Recursive Estimation with Implicit Constraints

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Abstract. Recursive estimation or Kalman filtering usually relies on explicit model functions, that directly and explicitly describe the effect of the parameters on the observations. However, many problems in computer vision, including all those resulting in homogeneous equation systems, are easier described using implicit constraints between the observations and the parameters. By implicit we mean, that the constraints are given by equations, that are not easily solvable for the observation vector.

We present a framework, that allows to incorporate such implicit constraints as measurement equations into a Kalman filter. The algorithm may be used as a black-box, simplifying the process of specifying suitable measurement equations for many problems. As a byproduct, the possibility of specifying model equations non-explicitly, some non-linearities may be avoided and better results can be achieved for certain problems.

1 Introduction

Recursive estimation or Kalman filtering is a classical technique [10] and has been widely used in computer vision [15] and photogrammetry [4]. All those recursive estimation schemes assume a functional model, where the observations are explained by an explicit function in the unknown parameters.

However, many problems encountered in computer vision naturally result in implicit constraints between the observations and the parameters [6,8,13,7]. For instance, all problems resulting in homogeneous equation systems fall into this class. Although it is always possible to reduce the solution of an implicit problem to the solution of an explicit problem [11, p.231ff], to our knowledge no recursive estimation scheme is readily available in this case. The main goal of this paper is to provide a recursive estimation scheme, that can be applied to such problems comprising of implicit constraints in a black-box manner thereby simplifying the task of recursive estimation from the modeling point of view. The scope is not to present a run-time optimized estimation scheme tailored specifically for the task of structure-from-motion, as the proposed method is a framework, which is applicable in a much broader context.

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The Kalman filter consists of two parts, namely a time update and a measurement update. The scope of our work is not the time update but the measurement update, for which we will present a solution based on implicit constraints.

Recently the Kalman filter based on the unscented transformation [9] has obtained a lot of attention, which aims at improving the stochastic properties of the filter. Our work on the other hand aims at simplifying the specification of measurement equations, which are often much easier and straightforward to derive as implicit functions. By allowing more freedom in the task of modeling a certain problem the effects arising from non-linearities in the model equations can possibly be reduced resulting in more stable algorithms.

We will demonstrate the applicability of our approach for the task of on-line structure-from-motion from image sequences, which may be modeled using explicit functions [2] as well as using implicit functions (see section 3.1). The two approaches will be compared in section 3.2.

We are aware that a lot of highly optimized non-linear methods for the task of on-line structure-from-motion from image sequences are available [14,5,3], which exploit the specific structure of the normal equation matrix. However, this is not the scope of our paper as the presented methods are applicable to a variety of problems beyond structure-from-motion, which can be specified using implicit functions. The structure-from-motion problem is only used to demonstrate the applicability of the proposed method, as it is well-known to many researchers and test-sequences are readily available.

In the following section a recursive estimation scheme based on implicit functions will be derived. Section 2.3 summarizes the results and presents an easily applicable algorithm based on the derived equations. Finally we will compare the presented method to [2] in section 3.

2 Recursive Estimation Using Implicit Functions

2.1 Estimation Using Implicit Functions

We will now derive a recursive estimation scheme for the case of implicit constraints, which are functions relating the parameters $\boldsymbol{p}$ and the observations $\boldsymbol{l}$ as

$$\boldsymbol{g}(\tilde{\boldsymbol{p}}, \tilde{\boldsymbol{l}}) = \boldsymbol{0} \; . \tag{1}$$

