Variance Component Estimation in Performance Characteristics applied to Feature Extraction Procedures

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Abstract. This paper proposes variance component estimation (VCE) for empirical quality evaluation in computer vision. An outline is given for the scope of VCE in the context of quality evaluation. The principle of VCE is explained and the approach is applied to results of low level feature extraction. Ground truth is only partly needed for estimating the precision, accuracy and bias of extracted points and straight lines. The results of diverse feature extraction modules are compared.

1 Introduction

Performance evaluation is essential for systems development. Building computer vision systems requires clear documentation of the quality of each algorithm.

This paper deals with algorithms resulting in quantities (e.g. lengths, angles, probabilities) which have a probability density function that can be parameterized by first and second order moments. Characterizing such algorithms can be based on the results on multiple data sets, either exploiting mutual constraints between different results or using ground truth, e. g. when using simulated data. Both scenarios are useful. We propose variance component estimation (VCE) for determining the quality in both cases (cf fig. 1).

VCE estimates parameters of the distribution of observed values from the residuals of a maximum likelihood estimation. Together with additional parameters in the estimation it is able to determine (1) the internal precision, (2) the external accuracy and (3) the bias, all three measures being the classical triad for characterizing measurement. In case of repeated observations the VCE simplifies, in case of given ground truth the estimation of the bias simplifies.

The paper is organized as follows: Section 2 defines "precision", "accuracy" and "bias" as concepts for specifying quality. Section 3 proposes VCE for estimating these measures based on the results of parameter estimation. In Section 4, VCE is specialized to the case of estimating the quality of feature extraction procedures for point and line extraction and in section 5, the approach is applied to the output of diverse point and edge extraction modules. The paper closes with a discussion and an outlook.

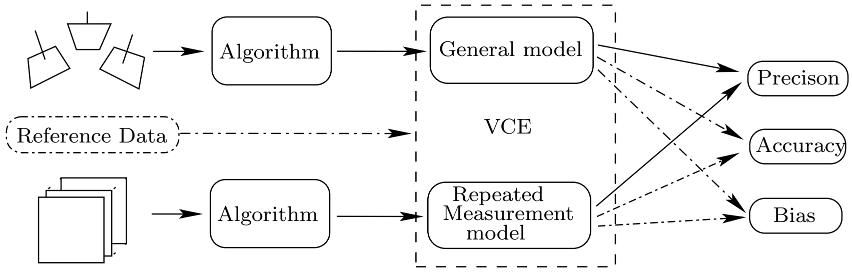

Fig. 1. Estimating precision, accuracy and bias by VCE in general and repeated measurement model.

Notation. We use Euclidean and homogeneous representation of entities in 2D. Euclidean entities are denoted with slanted letters, e. g. $\boldsymbol{x}$ and homogeneous entities are denoted with upright shaped letters, e. g. $\mathbf{x}$. Stochastic entities $\underline{\mathbf{x}}$ are underscored. "True" values $\widetilde{\mathbf{x}}$ are marked with a tilde and expectation values $\bar{\mathbf{x}}$ are marked with a bar. Estimated entities $\hat{\mathbf{x}}$ are labeled with a hat. Uncertainty of entities is represented by covariance matrices $\boldsymbol{\Sigma}_{xx}$, containing variances $\sigma_{xx} \doteq \sigma_x^2$ and covariances σ_{xy}.

2 Precision, bias and accuracy

In the following, processing results by applying an algorithm to data is interpreted as an observation process: The result of an algorithm is modeled as a stochastic variable $\underline{\boldsymbol{p}}$ (observation) with mean $\bar{\boldsymbol{p}} = E(\boldsymbol{p})$ and covariance matrix $\boldsymbol{\Sigma}_{pp}$. We assume that true values $\widetilde{\boldsymbol{p}}$ exist, representing the perfect result on given noisy data and use the following terms for characterization:

Precision. The *precision* of an observation $\underline{\boldsymbol{p}}$ is defined as the variance of $\underline{\boldsymbol{p}}$. It is represented by the covariance matrix

$$\boldsymbol{\Sigma}_{pp} = E[(\underline{\boldsymbol{p}} - \bar{\boldsymbol{p}})(\underline{\boldsymbol{p}} - \bar{\boldsymbol{p}})^{\mathsf{T}}]$$

and covers stochastic errors of the observation process.

