Specification and Propagation of Uncertainty

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Working Group Report

Wolfgang Förstner, Universität Bonn

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1 Participants

The working group consisted of the following participants: S. Bailey, H. de Boer, P. Faber, W. Förstner, R. M. Haralick, H. Haussecker, N. Ohta, J. Sporring.

2 Scope

The scope of the working group is to discuss the two questions:

1. How to specify the required performance of an algorithm?
2. How to perform propagation of uncertainty in algorithms?

3 Outline

We proceeded in a top down way to attack the problem. We hypothesize each algorithm can be specified by its input (x, Q_x), its output (y, Q_y) and the theory A of the transition between input and output. The input and output values x and y are uncertain, their quality is described by Q_x and Q_y. This allows to link different atomic algorithms to form an aggregated algorithm.

Specification is to pose constraints on the range of y and Q_y, which are task specific.

The discussion was intended to cover the following topics:

1. Prove the hypothesis. This is done by investigating a complicated enough task which contains enough representative subtasks.
2. Discuss the possible structures of input and output.
3. Discuss the propagation of uncertainty.

Figure 1: shows the general structure of an algorithm A with input (x, Q_x) and output (y, Q_y)

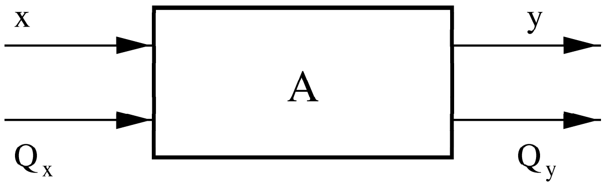
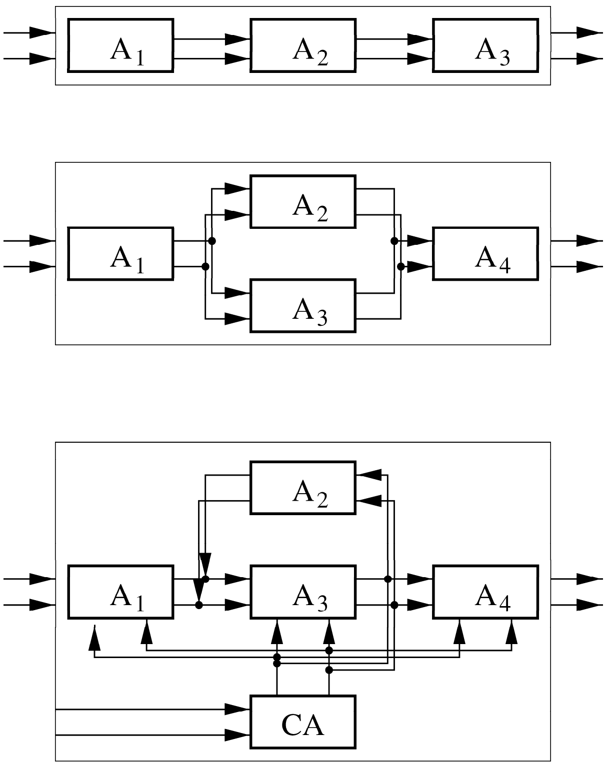


Figure 2: shows three examples of aggregated algorithms. The top example is a simple sequence. The middle example contains a bifurcation which may be guided by a decision made by the first algorithm. It also may represent the parallelity of the two algorithms A_2 and A_3. The bottom example contains a control algorithm CA which uses only the input data, and a loop of the algorithms A_2 and A_3, where the decision to stop is made by A_3. Observe each of the algorithms A_i may internally have a compsite stucture.



4 The Example

Given a conveyor belt with parts being observed by a stereo system with 3 cameras. The task is to locate and classify the parts depending on their 3D-geometry.

The task can be solved by a sequence of atomic tasks:

1. Extraction of linear features in the image, especially straight line segments and segments of conics.
2. Stereo matching leading to a set of match candidates.
3. 3D-reconstruction leading to a set of 3D line segments, namely straight line segments, conic segments and possibly other segments.
4. Indexing of 3D-line segments into object data base, leading to pose parameters and a classification.

5 Specification

5.1 The Complete Algorithm

We first checked whether the complete algorithm can be described in the hypothesized manner.

The input x is given by:

1. 3 images
2. orientation and calibration parameters. The calibration includes geoemtric and radiometric parameters.
3. tunnig parameters.
4. The geometry of the objects.
5. The a priori probability $P(\omega_i)$ of an object appearing.

The uncertainty Q_x of the input is given by

1. noise characteristics of image. This is given by the signal dependent noise standard deviation $\sigma_n(g) = \sqrt{a + bg}$ where g is the greyvalue.
2. Covariance matrix of the orientation and calibration parameters.
3. The quality of $P(\omega_i)$.

The output y is given by:

1. The proposed class label $\hat{\omega}$.

2. The vector of the probabilities $P(\omega_i)$ of class labels.

The uncertainty Q_y of the output is given by:

1. The uncertainty of the probabilities of the class labels.

It was not the task to specify the algorithm A in detail. The discussion suggests that the structure of an algorithm is applicable to the chosen complex task.