Note, that such implicit functions are often much easier derived than explicit functions of the form $\tilde{\boldsymbol{l}} = \boldsymbol{f}(\tilde{\boldsymbol{p}})$. The best linear unbiased estimate of the parameter vector $\hat{\boldsymbol{p}}$ given observations $\boldsymbol{l}$ together with their covariance matrix C_{ll} may be obtained iteratively by solving the linear normal equation system [4, p.85]

$$A^{\mathsf{T}}(B^{\mathsf{T}} C_{ll} B)^{-1} A \widehat{\Delta \boldsymbol{p}} = A^{\mathsf{T}}(B^{\mathsf{T}} C_{ll} B)^{-1} \boldsymbol{c}_g \tag{2}$$

using the Jacobians at appropriate initial values

$$A = \left. \frac{\partial \boldsymbol{g}(\boldsymbol{p}, \boldsymbol{l})}{\partial \boldsymbol{p}} \right|_{\hat{\boldsymbol{l}}, \hat{\boldsymbol{p}}} \qquad\qquad B = \left. \frac{\partial \boldsymbol{g}(\boldsymbol{p}, \boldsymbol{l})^{\mathsf{T}}}{\partial \boldsymbol{l}} \right|_{\hat{\boldsymbol{l}}, \hat{\boldsymbol{p}}} \tag{3}$$

the contradiction vector

$$\boldsymbol{c}_g = -\boldsymbol{g}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{l}}) - B^\mathsf{T}(\boldsymbol{l} - \hat{\boldsymbol{l}}) = -\boldsymbol{g}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{l}}) + B^\mathsf{T}\boldsymbol{v} \tag{4}$$

and the residual of the observations

$$\boldsymbol{v} = \hat{\boldsymbol{l}} - \boldsymbol{l} = C_{ll} B (B^\mathsf{T} C_{ll} B)^{-1} (\boldsymbol{c}_g - A\widehat{\Delta\boldsymbol{p}}) \ . \tag{5}$$

In the following we will analyze the effect additional observations have on this estimation scheme.

2.2 Recursive Estimation

The task of recursive estimation is now to incorporate additional observations into the model. Hence, the model equation is augmented by a second implicit constraint block

$$\begin{bmatrix} \boldsymbol{g}_1(\tilde{\boldsymbol{p}}, \tilde{\boldsymbol{l}_1}) \\ \boldsymbol{g}_2(\tilde{\boldsymbol{p}}, \tilde{\boldsymbol{l}_2}) \end{bmatrix} = \boldsymbol{0} \ . \tag{6}$$

Applying the same reasoning as before the solution of this new model equation may be obtained using the new normal equation system with

$$A^\mathsf{T}(B^\mathsf{T} C_{ll} B)^{-1} A = \begin{bmatrix} A_1^\mathsf{T} \\ A_2^\mathsf{T} \end{bmatrix} \left(\begin{bmatrix} B_{11}^\mathsf{T} & B_{21}^\mathsf{T} \\ B_{12}^\mathsf{T} & B_{22}^\mathsf{T} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{7}$$

on the left hand side and

$$A^\mathsf{T}(B^\mathsf{T} C_{ll} B)^{-1} \boldsymbol{c}_g = \begin{bmatrix} A_1^\mathsf{T} \\ A_2^\mathsf{T} \end{bmatrix} \left(\begin{bmatrix} B_{11}^\mathsf{T} & B_{21}^\mathsf{T} \\ B_{12}^\mathsf{T} & B_{22}^\mathsf{T} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} \boldsymbol{c}_{g_1} \\ \boldsymbol{c}_{g_2} \end{bmatrix} \tag{8}$$

on the right hand side with the respective Jacobians in the block matrices.

In the following we will assume that the two observation blocks are stochastically independent, i.e. $C_{12} = C_{21} = \mathit{0}$, as well as functionally independent, i.e. $B_{12} = B_{21} = \mathit{0}$. Observe that this is analogous to classical recursive estimation with explicit functions in the Kalman filter. Now we can reformulate the left hand side of the normal equation system

$$A^\mathsf{T}(B^\mathsf{T} C_{ll} B)^{-1} A = A_1^\mathsf{T}(B_1^\mathsf{T} C_{11} B_1)^{-1} A_1 + A_2^\mathsf{T}(B_2^\mathsf{T} C_{22} B_2)^{-1} A_2 \tag{9}$$

as well as the right hand side of the normal equation system

$$A^\mathsf{T}(B^\mathsf{T} C_{ll} B)^{-1} \boldsymbol{c}_g = A_1^\mathsf{T}(B_1^\mathsf{T} C_{11} B_1)^{-1} \boldsymbol{c}_{g_1} + A_2^\mathsf{T}(B_2^\mathsf{T} C_{22} B_2)^{-1} \boldsymbol{c}_{g_2} \ . \tag{10}$$