Bias. The *bias* $\boldsymbol{b}$ of an observation $\underline{\boldsymbol{p}}$ is the deviation

$$\boldsymbol{b} = \widetilde{\boldsymbol{p}} - \bar{\boldsymbol{p}}$$

of the expectation value $\bar{\boldsymbol{p}}$ from the true value $\widetilde{\boldsymbol{p}}$. It covers systematic errors of the observation process.

Accuracy. The *accuracy* of an observation $\underline{\boldsymbol{p}}$ is the variance of the observation $\boldsymbol{p}$ referring to the true value $\widetilde{\boldsymbol{p}}$. It is represented by the matrix of second moments

$$^{a}\boldsymbol{\Sigma}_{pp} = E[(\underline{\boldsymbol{p}} - \widetilde{\boldsymbol{p}})(\underline{\boldsymbol{p}} - \widetilde{\boldsymbol{p}})^{\mathsf{T}}]$$

and covers both, systematic and stochastic errors of the observation process.

The relation between precision accuracy, and bias is given by (cf [8])

$$^{a}\boldsymbol{\Sigma}_{pp} = \boldsymbol{\Sigma}_{pp} + \boldsymbol{b}\boldsymbol{b}^{\mathsf{T}}. \tag{1}$$

3 Estimating precision, bias and accuracy

This section presents a two step procedure for estimating the bias, precision and accuracy. In the first step, parameter estimation is carried out in a linear model which links the expectation values of the observations mutually and together with unknown parameters. In this model, the biases of the observations are treated as additional unknowns. Maximum likelihood estimation leads to optimal estimates for the biases and the other model parameters. In the second step, VCE is carried out based on the estimated observation residuals of step 1. This leads to optimal estimates for precision and accuracy of the observations.

3.1 Step 1: Parameter estimation for estimating the bias

The most simple case of parameter estimation in a linear model is the well known Gauß Markoff model

$$E(\boldsymbol{y}) = \sum_i \beta_i \boldsymbol{a}_i = \boldsymbol{A}\boldsymbol{\beta}, \tag{2}$$

where the goal is estimating unknown parameters $\boldsymbol{\beta} = (\beta_i)$ from given data $\boldsymbol{y} = (\boldsymbol{p}_1{}^{\mathsf{T}}, \ldots, \boldsymbol{p}_N{}^{\mathsf{T}})^{\mathsf{T}}$ via a given coefficient matrix $\boldsymbol{A} = (\boldsymbol{a}_i)$. For estimating the unknown parameters, the expectation values $E(\boldsymbol{y})$ of the observations $\boldsymbol{y}$ are formulated as a linear combination of the known vectors $\boldsymbol{a}_i$. A best unbiased estimation $\hat{\boldsymbol{\beta}}$ for the unknown weights β_i is obtained by minimizing the variance $V(\hat{\boldsymbol{\beta}})$ under $E(\hat{\boldsymbol{\beta}}) = \tilde{\boldsymbol{\beta}}$ (cf [8]).

If in (2) the unknown biases of the observations are introduced as additional parameters, the parameter estimation procedure may be used for optimally estimating the biases. This requires a measurement setup that reveals sufficient information for estimating the biases together with the other parameters.

3.2 Step 2: Variance component estimation for estimating precision

Variance component estimation (VCE) is a technique for estimating the precision of observations by analyzing the estimated residuals of the observations. For this purpose, systems with high redundancy are required.

