheme for the case of implicit measurement constraints. In complete analogy to the classical explicit Kalman filter we start with a parameter vector $\boldsymbol{p}_{11}$ (also known as state vector) and its covariance matrix Q_{11} resulting from some prediction step. This state should now be updated according to a newly acquired measurement vector $\boldsymbol{z}$, which implicitly constraints the parameter vector. By implicit we mean, that the measurement model is given by an implicit function

$$\boldsymbol{g}(\boldsymbol{p}, \boldsymbol{z}) = \boldsymbol{0} \tag{1}$$

relating the unknown parameter vector $\boldsymbol{p}$ to the observation vector $\boldsymbol{z}$. Such an implicit observation model equation is often much easier to obtain than an explicit function $\boldsymbol{z} = \boldsymbol{f}(\boldsymbol{p})$, which is required by the classical Kalman filter. Note, that every explicit function is easily made implicit by simple subtraction. Furthermore, we will assume a covariance matrix

$$D(\boldsymbol{z}) = \mathsf{C}_{zz} \tag{2}$$

supplied together with each measurement.

We start by analyzing, how a new parameter vector can be estimated from those observations alone, by looking at the Taylor expansion of the observation model equation

$$\begin{aligned} \boldsymbol{0} &\approx \boldsymbol{g}(\boldsymbol{p}^\nu, \boldsymbol{z}^\nu) + \mathsf{A}(\widehat{\boldsymbol{p}} - \boldsymbol{p}^\nu) + \mathsf{B}^\mathsf{T}(\widehat{\boldsymbol{z}} - \boldsymbol{z}^\nu) && (3) \\ &= \boldsymbol{g}(\boldsymbol{p}^\nu, \boldsymbol{z}^\nu) + \mathsf{A}\Delta\boldsymbol{p} + \mathsf{B}^\mathsf{T}(\widehat{\boldsymbol{z}} - \boldsymbol{z} + \boldsymbol{z} - \boldsymbol{z}^\nu) && (4) \end{aligned}$$

containing the Jacobians

$$\mathsf{A} = \left.\frac{\partial \boldsymbol{g}(\boldsymbol{p}, \boldsymbol{z})}{\partial \boldsymbol{p}}\right|_{\boldsymbol{z}^\nu, \boldsymbol{p}^\nu} \qquad \mathsf{B} = \left.\frac{\partial \boldsymbol{g}(\boldsymbol{p}, \boldsymbol{z})^\mathsf{T}}{\partial \boldsymbol{z}}\right|_{\boldsymbol{z}^\nu, \boldsymbol{p}^\nu} \tag{5}$$

Rearranging this equation we obtain

$$\mathsf{A}\Delta\boldsymbol{p} + \mathsf{B}^\mathsf{T}(\widehat{\boldsymbol{z}} - \boldsymbol{z}) = -\boldsymbol{g}(\boldsymbol{p}^\nu, \boldsymbol{z}^\nu) - \mathsf{B}^\mathsf{T}(\boldsymbol{z} - \boldsymbol{z}^\nu) \tag{6}$$

Given enough such observations, the maximum likelihood estimate of the parameter vector $\boldsymbol{p}$ is obtained by iteratively updating (cf. [3])

$$\widehat{\boldsymbol{p}} = \boldsymbol{p}^{\nu} + \Delta\boldsymbol{p} \tag{7}$$

with

$$\Delta\boldsymbol{p} = \mathsf{Q}\mathsf{A}^{\mathsf{T}}(\mathsf{B}^{\mathsf{T}}\mathsf{C}_{zz}\mathsf{B})^{-1}\boldsymbol{c}_g \tag{8}$$

using the covariance matrix

$$\mathsf{Q} = (\mathsf{A}^{\mathsf{T}}(\mathsf{B}^{\mathsf{T}}\mathsf{C}_{zz}\mathsf{B})^{-1}\mathsf{A})^{-1} \tag{9}$$

and the contradiction vector

$$\boldsymbol{c}_g = -\boldsymbol{g}(\boldsymbol{p}^{\nu}, \boldsymbol{z}^{\nu}) - \mathsf{B}^{\mathsf{T}}(\boldsymbol{z} - \boldsymbol{z}^{\nu}) \tag{10}$$

