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High Quality Photogrammetric Point Determination

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Summary

Photogrammetric blocktriangulation is a versatile tool for high quality point determination. The paper outlines the precision and reliability features of the method. Examples of controlled tests prove that accuracies of 3–5 ppm can be achieved on standard equipment and that proper planning guarantees results which are robust with respect to gross and systematic errors. Digital image correlation techniques will further increase the economy and the flexibility of the procedure.

Zusammenfassung

Die photogrammetrische Blocktriangulation ist ein vielseitiges Instrument zur genauen Punktbestimmung. Der Beitrag zeigt die Genauigkeit und die Zuverlässigkeit, die dieses Verfahren kennzeichnen, auf. Die Beispiele mit kontrollierten Tests beweisen, daß mit normalem Instrumentarium Genauigkeiten von 3–5 ppm erreichbar sind, und daß eine gute Planung Ergebnisse liefert, die robust gegenüber groben und systematischen Fehlern sind. Techniken zur digitalen Zuordnung und Korrelation von Bildern werden die Anpassungsfähigkeit und die Wirtschaftlichkeit dieses Verfahrens noch steigern.

1. Introduction

1.1 This paper wants to give a review on the theoretical and practical developments which enable photogrammetry to be an alternative tool for high quality point determination.

Photogrammetry found and still has its main application as data aquisition method for subsequent mapping. The need of terrestrial pass points for the absolute orientation of the models was the motivation for the development of aerial triangulation. The general trend to analytical methods together with the increased quality of the hardware opened the door to high precision application.

1.2 The progress of analytical photogrammetry began with studies about the theoretical accuracy of photogrammetric blocks with independent models (ACKERMANN, 1966) and with bundles (KUNJI, 1968; TALTS, 1968). Their aim was to provide guidelines for project planning especially with the aim to decrease the necessary number of control points. However, what is more important, they were the starting point for all further quality studies in photogrammetry and the development of efficient computer programs for the rigorous blockadjustment.

The basic idea was to determine the standard deviations or other accuracy measures of the new points of regular shaped photogrammetric blocks using the inverse of the normal equations. The underlying mathematical model was and still is very much simplified just using the perspective relations and treating the observations (model or image coordinates) as uncorrelated and usually of equal weight. Though the simplifications are rather crude and the resulting discrepancies, revealed by experiments lead to further refinements of the functional model to compensate for systematic image errors, the main results have been confirmed by numerous empirical tests and pushed aerial triangulation into new fields of application such as network densification or deformation analysis.

1.3 The results, however, not always were satisfying. The reliability of photogrammetric coordinates was queried by the land surveyors. Thus after having proved the excellent accuracy structure of photogrammetric point fields, reliability was the intrinsic problem of aerial triangulation. As a consequence photogrammetric point determination often was burdened with terrestrial checks, decreasing the economy of the procedure.

The situation has changed, due to the improvement of the hardware conditions as increased stability of the films, especially of colour films and the availability of high precision comparators, but also due to a better insight into the geometrical structure of photogrammetric blocks.

1.4 During the last few years quite some attempt has been made also to automate the process of stereo vision for measuring targets or for point transfer using techniques of digital image processing. Recent results prove the precision achieved with digital correlation techniques to be comparable to the precision obtained by human stereopsis. Moreover, the measuring process provides criteria for its evaluation, which may be used to improve the rigour of the subsequent adjustment.

1.5 The following sections provide the basic theoretical and practical results which are the foundation for the prerequisites being necessary to achieve photogrammetric coordinates of high quality. Section 2 discusses the accuracy structure of photogrammetric bundle blocks. The different aspects of reliability are treated in section 3. Section 4 discusses the basic properties of digital image correlation and their impact onto aerial triangulation. The empirical results selected for section 5 want to show that aerial triangulation is able to meet high quality conditions for point determination.

2. The theoretical precision of aerial triangulation

2.1 The studies on the theoretical precision of block triangulation are based on the adjustment of regular shaped blocks, as the first aim was to find out the dependence of the precision on different block parameters, especially
- the block size
- the overlap
- the control point density and distribution
- the precision of the photogrammetric measurements
- the image scale.

The average values μ and the maximum values σ_{max} of the standard errors σ_x, σ_y and σ_z of the new points determine the absolute accuracy. As the precision of the photogrammetric measurement σ_0 is practically independent of the image scale 1:s and the standard errors of the new coordinates proportionally increase with the scale factor the absolute accuracy is related to $\sigma_0 \cdot s$, i.e. always the ratios $\mu/(\sigma_0 \cdot s)$ are given.

2.2 The main results of the investigations can be summarized as follows: The accuracy is very homogeneous in the interior of the blocks and in the order of the precision of the photogrammetric measurements and nearly independent of block size and form. It only increases with the logarithm of block size (cf. EBNER, 1972; MEISSL, 1976). Thus even in very large blocks the standard errors of the new points do not exceed the 1.5- or 2-fold standard errors σ_0 of the photogrammetric measurements, referring to image scale.

Therefore on one hand large areas can be covered which is of utmost importance for the determination of pass points in small scale mapping. On the other hand, as the precision of the measurements is practically independent on the photo scale, the precision of the new points can be increased to nearly any level by increasing the image scale, being the basis for high precision point determination or industrial application. The maximum standard errors occur at the perimeter of the blocks between the control points. They can be decreased by using dense perimeter control or by extending the block by half a base length, thus using only the interior of the enlarged block.