Analogous to the Gauß-Markoff-Model (2), the model of VCE component estimation is given by

$$E(\boldsymbol{\Sigma}) = \sum_c \sigma_{0,c}^2 \boldsymbol{\Sigma}_c^{(0)},$$

where $\boldsymbol{\Sigma}_c^{(0}$ are given matrices and the goal is estimating the unknown variance factors $\sigma_{0,c}^2$. Here it is the expectation value $E(\boldsymbol{\Sigma})$ of the covariance matrix $\boldsymbol{\Sigma}$ of observations that is formulated as linear combination of given matrices $\boldsymbol{\Sigma}_c^{(0)}$. A best estimation for the variance factors $\sigma_{0,c}^2$ is obtained by minimizing $V(\hat{\boldsymbol{\Sigma}})$. For estimating the unknown variance factors, the expectation value $E(\boldsymbol{\Sigma})$ of the covariance matrix $\boldsymbol{\Sigma}$ is approximated by $\boldsymbol{\Sigma}_0 = \sum_c \boldsymbol{\Sigma}_c^{(0)}$.

The principle of VCE can be sketched as follows: Let $\hat{\boldsymbol{y}}$ be the vector containing the estimated expectation values of the observations $\boldsymbol{y}$ and let $\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{y}} - \boldsymbol{y} =$

$\boldsymbol{D y}$ with $\boldsymbol{D} = \boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^{\mathsf{T}}\boldsymbol{\Sigma}_0^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathsf{T}}\boldsymbol{\Sigma}_0^{-1}$ (cf [8]) be the vector of the estimated observation residuals resulting from parameter estimation in a Gauß Markoff model. In the case of a diagonal covariance matrix $\boldsymbol{\Sigma}_0 = \mathrm{Diag}\,(\sigma_i^2)$, the estimated variance factor of the observations is given by

$$\hat{\sigma}_0^2 = \frac{\hat{\boldsymbol{\varepsilon}}^{\mathsf{T}}\boldsymbol{\Sigma}_0^{-1}\hat{\boldsymbol{\varepsilon}}}{R} = \frac{\hat{e}_1^2/\sigma_1^2 + \hat{e}_2^2/\sigma_2^2 + \ldots + \hat{e}_k^2/\sigma_k^2 + \; \Big| \; \hat{e}_{k+1}^2/\sigma_{k+1}^2 + \ldots + \hat{e}_K^2/\sigma_K^2}{\underbrace{r_1 + r_2 + \ldots + r_k}_{\to\hat{\sigma}_{01}^2} \; \Big| \; \underbrace{+ \; r_{k+1} + \ldots + r_K}_{\to\hat{\sigma}_{02}^2}}, \quad (3)$$

where r_i is the contribution of observation i to the total redundancy R. If different variance factors $\sigma_{0,1}^2$ and $\sigma_{0,2}^2$ are expected for e.g. two different types of observations, the fraction in (3) can formally be partitioned into two parts. Analyzing each part leads to separate estimations $\hat{\sigma}_{0,i}^2$ and $\hat{\sigma}_{0,2}^2$ of the variance factors σ_{01}^2 and σ_{02}^2.

For a general covariance matrix $\boldsymbol{\Sigma}_0$, best estimations $\hat{\sigma}_{0,c}^2$ of the variance components $\sigma_{0,c}^2$ are given by (cf. [1], [8])

$$\hat{\sigma}_{0,c}^2 = \frac{\hat{\boldsymbol{\varepsilon}}^{\mathsf{T}}\boldsymbol{\Sigma}_0^{-1}\boldsymbol{\Sigma}_c^{(0)}\boldsymbol{\Sigma}_0^{-1}\hat{\boldsymbol{\varepsilon}}}{\mathrm{tr}(\boldsymbol{\Sigma}_0^{-1}\boldsymbol{D}\boldsymbol{\Sigma}_c)},$$