We can also compute the residuals of the observations

$$\boldsymbol{v} = \widehat{\boldsymbol{z}} - \boldsymbol{z} = \mathsf{C}_{zz}\mathsf{B}(\mathsf{B}^{\mathsf{T}}\mathsf{C}_{zz}\mathsf{B})^{-1}(\boldsymbol{c}_g - \mathsf{A}\widehat{\Delta\boldsymbol{p}}) \tag{11}$$

yielding the linearization point for the next iteration

$$\boldsymbol{z}^{\nu+1} = \widehat{\boldsymbol{z}} = \boldsymbol{z} + \boldsymbol{v} \qquad \boldsymbol{p}^{\nu+1} = \widehat{\boldsymbol{p}} \tag{12}$$

Now we combine this estimation scheme with the state vector from the prediction step. To do so we note that the prediction is equivalent to a direct observation of the new state vector, which fits into the above framework using the model equation

$$\mathbf{0} = \boldsymbol{g}_1(\boldsymbol{p}, \boldsymbol{z}_1) = \boldsymbol{p} - \boldsymbol{z}_1 \tag{13}$$

and the observation $\boldsymbol{z}_1 = \boldsymbol{p}_1$ having the covariance matrix $\mathsf{C}_{zz_{11}} = \mathsf{Q}_{11}$. Because this constraint is linear, the Jacobians are in this case simply $\mathsf{A}_1 = \mathsf{I}$ and $\mathsf{B}_1 = -\mathsf{I}$ independent of the linearization point. Considering the prediction of the state vector alone we would obtain $\widehat{\boldsymbol{p}} = \boldsymbol{p}_1$, so that $\boldsymbol{c}_{g_1} = \mathbf{0}$. As the measurement update is supposed to influence the state vector and thereby the joint linearization point, we have to cope with the change of the contradiction incurred by this change, which we will call $\Delta\boldsymbol{c}_{g_1}$ in the following to reflect this important property. Plugging the direct measurement equation (13) and its Jacobians into equation (10), yields the contradiction

$$\Delta\boldsymbol{c}_{g_1} = \boldsymbol{p}_1 - \boldsymbol{p}^{\nu} - \boldsymbol{v}_1^{\nu-1} \tag{14}$$

Further note, that because $\mathsf{A} = \mathsf{I}$ also the following equation holds

$$\mathsf{Q}_{11}^{-1}\Delta\boldsymbol{p}_1 = \mathsf{Q}_{11}^{-1}\Delta\boldsymbol{c}_{g_1} \tag{15}$$

which will become useful in the following.

We are now ready to formulate the recursive estimation as a weighted mean process of two variables being the predicted state $\boldsymbol{p}_1$ on the one hand and the state estimated from the novel observations $\boldsymbol{p}_2$ on the other hand. Hence, the state update is given by

$$\widehat{\Delta \boldsymbol{p}} = (\mathsf{Q}_{11}^{-1} + \mathsf{Q}_{22}^{-1})^{-1}(\mathsf{Q}_{11}^{-1}\Delta \boldsymbol{p}_1 + \mathsf{Q}_{22}^{-1}\Delta \boldsymbol{p}_2) \tag{16}$$

Substituting equation (15) and equation (8) into this weighted mean update we obtain

$$\widehat{\Delta \boldsymbol{p}} = \underbrace{(\mathsf{Q}_{11}^{-1} + \mathsf{Q}_{22}^{-1})^{-1}}_{\mathsf{Q}_{pp}}(\mathsf{Q}_{11}^{-1}\Delta \boldsymbol{c}_{g_1} + \mathsf{A}_2^{\mathsf{T}}(\mathsf{B}_2^{\mathsf{T}}\mathsf{C}_{zz}\mathsf{B}_2)^{-1}\boldsymbol{c}_{g_2}) \tag{17}$$

Now using the well known matrix inversion identity

$$(K + LN^{-1}M)^{-1} = K^{-1} - K^{-1}L(N + MK^{-1}L)^{-1}MK^{-1} \tag{18}$$

we can reformulate equation (17) and finally get

$$\widehat{\Delta \boldsymbol{p}} = \mathsf{F}\boldsymbol{c}_{g_2} + (\mathsf{I} - \mathsf{F}\mathsf{A}_2)\Delta \boldsymbol{c}_{g1} \tag{19}$$

with the substitution

$$\mathsf{F} = \mathsf{Q}_{11}\mathsf{A}_2^{\mathsf{T}}(\mathsf{B}_2^{\mathsf{T}}\mathsf{C}_{zz}\mathsf{B}_2 + \mathsf{A}_2^{\mathsf{T}}\mathsf{Q}_{11}\mathsf{A}_2)^{-1} \tag{20}$$