Using the substitution $W = B^\mathsf{T} C_{ll} B$ the final solution, that incorporates both observations $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$, may be obtained iteratively as

$$\widehat{\Delta\boldsymbol{p}} = (A_1^\mathsf{T} W_{11}^{-1} A_1 + A_2^\mathsf{T} W_{22}^{-1} A_2)^{-1} (A_1^\mathsf{T} W_{11}^{-1} \boldsymbol{c}_{g_1} + A_2^\mathsf{T} W_{22}^{-1} \boldsymbol{c}_{g_2}) \ . \tag{11}$$

In the following the dependence on the first set of observation $\boldsymbol{l}_1$ should be removed.

The goal of recursive estimation is now to derive such a solution $\widehat{\Delta \boldsymbol{p}_2}$ for the combined constraints using the solution of the first constraint block $\boldsymbol{g}_1$ represented by $\widehat{\Delta \boldsymbol{p}_1}$ and its covariance matrix $Q_{\hat{p}\hat{p}_{11}}$ as well as the new constraint block $\boldsymbol{g}_2$ together with the new observations $\boldsymbol{l}_2$ and their covariance matrix C_{22}. In order to achieve this goal equation (11) may be re-written as

$$\widehat{\Delta \boldsymbol{p}_2} = \underbrace{(\underbrace{A_1^\mathsf{T} W_{11}^{-1} A_1}_{Q_{\hat{p}\hat{p}_{11}}^{-1}} + A_2^\mathsf{T} W_{22}^{-1} A_2)^{-1}}_{Q_{\hat{p}\hat{p}_{22}}} (\underbrace{A_1^\mathsf{T} W_{11}^{-1} \bar{\boldsymbol{c}}_{g_1}}_{Q_{\hat{p}\hat{p}_{11}}^{-1} \widehat{\Delta \boldsymbol{p}_1}} + A_2^\mathsf{T} W_{22}^{-1} \boldsymbol{c}_{g_2} + A_1^\mathsf{T} W_{11}^{-1} \Delta \boldsymbol{c}_{g_1}) \tag{12}$$

with the contradictions being separated into

$$\boldsymbol{c}_{g_1} = \bar{\boldsymbol{c}}_{g_1} + \Delta \boldsymbol{c}_{g_1} \; . \tag{13}$$

Observe that the contradictions for the first contradiction block $\boldsymbol{g}_1$ change due to the change of parameters resulting from the new contradiction block $\boldsymbol{g}_2$, due to the dependence on $\hat{\boldsymbol{p}}$ of equation (4). As a consequence the residuals for the observations of the first contradiction block change as well

$$\boldsymbol{v}_1 = C_{11} B_1 W_{11}^{-1} (\bar{\boldsymbol{c}}_{g_1} + \Delta \boldsymbol{c}_{g_1} - A_1 \widehat{\Delta \boldsymbol{p}_2}) \tag{14}$$

$$= \underbrace{C_{11} B_1 W_{11}^{-1} (\bar{\boldsymbol{c}}_{g_1} - A_1 \widehat{\Delta \boldsymbol{p}_1})}_{\bar{\boldsymbol{v}}_1} + \underbrace{C_{11} B_1 W_{11}^{-1} (\Delta \boldsymbol{c}_{g_1} - A_1 (\widehat{\Delta \boldsymbol{p}_2} - \widehat{\Delta \boldsymbol{p}_1}))}_{\Delta \boldsymbol{v}_1} . \tag{15}$$