$tr(\#)$ denoting the trace operator. The estimated covariance matrix of observations is given by

$$\widehat{\boldsymbol{\Sigma}} = \sum_c \hat{\boldsymbol{\Sigma}}_c \qquad \text{with} \qquad \hat{\boldsymbol{\Sigma}}_c = \hat{\sigma}_{0,c}^2\boldsymbol{\Sigma}_c^{(0)}.$$

Observe that the estimated covariance matrix $\hat{\boldsymbol{\Sigma}}$ depends on the approximation $\boldsymbol{\Sigma}_0$. Therefore VCE is applied iteratively with $\boldsymbol{\Sigma}_0^{(\nu+1)} = \sum_c \hat{\boldsymbol{\Sigma}}_c^{(\nu)}$ and $\hat{\boldsymbol{\Sigma}}_c^{(\nu)} := (\hat{\sigma}_{0,c}^2)^{(\nu)}\hat{\boldsymbol{\Sigma}}_c^{(\nu-1)}$. In the case of convergence, it holds $(\hat{\sigma}_{0,c}^2)^{(\nu)} \to 1$ for all factors $\sigma_{0,c}^2$.

3.3 Special case: repeated measurement with ground truth available

In the case that an algorithm is applied to N noisy versions of a data set, resulting in the observations $(\boldsymbol{p}_{1n}, \ldots, \boldsymbol{p}_{In})$ on the nth data set, and that ground truth $\tilde{\boldsymbol{p}}_{in}$, $i \in \{1, \ldots, I\}$ is available, parameter estimation and VCE lead to the following trivial results:

Bias. If the observations are weighted equally, the estimated expectation value of N observations $\boldsymbol{p}'_{in}$ is given by their mean $\hat{\boldsymbol{p}}_i = \frac{1}{N}\sum_{n=1}^N \boldsymbol{p}_{in}$. With the true value $\tilde{\boldsymbol{p}}_i$ of the observations $\boldsymbol{p}_{in}$, the bias of observations $\boldsymbol{p}_{in}$ is obtained by

$$\hat{\boldsymbol{b}}_{p_i} = \hat{\boldsymbol{p}}_i - \tilde{\boldsymbol{p}}_i \qquad \text{with} \qquad \hat{\boldsymbol{p}}_i = \tfrac{1}{N}\textstyle\sum_{n=1}^N \boldsymbol{p}_{in}. \quad (4)$$

Precision. The estimated covariance matrix and therefore the precision of the observations are given by

$$\widehat{\boldsymbol{\Sigma}}_{p_i p_i} = \tfrac{1}{N-1}\textstyle\sum_{n=1}^N (\boldsymbol{p}_{in} - \hat{\boldsymbol{p}}_i)(\boldsymbol{p}_{in} - \hat{\boldsymbol{p}}_i)^{\mathsf{T}}. \quad (5)$$

Accuracy is obtained by replacing in (5) the estimated point coordinates $\hat{\boldsymbol{p}}_i$ are replaced by their error free values $\tilde{\boldsymbol{p}}_i$, leading to the matrix of second moments

$$^{a}\widehat{\boldsymbol{\Sigma}}_{p_ip_i} = \tfrac{1}{N-1}\sum_{n=1}^{N}(\boldsymbol{p}_{in} - \tilde{\boldsymbol{p}}_i)(\boldsymbol{p}_{in} - \tilde{\boldsymbol{p}}_i)^{\mathsf{T}} \tag{6}$$

If the sum (6) is taken not over n but over i, j and divided by $(I-1)$, where I is the total number of observations in image n, then the mean accuracy of the observations in image n is obtained.