The residuals are computed using equation (11)

$$\boldsymbol{v}_1 = -\Delta \boldsymbol{c}_{g_1} + \widehat{\Delta \boldsymbol{p}} \tag{21}$$

$$\boldsymbol{v}_2 = \mathsf{C}_{zz}\mathsf{B}_2(\mathsf{B}_2^{\mathsf{T}}\mathsf{C}_{zz}\mathsf{B}_2)^{-1}(\boldsymbol{c}_{g_2} - \mathsf{A}_2\widehat{\Delta \boldsymbol{p}}) \tag{22}$$

allowing to compute the contradiction for the next iteration

$$\boldsymbol{c}_{g_2} = -\boldsymbol{g}_2(\widehat{\boldsymbol{p}}, \widehat{\boldsymbol{z}}_2) + \mathsf{B}_2^{\mathsf{T}}\boldsymbol{v}_2 \tag{23}$$

Finally note, that the new covariance matrix of the state vector is given by

$$\mathsf{Q}_{pp} = (\mathsf{I} - \mathsf{F}\mathsf{A}_2)\mathsf{Q}_{pp_{11}} \tag{24}$$

The presented algorithm is based on least squares optimization, which is known to be very sensitive to outliers. In the following chapter we will show, how the robustness of the presented method can be increased by re-weighting the observations.

2.2 Robustification by re-weighting

The estimation scheme presented so far minimizes the squared residuals of the observations, which is known to be extremely sensitive to outliers. We will now show, how single outliers may be detected by looking at the plausibility of the computed residuals with respect to the expected uncertainty. By reducing the influence of such observations on the estimation the robustness can be increased.

The weighted mean process is mainly influenced by two error effects. First, an erroneous dynamic model results in an erroneous prediction, and second, noisy observations yield a correction effect to the estimated state.

In [17] a robust outlier detection is presented for the classical Kalman filter. We will adapt this technique with a better re-weighting method proposed in [8]. Assuming an error free prediction, the improvement of the observations in $\boldsymbol{v}_2$ is normal distributed with zero mean. In this case we are able to detect outliers by simply normalizing v_2 with the inverse observations covariance and reweigh the observations accordingly.

However, in realistic applications the prediction model does not always hold true. Its effect on the improvement of the observations in $\boldsymbol{v}_2$ cannot be modeled in general and depends on the system noise of the dynamic model. In the structure-from-motion problem, for instance, an error in the camera position orthogonal to the viewing direction results in a consistent translation fraction, or a rotation around the viewing direction results in a more complex deformation in the image coordinates.

One common way to solve this problem is to approximate the complex deformation of the estimated observation $\widehat{\boldsymbol{z}}$. This can be done by choosing an approximation function depending on the expected deformations. In the case of image observations a homography could be a good choice. The robust estimation of this function can then be done by a RANSAC based approach or by a robustified least square solution. However, such a procedure is often quite expensive.

From another point of view, the influence of the erroneous prediction is small, if the system noise is large enough to compensate for the prediction error, which should be the case for a well approximating dynamic model. Then we are able to robustify the update by reweighing the observations in the following sense.

We first normalize the residual vector $\boldsymbol{v}_2$ with the observations covariance matrix to get a standard normal distributed test vector

$$\boldsymbol{c} = \boldsymbol{C}_{zz}^{(0)^{-1}} \boldsymbol{v}_2 \tag{25}$$

The absolute values of the entries of $\boldsymbol{c}$ allow to decide for each single observation, if there is reason to consider it as an outlier. We then compute for each observation a variance factor w_j according to [8]

$$w_j = \begin{cases} 1 & \text{if } \|c_j\| \leq k \\ \frac{\|c_j\|}{k} & \text{if } \|c_j\| > k \end{cases} \tag{26}$$

which does not alter observations withing the range of k times the expected standard deviation and reduces the effect of observations outside this range on the estimation. To perform the desired re-weighting, we use in each iteration the observation covariance matrix

$$\boldsymbol{C}_{zz}^{(\nu)} = diag(\boldsymbol{w})\boldsymbol{C}_{zz}^{(0)} \tag{27}$$

instead of the initially given covariance matrix $\boldsymbol{C}_{zz}^{(0)}$.