The expression $Q_{\widehat{p}\widehat{p}_{22}}$ in equation (12) is the inverse of a sum and can be decomposed as follows [11, p.37]

$$Q_{\widehat{p}\widehat{p}_{22}} = Q_{\widehat{p}\widehat{p}_{11}} - Q_{\widehat{p}\widehat{p}_{11}} A_2^\mathsf{T} (W_{22} + A_2^\mathsf{T} Q_{\widehat{p}\widehat{p}_{11}} A_2)^{-1} A_2 Q_{\widehat{p}\widehat{p}_{11}} \tag{16}$$

$$= Q_{\widehat{p}\widehat{p}_{11}} - F A_2 Q_{\widehat{p}\widehat{p}_{11}} \tag{17}$$

$$= (I - F A_2) Q_{\widehat{p}\widehat{p}_{11}} \tag{18}$$

with F being the well known gain matrix. Note that this update does not involve the inversion of the full normal equation matrix. Substituting this back into equation (12) we obtain

$$\widehat{\Delta \boldsymbol{p}_2} = (I - F A_2) Q_{\widehat{p}\widehat{p}_{11}} Q_{\widehat{p}\widehat{p}_{11}}^{-1} \widehat{\Delta \boldsymbol{p}_1} + (I - F A_2) Q_{\widehat{p}\widehat{p}_{11}} A_2^\mathsf{T} W_{22}^{-1} \boldsymbol{c}_{g_2} + (I - F A_2) Q_{\widehat{p}\widehat{p}_{11}} A_1^\mathsf{T} W_{11}^{-1} \Delta \boldsymbol{c}_{g_1} \tag{19}$$

$$= \widehat{\Delta \boldsymbol{p}_1} - F A_2 \widehat{\Delta \boldsymbol{p}_1} + F \boldsymbol{c}_{g_2} + (I - F A_2) Q_{\widehat{p}\widehat{p}_{11}} A_1^\mathsf{T} W_{11}^{-1} \Delta \boldsymbol{c}_{g_1} \tag{20}$$

using the identity [11, p.37]

$$F = (I - F A_2) Q_{\widehat{p}\widehat{p}_{11}} A_2^\mathsf{T} W_{22}^{-1} \; . \tag{21}$$

The only remaining part still depending on $\boldsymbol{l}_1$ is now the change of the contradictions (see equation (4))

$$\Delta \boldsymbol{c}_{g_1} = \boldsymbol{c}_{g_1} - \bar{\boldsymbol{c}}_{g_1} = -\boldsymbol{g}_1(\widehat{\boldsymbol{p}_2}, \widehat{\boldsymbol{l}_1}) + B_1^\mathsf{T} \boldsymbol{v}_1 + \boldsymbol{g}_1(\widehat{\boldsymbol{p}_1}, \widehat{\boldsymbol{l}_1}) - B_1^\mathsf{T} \bar{\boldsymbol{v}}_1 \; . \tag{22}$$

In order to get rid of this remaining dependence on the previous observations observe, that the whole first contradiction block in the Kalman filter is encoded in the first two moments of the parameter vector only. We therefore replace the first constraint block by a direct observation of the parameters itself, i.e. $\boldsymbol{l}_1 = \widehat{\boldsymbol{p}_1}$ and $C_{11} = Q_{\widehat{p}\widehat{p}_{11}}$, so that

$$\boldsymbol{g}_1(\widehat{\boldsymbol{p}_1}, \boldsymbol{l}_1) = \widehat{\boldsymbol{p}_1} - \boldsymbol{l}_1 = \mathbf{0} \tag{23}$$

immediately fulfills the constraint and therefore $\bar{\boldsymbol{c}}_{g_1} = \mathbf{0}$, $\bar{\boldsymbol{v}}_1 = \mathbf{0}$ and $\widehat{\Delta\boldsymbol{p}_1} = \mathbf{0}$. Furthermore the Jacobians are given by $A_1 = I$ and $B_1 = -I$.