4 Precision, accuracy and bias of points and straight lines

4.1 Representation of points and straight lines in 2D

Points. A point in 2D and its uncertainty is represented by its Euclidean coordinate vector $\boldsymbol{x} = (x, y)^{\mathsf{T}}$ and its 2×2 covariance matrix $\boldsymbol{\Sigma}_{xx}$, given by

$$\boldsymbol{\Sigma}_{xx} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} = \boldsymbol{R}_\psi \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_t^2 \end{pmatrix} \boldsymbol{R}_\psi{}^{\mathsf{T}} \text{ with } \boldsymbol{R}_\psi = \begin{pmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{pmatrix}.$$

Herein, σ_r^2 and σ_t^2 are the variances of the point in the two main directions of its confidence ellipse; ψ represents the direction of the main semi-axis of the confidence ellipse in the image coordinate system. (cf. fig. 2).

In homogeneous coordinates, an uncertain point is represented by a 3×1 coordinate vector $\mathbf{x}$ and its rank 2 covariance matrix $\boldsymbol{\Sigma}_{\mathrm{xx}}$, for example

$$\mathbf{x} = (\boldsymbol{x}^{\mathsf{T}}, 1)^{\mathsf{T}} \text{ and } \boldsymbol{\Sigma}_{\mathrm{xx}} = \mathrm{Diag}\left(\boldsymbol{\Sigma}_{xx}, 0\right). \tag{7}$$

Straight lines. Following ([3]), straight lines $\boldsymbol{l} = (\phi, d)^{\mathsf{T}}$ in 2D are represented by their normal direction ϕ and their distance d to the origin of the image coordinate system (cf fig. 2). With the coordinates $\begin{pmatrix} s \\ d \end{pmatrix} = \begin{pmatrix} -\sin(\phi) & \cos(\phi) \\ \cos(\phi) & \sin(\phi) \end{pmatrix} \begin{pmatrix} x_g \\ y_g \end{pmatrix}$ of the center of gravity (x_g, y_g) of the line in the uv-coordinate system of fig. 2, the uncertainty of the line is given by the covariance matrix (cf [3])

$$\boldsymbol{\Sigma}_{ll} = \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix} \begin{pmatrix} \sigma_\phi^2 & 0 \\ 0 & \sigma_q^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}^{\mathsf{T}}.$$

Herein, σ_ϕ^2 denotes the variance of the line direction and σ_q^2 is the variance representing the uncertainty of the center of gravity in the direction across the line.

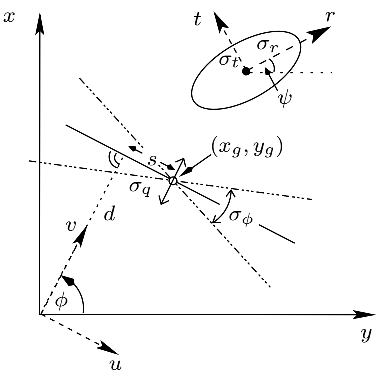

Fig. 2. Representation of uncertain points and straight lines in 2D.

With the 3×1 vector $\mathbf{a} = (\sin(\phi), -\cos(\phi), t)^{\mathsf{T}}$, homogeneous coordinates $\mathbf{l}$ of the line and their 3×3 covariance matrix $\boldsymbol{\Sigma}_{\mathrm{ll}}$ of rank 2 are given by (cf [7])

$$\mathbf{l} = (\sin(\phi), \cos(\phi), -s)^{\mathsf{T}} \text{ and } \boldsymbol{\Sigma}_{\mathrm{ll}} = \sigma_\phi^2\, \mathbf{a}\mathbf{a}^{\mathsf{T}} + \sigma_d^2\, \mathrm{Diag}(0,0,1). \tag{8}$$

Observe that the covariance matrix $\boldsymbol{\Sigma}_{\mathrm{ll}}$ can be decomposed into a sum of a matrix that only depends on the uncertainty σ_ϕ^2 of the direction and a matrix that only depends on the uncertainty σ_q^2 of the center of gravity across the line.

4.2 Procedures for estimating bias, precision and accuracy of extracted points and straight line segments

According to the explanations in section 3, we follow two approaches to estimating the bias, precision and accuracy of points and straight line segments provided by feature extraction procedures.