Following the experimental validation of [17] we also demonstrate the robustification on the one dimensional estimation of a cosinus curve containing some outliers. In figure 1 the noisy observations with 5% of outliers are shown. Figure 2 shows the non robust and the robust version of the estimated curve parameters. The robustification yields a much smoother estimate not being perturbed by the outliers.

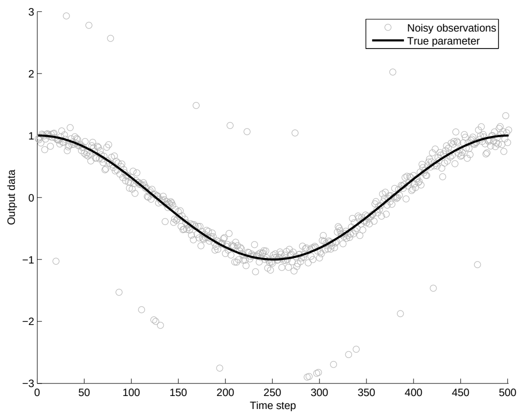

Figure 1: Full cosinus wavelength 2π, sampled with 500 samples, noise is 0.05, system noise 0.01, 5 percent outliers with strength of 2, iteration to convergence

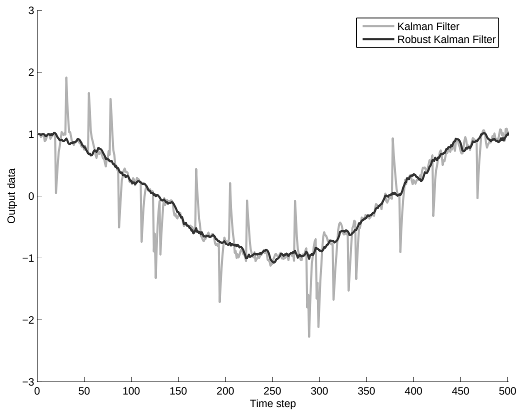

Figure 2: Recursive estimation using the non-robustified and the robustified version of the Kalman filter.

After having derived the required equations and robustification, we will summarize the complete algorithm in the following section.

2.3 The final algorithm

We will now summarize the recursive estimation algorithm, which can be applied as a black-box if only the Jacobians of the implicit model function are supplied. From a previous estimation or prediction step of the filter, a current state vector $\boldsymbol{p}_1$ together with its covariance Q_{11} is known. We now gather additional observations $\boldsymbol{z}_2$ together with their covariance matrix C_{22} in a subsequent measurement step. The following algorithm may then be applied to update the state vector accordingly

1. set $\widehat{\Delta \boldsymbol{p}} = \mathbf{0}$
2. set $\widehat{\boldsymbol{p}} = \boldsymbol{p}_1$
3. set $\boldsymbol{v}_1 = \mathbf{0}$
4. set $\boldsymbol{v}_2 = \mathbf{0}$, hence $\widehat{\boldsymbol{z}}_2 = \boldsymbol{z}_2$
5. Iterate until $\widehat{\Delta \boldsymbol{p}}$ is sufficiently small
   (a) compute Jacobians A_2 and B_2 at $\widehat{\boldsymbol{p}}$ and $\widehat{\boldsymbol{z}}_2$
   (b) compute the gain matrix F according to equation (20)
   (c) compute $\boldsymbol{c}_{g_2}$ according to equation (23)
   (d) compute $\Delta \boldsymbol{c}_{g_1}$ according to equation (14)

(e) compute $\widehat{\Delta \boldsymbol{p}}$ according to equation (19)

(f) update $\widehat{\boldsymbol{p}}_2$ with $\widehat{\Delta \boldsymbol{p}}$

(g) compute $\boldsymbol{v}_1$ according to equation (21)

(h) compute $\boldsymbol{v}_2$ according to equation (22)

(i) update $\widehat{\boldsymbol{z}}_2$ with $\boldsymbol{v}_2$

(j) compute normalized test values according to equation (25)

(k) compute variance factor for all observations with equation (26)

(l) compute reweighted observation covariance matrix for the next iteration

6. compute $\boldsymbol{Q}_{\widehat{p}\widehat{p}}$ according to equation (24)

After the algorithm is converged we finally obtained the updated state vector $\widehat{\boldsymbol{p}}$ together with its covariance matrix $\boldsymbol{Q}_{pp}$. The only problem specific part is the computation of the Jacobians in step 5a, which has to be adapted by the user. This completes the measurement update using the implicit constraint and a subsequent time update may be performed. Also note, that for implicit measurement equations obtained directly from explicit equations by subtraction, the presented algorithm yields the same results as the classical iterated extended Kalman filter.