Now equation (20) simplifies to

$$\widehat{\Delta\boldsymbol{p}_2} = F\boldsymbol{c}_{g_2} + (I - FA_2)\Delta\boldsymbol{c}_{g_1} \tag{24}$$

with

$$\Delta\boldsymbol{c}_{g_1} = -\boldsymbol{g}_1(\widehat{\boldsymbol{p}_2}, \widehat{\boldsymbol{l}_1}) - \boldsymbol{v}_1 \tag{25}$$

and equation (14) boiling down to

$$\boldsymbol{v}_1 = -\Delta\boldsymbol{c}_{g_1} + \widehat{\Delta\boldsymbol{p}_2}\ . \tag{26}$$

For the second contradiction block we can compute the residuals

$$\boldsymbol{v}_2 = C_{22}B_2W_{22}^{-1}(\boldsymbol{c}_{g_2} - A_2\widehat{\Delta\boldsymbol{p}_2}) \tag{27}$$

and the contradictions

$$\boldsymbol{c}_{g_2} = -\boldsymbol{g}_2(\widehat{\boldsymbol{p}_2}, \widehat{\boldsymbol{l}_2}) + B_2^{\mathsf{T}}\boldsymbol{v}_2\ . \tag{28}$$

We now have derived all required equations for incorporating an additional implicit constraint into an estimation. In the following section those equations will be summarized and put together into an easily applicable algorithm.

2.3 The Final Algorithm

We will now summarize the recursive estimation algorithm, which can be applied as a black-box if only the Jacobians of the implicit model function are supplied. From a previous estimation or prediction step of the filter, a current state vector $\boldsymbol{p}_1$ together with its covariance $\boldsymbol{Q}_{p_1p_1}$ is known. We now gather additional observations $\boldsymbol{l}_2$ together with their covariance matrix C_{22} in a subsequent measurement step. The following algorithm may then be applied to update the state vector accordingly.

1. set $\widehat{\Delta\boldsymbol{p}_2} = 0$
2. set $\widehat{\boldsymbol{p}_2} = \boldsymbol{p}_1$
3. set $\boldsymbol{v}_1 = \mathbf{0}$
4. set $\boldsymbol{v}_2 = \mathbf{0}$, hence $\widehat{\boldsymbol{l}_2} = \boldsymbol{l}_2$

5. iterate until $\widehat{\Delta \boldsymbol{p}_2}$ is sufficiently small
   (a) compute Jacobians A_2 and B_2 at $\widehat{\boldsymbol{p}_2}$ and $\widehat{\boldsymbol{l}_2}$
   (b) compute the gain matrix F as shown in equation (17)
   (c) compute $\boldsymbol{c}_{g_2}$ according to equation (28)
   (d) compute $\Delta \boldsymbol{c}_{g_1}$ according to equation (25)
   (e) compute $\widehat{\Delta \boldsymbol{p}_2}$ according to equation (24)
   (f) update $\widehat{\boldsymbol{p}_2}$ with $\widehat{\Delta \boldsymbol{p}_2}$
   (g) compute $\boldsymbol{v}_1$ according to equation (26)
   (h) compute $\boldsymbol{v}_2$ according to equation (27)
   (i) update $\widehat{\boldsymbol{l}_2}$ with $\boldsymbol{v}_2$
6. compute $Q_{\widehat{p}\widehat{p}_{22}}$ according to equation(18)

After the algorithm is converged we finally obtained the updated state vector $\widehat{\boldsymbol{p}_2}$ together with its covariance matrix $Q_{\widehat{p}\widehat{p}_{22}}$. The only problem specific part is the computation of the Jacobians in step 5a, which has to be adapted by the user. This completes the measurement update using the implicit constraint and a subsequent time update may be performed.

3 Results

The algorithm presented in the previous section is applicable to a broad range of problems. In the following we will demonstrate the applicability of the framework for the task of structure-from-motion using a single camera [2]. We will first briefly sketch the involved model equations and then give some results on a test sequence, where we will compare our approach to [2].

3.1 Model Equations

In [2] a popular model for on-line structure from motion using a single camera is presented, which uses the following state vector

$$\boldsymbol{p} = \begin{pmatrix} \boldsymbol{r}^W \\ \mathbf{q}^{RW} \\ \boldsymbol{v}^W \\ \boldsymbol{\omega}^R \\ \boldsymbol{X}_1 \\ \vdots \\ \boldsymbol{X}_i \end{pmatrix} \tag{29}$$

comprising of the camera state followed by a set of features parameters. The uncertainty is coded in the covariance matrix Q_{pp}, which is a square matrix of equal dimension. The camera trajectory is represented by its actual position $\boldsymbol{r}^W$, orientation quaternion $\mathbf{q}^{RW}$, velocity vector $\boldsymbol{v}^W$ and angular velocity vector $\boldsymbol{\omega}^R$. The 3d point coordinates are represented by their Euclidean points $\boldsymbol{X}_i$.