1. Parameter estimation and VCE in a general model without ground truth at hand
2. Estimating bias, precision and accuracy from repeated measurement with ground truth at hand

General approach. Given homologous points and straight lines extracted from $N \geq 2$ projective images of an object, in a first step bundle adjustment for camera orientation is carried out (cf [6]). The biases of the observed points and line segments are treated as additional unknowns and they are optimally estimated together with the camera orientation parameters.[1] In the second step, VCE is carried out for estimating the precision. Accuracy is estimated using (1).

Within the VCE procedure, we assume stochastic independence of points and straight lines and decompose the covariance matrix $\boldsymbol{\Sigma}$ of the observations $\boldsymbol{y}$ into

$$\boldsymbol{\Sigma} = \sigma_{0,\mathrm{x}'}^2 \underbrace{\begin{pmatrix} \boldsymbol{\Sigma}_{\mathrm{x}'\mathrm{x}'} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}}_{\boldsymbol{\Sigma}_1^{(0)}} + \sigma_{0,\phi}^2 \underbrace{\begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\mathrm{l}'\mathrm{l}',\phi} \end{pmatrix}}_{\boldsymbol{\Sigma}_2^{(0)}} + \sigma_{0,q}^2 \underbrace{\begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\mathrm{l}'\mathrm{l}',d} \end{pmatrix}}_{\boldsymbol{\Sigma}_3^{(0)}}$$

with the unknown variance factors $\sigma_{0,\mathrm{x}'}^2$, $\sigma_{0,\phi}^2$ and $\sigma_{0,q}^2$. The first matrix $\boldsymbol{\Sigma}_1^{(0)}$ represents the uncertainty of points. It contains the $3I \times 3I$ block diagonal matrix $\boldsymbol{\Sigma}_{\mathrm{x}'\mathrm{x}'} = \mathrm{Diag}\left(\boldsymbol{\Sigma}_{\mathrm{x}_1'\mathrm{x}_1'}, \ldots, \boldsymbol{\Sigma}_{\mathrm{x}_I',\mathrm{x}_I'}\right)$ with the elements given by (7), assuming independent observations of equal precision $\sigma_x = \sigma_y = 1$ and $\sigma_{xy} = 0$.

The $3J \times 3J$ structure matrices $\boldsymbol{\Sigma}_2^{(0)}$ and $\boldsymbol{\Sigma}_3^{(0)}$ represent the uncertainty of lines. Their sub-matrices $\boldsymbol{\Sigma}_{\mathrm{l}'\mathrm{l}',\phi} = \mathrm{Diag}\left(\boldsymbol{\Sigma}_{\mathrm{l}_1'\mathrm{l}_1',\phi}, \ldots, \boldsymbol{\Sigma}_{\mathrm{l}_J'\mathrm{l}_J',\phi}\right)$ and $\boldsymbol{\Sigma}_{\mathrm{l}'\mathrm{l}',q} = \mathrm{Diag}\left(\boldsymbol{\Sigma}_{\mathrm{l}_1'\mathrm{l}_1',q}, \ldots, \boldsymbol{\Sigma}_{\mathrm{l}_J'\mathrm{l}_J',q}\right)$ are obtained from (8) with $\sigma_\phi = 1[\mathrm{rad}]$, $\sigma_q = 1[\mathrm{pel}]$.

[1] In the case of a planar object, bundle adjustment can be replaced by estimating planar homographies between the object and the images (cf [7]), including the biases as additional unknowns.

Repeated measurement approach. Given points and straight lines extracted from N noisy versions of the same image and having ground truth at hand, bias, precision and accuracy of the points and straight lines is estimated by employing the equations (4), (5) and (6) with points parameterized by $\boldsymbol{x} = (x, y)^{\mathsf{T}}$ and lines parameterized by $\boldsymbol{l} = (\phi, d)^{\mathsf{T}}$.

5 Experiments

Primarily, our experiments are intended to verify the usability of our approach for evaluating the precision, accuracy and bias of points and straight lines. Secondary, we wanted to compare the quality of feature extraction modules on various levels of image noise.