3 Results

The algorithm presented in the previous section is applicable to a broad range of applications. In the following we will demonstrate the applicability of the framework for the task of structure-from-motion using a single camera [1]. We will first briefly sketch the involved model equations and then give some results on a real test sequence acquired from an UAV.

3.1 UAV Hardware

The real data experiment shown below is based on image sequences taken with the UAV produced by Microdrones GmbH, which is depicted in figure 3. This drone is an electric powered quad-copter, which was at the time being manually operated. It can carry up to approximately 200 g of payload and is equipped with a Panasonic Lumix camera with a resolution of 848 x 480 pixels, a viewing angle of approximately $90°$ and a frame rate of 30 Hz in video mode. The camera can be tilted from $0°$ to $90°$ nadir angle. The battery allows a flying time up to approximetely 30 minutes.

Figure 3: Used hardware. Drone MD 4-200 from Microdrones equipped with a Panasonic Lumix video camera

3.2 Model equations

Following the approach of [1] the motion of a single camera can be described by the following state vector

$$\boldsymbol{p} = \begin{pmatrix} \boldsymbol{r} \\ \mathbf{q} \\ \boldsymbol{v} \\ \boldsymbol{\omega} \\ \boldsymbol{X}_1 \\ \vdots \\ \boldsymbol{X}_i \end{pmatrix} \tag{28}$$

comprising of the camera state followed by a set of feature parameters. The uncertainty is coded in the covariance matrix $\boldsymbol{Q}_{pp}$, which is a square matrix of equal dimension. The camera trajectory is represented by its actual position $\boldsymbol{r}$, its orientation quaternion $\mathbf{q}$, its velocity vector $\boldsymbol{v}$ and its angular velocity vector $\boldsymbol{\omega}$. The 3d point coordinates are represented by their Euclidean points $\boldsymbol{X}_i$. The interior camera parameters are assumed to be known in this paper.

At the moment, our approach uses the same camera and structure representation. We assume a linear time update model, which can easily be computed by

$$\boldsymbol{p}_{t+1} = \begin{pmatrix} \boldsymbol{r}_{t+1} \\ \mathbf{q}_{t+1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{r}_t + \boldsymbol{v}_t \Delta t \\ \mathbf{q}_t \times \boldsymbol{q}_t(\boldsymbol{\omega}_t \Delta t) \end{pmatrix} \tag{29}$$

where velocity, angular velocity and Euclidean points do not change. The uncertainty of the predicted state is computed using error propagation and by adding some system noise (cf. [1]).

In the approach of [1] the measurement model is based on the co-linearity equations, which can be written as homogeneous equations

$$\mathbf{x}_i = \lambda_i \mathsf{P} X_i \qquad \text{with} \qquad \mathsf{P} = \mathsf{K} R(\mathbf{q}) \left[I_{3\times 3} | - \boldsymbol{r} \right] \tag{30}$$

As our approach is able to cope with implicit functions, we formulate the co-linearity constraint using the cross-product as follows: Introducing the matrix

$$S(\boldsymbol{t}) = \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \end{pmatrix} \tag{31}$$

the co-linearity equations can be stated as implicit equation

$$S(\mathbf{x}_i)\mathsf{P}\mathbf{X}_i = -S(\mathsf{P}\mathbf{X}_i)\mathbf{x}_i = \mathbf{0} \tag{32}$$

Obviously, those implicit constraints are equivalent to the explicit constraints used in [1]. Also observe that they are also non-linear in the camera pose parameters.

3.3 Experimental evaluation

We compared our Kalman filter based approach with the results obtained from a bundle adjustment on an image sequence acquired with the UAV. The average flying height was approximately 30 m. The image sequence consists of 600 vertical views. It contains a building and vineyards and some images of the sequence are shown in figure 4. The camera was calibrated offline and nonlinear distortions were assumed to be zero.

We tracked features across the image sequence using the KLT algorithm and estimated the camera trajectory using the proposed recursive scheme as well as an overall bundle adjustment using the same measurement model equations and observations. The bundle adjustment solution is computed using every 10th frame of the image sequence and is based on approximately 950 object points. The datum is defined automatically by choosing that frame, which yields an optimum accuracy for the coordinate system. The estimated σ_0 was approximately 0.5, indicating that the tracker yields a standard deviation of 0.5 pixels.