The time update for the camera position and orientation can easily be compute as

$$\widehat{\boldsymbol{p}} = \begin{pmatrix} \boldsymbol{r}^W_{new} \\ \mathbf{q}^{RW}_{new} \end{pmatrix} = \begin{pmatrix} \boldsymbol{r}^W + \boldsymbol{v}^W \Delta t \\ \mathbf{q}^{RW} \times \mathbf{q}(\boldsymbol{\omega}^R \Delta t) \end{pmatrix} \tag{30}$$

where velocity, angular velocity and Euclidean points do not change. The uncertainty of the predicted state is computed using error propagation and adding some system noise (see [2]). We will use this time update model in both approaches we compare.

In the approach of [2] the measurement model is based on the co-linearity equations, which can be written as homogeneous equations

$$\mathbf{x}_i = \lambda_i \mathsf{P} X_i \qquad \text{with} \qquad \mathsf{P} = \mathsf{K} R(\mathbf{q})\,[I_{3\times3} | -\boldsymbol{r}]\ . \tag{31}$$

Rewriting this in Euclidean coordinates, we get $\{u_i, v_i\}$ as image coordinate observations

$$u_i = \frac{P_{11}X_i + P_{12}Y_i + P_{13}Z_i + P_{14}}{P_{31}X_i + P_{32}Y_i + P_{33}Z_i + P_{34}} \tag{32}$$

$$v_i = \frac{P_{21}X_i + P_{22}Y_i + P_{23}Z_i + P_{24}}{P_{31}X_i + P_{32}Y_i + P_{33}Z_i + P_{34}} \tag{33}$$

which are explicit functions in the observations as required by the classical Kalman filter. The fraction introduces a degree of non-linearity into the model equations, that could be avoided using implicit functions.

As our approach is able to cope with implicit functions, we re-formulate the co-linearity constraint using the cross-product as follows: Introducing the matrix

$$\mathsf{S}(\boldsymbol{t}) = \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \end{pmatrix} \tag{34}$$

the co-linearity equations can be stated as implicit equation

$$\mathsf{S}(\mathbf{x}_i)\mathsf{P}\mathbf{X}_i = -\mathsf{S}(\mathsf{P}\mathbf{X}_i)\mathbf{x}_i = \mathbf{0}\ . \tag{35}$$

Obviously, those implicit constraints are equivalent to the explicit constraints. Also observe that they are also non-linear in the camera pose parameters. However, there is no fraction involved, so that the effects introduced by the non-linearity turn out to be reduced, as will be seen in the next section.

3.2 Experimental Evaluation

In order to assess the performance of the presented technique for the non-linear structure-from-motion problem, we used the well-known rotating dinosaur sequence depicted in figure 1, where ground-truth camera calibration and orientation data were available. We extracted point features and tracked them across the sequence.

Because the initialization of a Kalman filter based reconstruction approach is known to influence the result significantly (see [12], [1]), we used the result of a bundle adjustment of the first five frames for initialization of both approaches. New points, that were introduced into the estimation, were initialized at the centroid of the point cloud and given a large initial covariance matrix.