5.1 Experimental setup

In the experiment, we involved the Harris corner detector (cf [5]), the Förstner window operator (cf [2]) and the Förstner point operator (cf [2]) as procedures for point extraction and the feature extraction software FEX (cf [3]) and the Schickler - Operator (cf [9]) as procedures for straight line extraction. Each operator was applied to 11 synthetic image pairs, each consisting of two noisy versions of the 500×500 image that is shown in fig. 5. On a dark background, the image contains 25 bright squares in various rotations. The side length of each square is $50[pel]$ and the image contrast is $\Delta g = 85[gr]$. Image noise σ_n was chosen in 12 steps in the range of $\sigma_n \in \{0,\ 0.7,\ 1.4,\ 2.8,\ 4.2,\ 5.7,\ 7.1,\ 8.8,\ 11.3,\ 16.7,\ 30.2\}[gr]$.

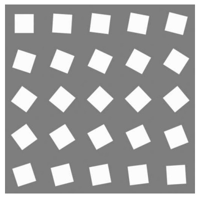

Fig. 3. Test Image

The tuning parameters σ_1 and σ_2 (cf [3]) of each operator were chosen to $\sigma_1 = 1.0$ and $\sigma_2 = 3.0$ for point extraction and $\sigma_1 = 1.0$ and $\sigma_2 = 2.0$ for straight line extraction. Only the noisiest image was smoothed with $\sigma_1 = 2.0$.

VCE was used in the general approach for estimating the precision of extracted points and lines. Accuracy and bias were analyzed using the repeated measurement approach with ground truth.

5.2 Results

The experiment proves the usability of VCE for estimating precision, bias and accuracy of points and straight lines. The results of VCE in the general and in the repeated measurement approach are consistent and plausible and allow a comparison of feature extraction modules with regard to precision, bias and accuracy.

Quality of points. In fig. 4, the estimated precision, accuracy and bias of points is depicted as function of the image noise σ_n.

For noise in the range $0 - 8[gr]$, the Harris operator and the Förstner window operator have the same characteristic in precision, accuracy and bias. This is to be expected because their theory is very similar. Bias and accuracy are about 3–4 pixels and thus quite bad – a fact that is plausible because both operators do not provide optimal *points* but *optimal positions of search windows* for

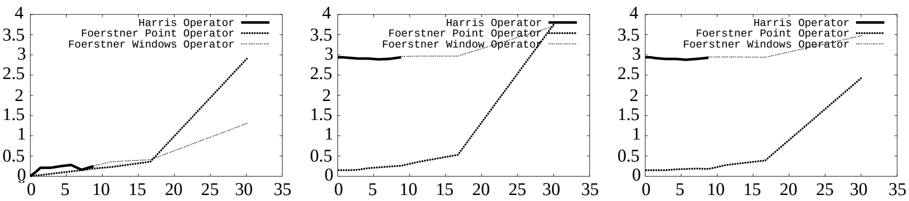

Fig. 4. Noise dependence of precision, accuracy and bias of points from the Harris corner detector (thick), the Förstner window operator (thin) and the Förstner point operator (dashed). *Left:* Precision $\hat{\sigma}_p = \sqrt{\sigma_{x'x'} + \sigma_{y'y'}}$ $[pel]$. *Center:* Accuracy ${}^a\hat{\sigma}_p = \sqrt{{}^a\hat{\sigma}_{x'x'} + {}^a\hat{\sigma}_{y'y'}}$ $[pel]$. *Right:* Bias $\hat{b} = \sqrt{{}^a\hat{\sigma}_p^2 - \hat{\sigma}_p^2}$ $[pel]$. In each graph, the first axis is labeled with the standard deviation σ_n $[gr]$ of the image noise.

point extraction (cf [2]). For larger noise ($\sigma_n > 8$), in our example no points are detected by the Harris operator.