In case of the Kalman filter all frames, rather than every 10th, are used and the datum is typically defined by the first camera position and orientation. In order to be able to compare the Kalman filter solution to the bundle adjustment, we initialized the Kalman filter by using the estimated object points and camera orientation from the first image of the bundle adjustment solution. The initialization of new object points in the Kalman filter is known to influence the result significantly (cf. [12]). One way to solve this problem is the use the inverse depth representation for newly introduced object points to achieve Gaussian distribution in case

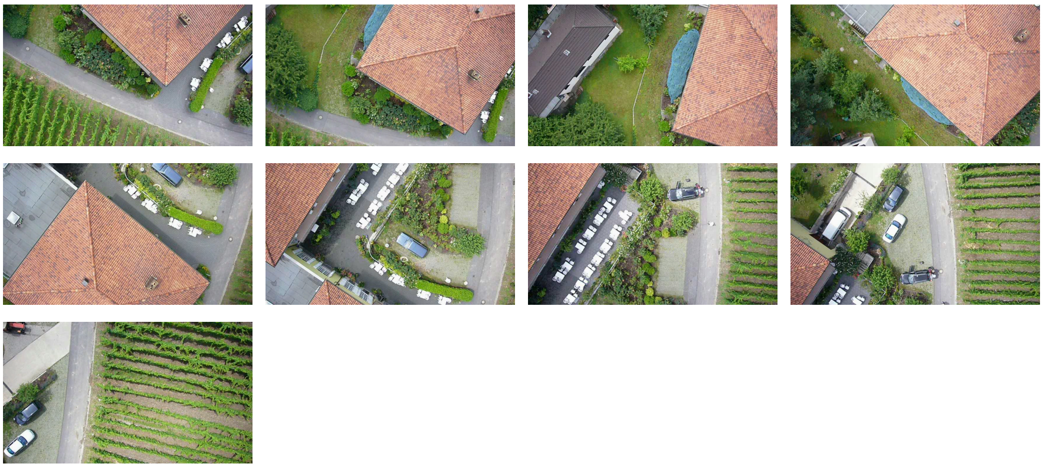

Figure 4: Every 75th of the 600 images of the real image sequence taken with the UAV at a height of approximately 30 m.

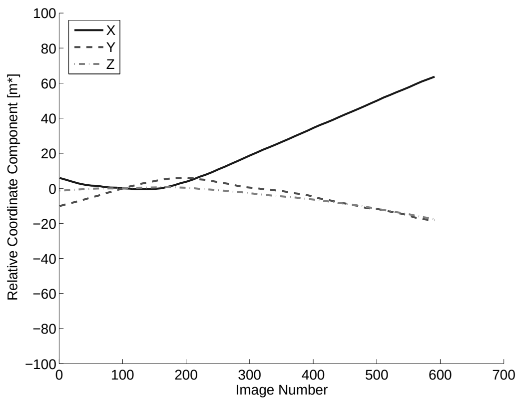

Figure 5: Estimated camera positions obtained from the bundle adjustment approach using every 10th frame of the image sequence.

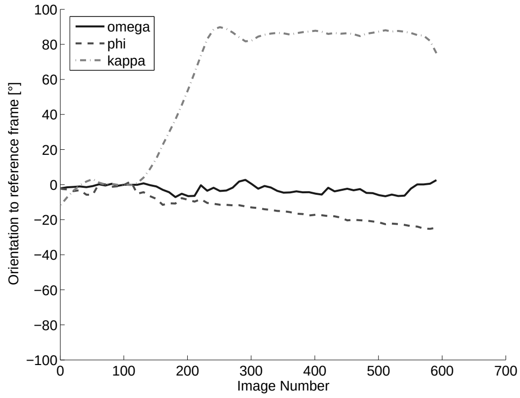

Figure 6: Estimated camera orientations in Euler representation obtained from the bundle adjustment approach using every 10th frame of the image sequence.