This shows that x may be simple or have a composed structure with subparts being:

- integers, reals, labels
- sets, lists, vectors, matrices, tensors
- graphs, trees, relational structures

The *problem* therefore is *to establish a theory of quality measures for the different types of data.*

It appears that statistics provides the appropriate tools. However, the task of modelling the uncertainty, of estimating the free parameters, of propagating the uncertainty and of testing the validity of the chosen models is by far not worked out for the basic tasks in image analysis. The numereous examples from literature however show, that the tools are available and suggest the statistical framework to be applicable.

5.2 The First Step: Extracting Linear Features

We discussed the first step as an example for a subtask.

The input x is given by:

1. The image.
2. The expected scale of the edges. This parameter of course could be determined from the data, leading to a more complex algorithm.
3. A threshold for distinguishing edge pixels from non-edge pixels. As thresholding can be interpreted as a statistical test, the threshold may be made dependent on a presecified significance level.

The quality Q_x of the input is given by:

1. The noise characteristics of the image. It can be approximated by the signal dependent noise standard deviation $\sigma_n(g) = \sqrt{a + bg}$.

The output y is given by:

1. List of straight line segments (4 parameters = 2 for the line + 2 for the end points)

2. List of conic line segments (7 parameters = 5 for the conic + 2 for the end points)

3. Possibly, attributes to the line segments as contrast and scale.

The quality Q_y of the output is given by:

1. Covariance matrices $\boldsymbol{\Sigma}$ for the parameters of the line segments. They can be assumed to be independent. However, the parameters describing the line segments certainly are correlated which prevents the covariance matrices to be of diagonal structure.

2. Propabilities or likelihoods $P(\hat{s})$ the line segment represents a true line segment.

3. Probabilities or likelihoods $P(s_j|\hat{s}_i)$ the found line segment $\hat{s}_i$ actually represents another one. Together with the previous probabilities, this information can be collected in a confusion matrix.

4. Probabilities or likelihoods the line segments are spurious.

Obviously there is no information available on missing edges.

There seems to be no reason to reject the hypthesis of the clear setup of algorithms as input-output relation.

6 Uncertainty Representation

We collected some well known representations from statistics.

- *real values, or vectors with real valued elements*: variance, covariance, correlation, bias. The number N of elements to represent the uncertainty then increases with the square of the number n of real valued elements: $N = O(n^2)$, mainly contained in the covariance matrix, being a complete representation for normally distributed variables.

- *integer values, or vectors with integer valued elements*: propability, joint probability. The number of elements to represent the uncertainty increases exponentially with the number n of integer valued elements: $N = O(2^n)$, as in general the probability of all combinations need to be specified.

- *class labels* as a result of a classification step: confusion matrix.

- For *sets* we hat no simple representation at hand. An example, which seems to be representative would be the probability $P(\{\hat{a}, \hat{b}, \hat{c}\}|\{a, b, e, f\})$ an algorithm would find the set $\{\hat{a}, \hat{b}, \hat{c}\}$ while the true result would be $\{a, b, e, f\}$. In case of independent elements this propability might be decomposable into $P(\hat{a}|a)P(\hat{b}|b)P(\hat{c}|\neg c)P(\neg\hat{e}|e)P(\neg\hat{f}|f)$. The probabilities might be derivable from a confusion matrix.

- The statistic of *lists* may be represented by Markov-chains, well known in speach analysis. The Leveshtein or Hamming distance of two strings is a well known quality measure, which weighs every symbol equally and assumes independence of the symbols.

- The statistics of *graph* structures is developping rapidly. For *undirected* graphs Markov-Random-fields are well established. For *directed* graphs Bayesian networks are applicable. For *graphs with directed and undirected edges and real and integer valued variables* graphical model are under development, unifying Markov-Random fields and Bayesian networks. In all cases independence assumptions simplify the model, the estimation and the evaluation.

In all cases classical statistics provides the means for *propagating uncertainty*.

7 Open Questions

We identified some open problems, which we did not see whether the theory is worked out:

- Algorithms may contain *feed back loops* within a sequence of subalgorithms. It is not clear how in a general setup uncertainty of such a loop can be handled. One certainly has to distinguish two cases. In case the loop results from an iterative solution to a global optimization problem an analysis should be feasible. Otherwise, as e. g. in diffusion techniques, the quality may not be derivable.

- In image processing and feature extraction one needs to make a *decision* on whether one starts at a *continuous model*, which then is discretized or one may directly start at the *grid*. In the last case it appears difficult to define and use properties of the image which refer to the embedding of the the grid into the continuous plane, as e. g. the straightness of an edge not lying in one of the principle directions.

- The propagation of uncertainty is worked out for quite some of the above mentioned representations. However, complex systems cannot be modelled rigorously. It is a practical and a hard theoretical problem to identify and evaluate *approximate propagation schemes*, especially schemes which neglect certain dependencies.