For image noise in the range of $\sigma_n < 16$ $[gr]$, the Förstner point operator provides points with accuracy ${}^a\sigma_p < 0.5[pel]$. In this noise range, precision bias and accuracy increase nearly linearly with the noise. The bias is small ($< 0.3[pel]$). Larger noise worsens heavily the quality of the results.

Quality of straight lines. The estimated quality of the results of line extraction is depicted in fig. 5. Referring to precision, accuracy and bias of extracted features, for lower noise ($\sigma_n < 17[gr]$) the feature extraction FEX is superior to the Schickler operator both in precision and accuracy. For noise $\sigma_n > 17[gr]$, precision and accuracy decrease heavily. This is caused by the fact that with increasing noise straight lines are broken up into smaller pieces with worse quality. Concerning the uncertainty of the center of gravity of lines in the direction across the line, both FEX and the Schickler operator behave similar and the uncertainty increases linearly with increasing noise.

6 Conclusions and outlook

In this paper we have proposed VCE for estimating the quality of results drawn from computer vision algorithms. The application area of VCE in the context of quality evaluation has been outlined and its basic principles have been explained. After a specialization to the case of repeated measurement, VCE was carried out exemplarily for estimating the precision, accuracy and bias of points and straight lines.

The results are consistent and plausible and show the usability of VCE for evaluating quality. A more meaningful investigation of precision, accuracy and bias of feature extraction modules will have to take into account synthetic and real images of various quality and content. Furthermore, investigating precision, accuracy and bias of features is not enough for characterizing feature extraction algorithms (cf [4]). We also will evaluate other properties of feature extraction extraction algorithms, especially the coverage of lines, and the effect onto relations between features needed for grouping.

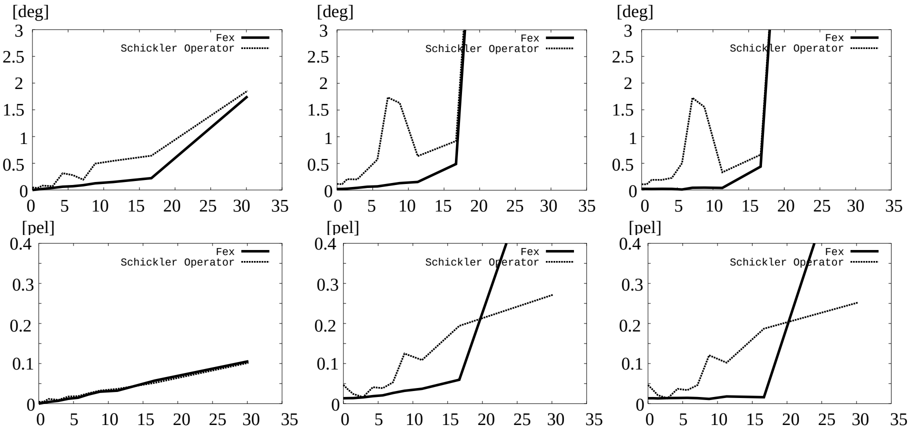

Fig. 5. Noise dependence of precision, accuracy and bias of lines from FEX and from the Schickler-Operator. **Top row:** Uncertainty in line direction *Left:* Precision $\hat{\sigma}_\phi$. *Center:* Accuracy ${}^a\hat{\sigma}_\phi$. *Right:* Bias $\hat{\sigma}_\phi = \sqrt{{}^a\hat{\sigma}^2_\phi - \hat{\sigma}^2_\phi}$. **Bottom row:** Uncertainty of the center of gravity across the line. *Left:* Precision $\hat{\sigma}_d$. *Center:* Accuracy ${}^a\hat{\sigma}_d$. *Right:* Bias $\hat{\sigma}_d = \sqrt{{}^a\hat{\sigma}^2_d - \hat{\sigma}^2_d}$. In each graph, the first axis is labeled with the standard deviation σ_n $[gr]$ of the image noise.

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