We estimated the camera trajectory and the 3d point cloud using the approach based on explicit functions presented in [2] as well as using our own approach

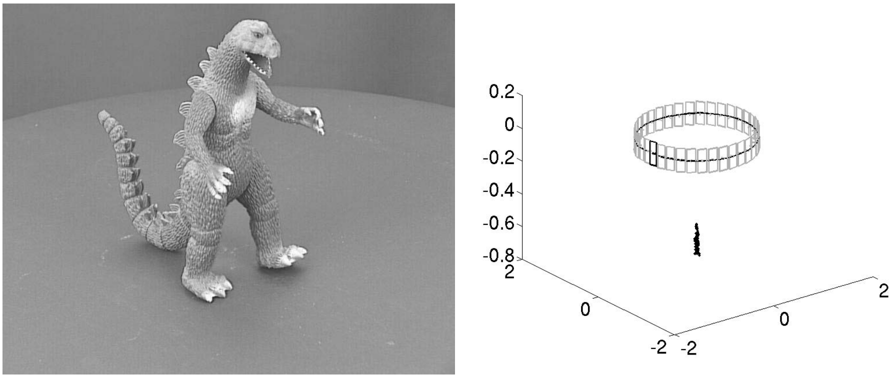

Fig. 1. *Left:* A single frame of the well-known rotating dinosaur sequence. The sequence consists of 36 images rotated in 10° steps around the dinosaur. Ground-truth for the camera calibration, position and rotation is available and will be used to quantify the performance of the presented methods. *Right:* Camera positions and orientations computed using a bundle adjustment of tracked feature points. The frame marked in black is the last in the sequence. Note that the features were tracked, so that no correspondences between the first and the last frame were used to close the loop.

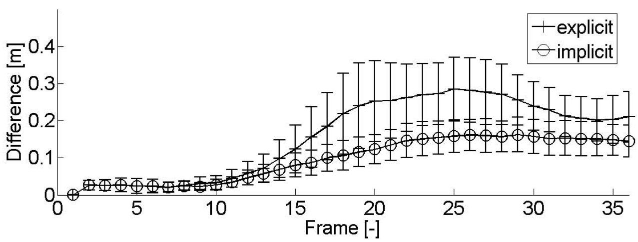

Fig. 2. The mean distances and standard deviations of the estimated projection centers to the ground truth for both methods with simulated noise plotted against the frame number

based on implicit functions as described in the previous section. Both algorithms were initialized using the same values, and the system noise and time update model were identical. Furthermore we iterated the measurement update until convergence for both approaches unlike proposed in [2], where only one iteration is performed.

To evaluate the new algorithm, we added noise to the ground truth observations, estimated the projection centers based on the noisy data with both methods and compared the results with the ground truth projection centers. We ran the experiment 20 times. The results can be observe in figure 2. We see that the proposed approach improves the accuracy of the estimated parameters.

Figure 3 shows the distance of the estimated projection centers to the ground truth projection centers plotted against the frame number of the real data.

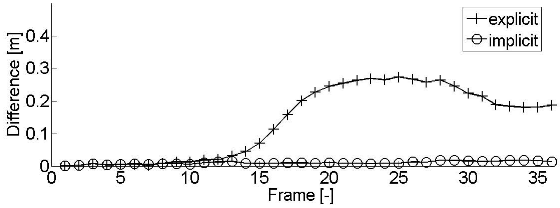

Fig. 3. The distances of the estimated projection centers to the ground truth for both methods plotted against the frame number

Observe, that the recursive estimation scheme based on the implicit function performs slightly better after the 15th frame.

4 Conclusion

We presented a new type of recursive estimation framework in a Kalman filter approach, which enables us to use implicit constraint functions, rather than being restricted to explicit ones. By allowing implicit constraints, not only the task of modeling recursive estimation schemes is eased significantly, but also those could lead to more linear models in the estimation part of a Kalman filter, which improves the robustness of such approaches.

We demonstrated the feasibility of this new algorithm for the task of structure-from-motion from monocular image sequences. The proposed implicit constraints turned out to be more robust than the explicit model used by [2] on our test sequence.

The presented method is applicable to a broad range of computer vision problems, including all those resulting in homogeneous equation systems, so that a lot of estimation task might benefit, which is a topic of further research. Furthermore it might be interesting, how the proposed measurement update might improve the performance of recursive estimation tasks in combination with the unscented transformation in the time update equations.

A MATLAB reference implementation of the presented estimation algorithm is available at `www.ipb.uni-bonn.de/∼richard/imEKF/`.

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