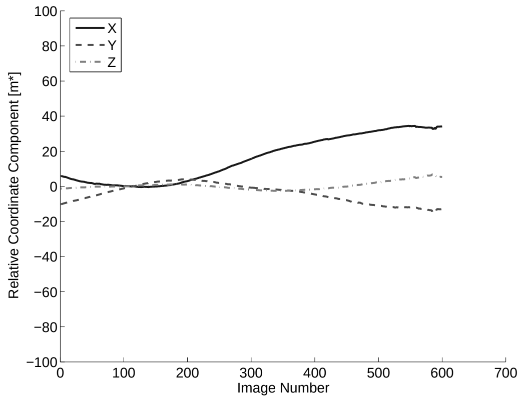

Figure 7: Estimated camera position obtained from our Kalman filter approach using all images.

of small parallaxes. However, we used the approach presented in [16] for a stable initialization instead. Because this initialization discards weak points, we end up with approximately 60 % of the object points used in the bundle adjustment.

Figures 5 and 6 show the estimated relative camera trajectory plotted against the frame number of the image sequence. We used this bundle adjustment solution to compare it to our Kalman filter results depicted in figures 7 and 8. The differences are shown in figures 9 and 10.

It can be seen in figures 9 and 10 the differences in the coordinates for the Y and Z component as well as for the ω and κ angle are small. We observe a significant drift in X and ϕ, though. This behavior is also known from classical aero-triangulation, where the orientation components X and ϕ for image stripes are known to be highly correlated. This effect is typically only reversible by loop

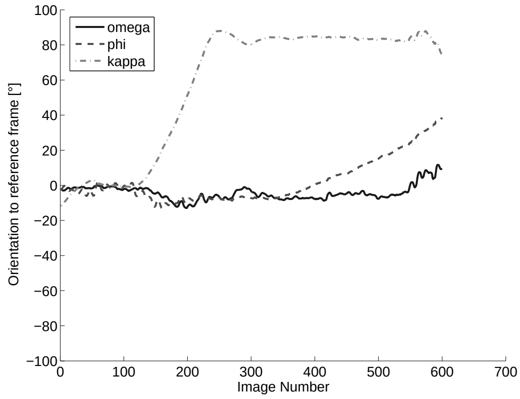

Figure 8: Estimated camera orientation in Euler representation obtained from our Kalman filter approach using all images.

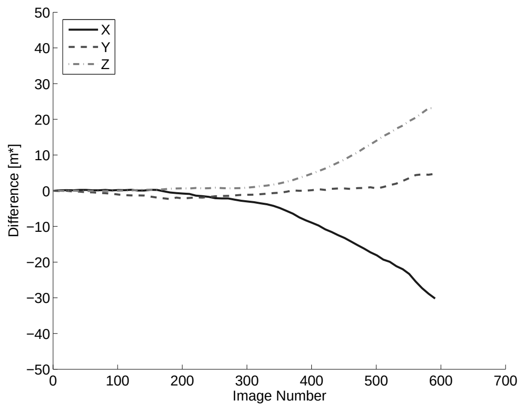

Figure 9: Differences of the bundle adjustment solution and the Kalman filter approach for the estimated camera position.

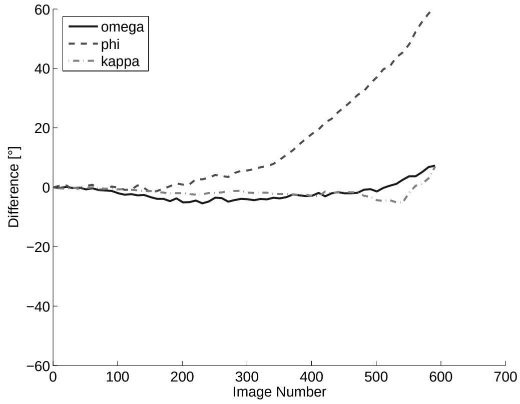

Figure 10: Differences of the bundle adjustment solution and the Kalman filter approach for the estimated camera orientation in Euler representation.

closing.

4 Conclusion

We presented a novel derivation of a recursive estimation framework in a Kalman filter approach, which enables us to use implicit measurement constraint equations, rather than being restricted to explicit ones. By allowing implicit constraints the task of modeling recursive estimation schemes is eased significantly. Furthermore, we presented an improvement to the framework in order to deal with outliers in the observations. Instead of the elimination of this observations, we used a re-weighting method, which leads to smoother results.

We demonstrated the feasibility of this new algorithm for the task of structure-from-motion from monocular image sequences. The computational complexity is approximately equal to the classical iterated extended Kalman filter in case of the same update model.

The presented method is applicable to a broad range of time driven estimation problems, including all those resulting in homogeneous equation systems, so that a lot of estimation task might benefit, which will be the topic of future research.

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