For planimetry a dense control of the border of the block is necessary. Control points in the interior of the block are not needed as they have only local effect on the precision. This surely is the most important result, as together with the high homogeneity it enables photogrammetry to densify large areas in a very economical way, transferring the precision of the ground control practically without any loss of accuracy. Of course, if additional control points are available, they should be used, especially to stabilize the determination of additional parameters for the compensation of systematic errors.

The situation is quite different for the height. Here control points are necessary also in the interior of the blocks, the optimal pattern being different for 20% and 60% side lap requiring chains or a regular grid of height control points resp. The precision here mainly depends on the control point interval.

2.3 The general results hold for the independent model and the bundle method. We will restrict the discussion to the determination of the planimetric coordinates using the bundle method. The following accuracy models, taken from EBNER, KRACK and SCHUBERT (1977) are valid also for very large blocks, thus generalize the results by KUNJI (1968) and TALTS (1968).

Table 1 gives the average standard deviation μ_{xy} of the planimetric coordinates of square shaped blocks with side lengths of n baselengths, dependent on the control point pattern and the overlap. The dense perimeter control with a control point distance of 2 baselengths is remarkable (pattern C1) as the accuracy is below the precision of the photogrammetric measurements, i. e. $0.9\ \sigma_0 \cdot s$ and $0.6\ \sigma_0 \cdot s$ for 20% and 60% sidelap resp.

Table 1 Accuracy models for bundle blocks (from EBNER, KRACK, SCHUBERT, 1977)
Average standard errors μ_{xy} of planimetric coordinates for square shaped bundle blocks of size $n \times n$ baselengths in dependency on the standard deviation σ_0 of the measured image coordinates, the photo scale 1:s, the side lap and the control point pattern.
Maximum standard deviations at the border are 1.5–2.0 times larger

Control points		Absolute accuracy $\mu_{xy}/(\sigma_0 \cdot s)$ coverage/side lap	
pattern	number	2-fold/20%	4-fold/60%
C 1	2 n	0.9	0.6
C 2	16	0.5 + 0.025 n	0.4 + 0.015 n
C 3	8	0.3 + 0.08 n	0.3 + 0.04 n
C 4	4	0.26 n	0.14 n

For example, assuming an accuracy of $\sigma_0 = 5\ \mu m$ and a photo scale of $1:s = 1:5000$, i. e. a flying height over ground of $h_g = 750$ m a precision of 1.5 cm can be achieved using dense control with fourfold overlap. Blocks covering an area of 4.6×4.6 km or 9.2×9.2 km consist of 121 or 441 photos resp., needing only 20 or 40 groups (cf. sect. 3) of horizontal control points at the perimeter of the block. 15 or 35 evenly distributed groups of vertical control points are sufficient to stabilize the height.

3. The reliability of aerial triangulation

3.1 With increasing accuracy and automation the control on the reliability of the photogrammetric procedure for point determination became necessary. In our context the notion reliability covers two aspects which interrelate to some extent:
- the problem of point identification
- the problem of the proper choice of the computational model.

3.2 To reach the obtainable accuracy in point determination all points have to be targeted. The identification of the points is done at two places: during targeting and during measuring. As a consequence the photogrammetric measuring process has to cope with three error sources:
1. unvisible targets (lost or hidden)
2. misidentification of targets, due to target-similar objects near to unvisible targets and
3. centering errors, due to illumination effects.

These error sources may be reduced by

1. using cameras with larger focal length, e.g. in areas covered with buildings.
2. changing the type of targeting.
   Good experiences were made with luminous colour (orange/red) used for the survey mark possibly together with an adequate optical filter increasing the reliability of identification in colour film.
3. using groups of targets with known distances in addition to indication-targets. They improve the security of identification and provide a check on the targeting.

3.3 This immediately leads to the second aspect of reliability, namely the adequate representation of reality by the used computational model which in the first phase of high precision photogrammetry reflected the simplified assumptions used for the theoretical accuracy models. Thus only the perspective relations and uncorrelated measuring errors are assumed, disregarding systematic errors, blunders and errors in the control points.

Therefore the refinement of the computational model was indispensable in order to be able to compensate for systematic errors of the image coordinates. Selfcalibration, i.e. the simultaneous determination of coordinates, orientation and additional parameters (cf. BAUER, MÜLLER, 1972; BROWN, 1974; EBNER, 1976; ACKERMANN, 1980) is superior to testfield calibration, being more economic and on an average producing better results (KILPELÄ, 1980). Also the use of reseau cameras is not commonly accepted due to the additional measuring effort which did not consistantly prove to yield better results (but cf. SLAMA, 1978). Though the evaluation of the additional parameters is not fully automized in case the requirements with regard to precision are very high, this method is commonly used. Compensating systematic errors leads to an improvement of the precision by a factor 2 or more and to a rather good agreement between the theoretical accuracy models and experimental results.

On the other hand it is pleasing and at the same time amazing that the gross error problem mainly seemes to be a question of a powerful error detection strategy than a question of a strong geometry, suggesting that the local redundancy of blocks for network densification is high.

3.4 Studies on the theoretical reliability according to BAARDA performed in the late 70s (FÖRSTNER (1980, 1985); GRÜN (1980); BOULOUCOS (1979); BOUW, MOLENAAR (1980)) have confirmed this. They were based on the same simplified assumptions as the accuracy studies and also used regular block configurations. Reliability usually is described by the controllability of the observations, namely lower bounds for detectable gross errors (internal reliability) and by the sensitivity of the result coordinates, namely the maximum influence of non-detectable gross errors onto the adjusted coordinates (external reliability).

For project planning the sensitivity values $\bar{\delta}_{oi}$ are decisive. They can be determined for all observations l_i. Their influence on the coordinates x with standard-deviation σ_x is bounded by $\bar{\delta}_{oi} \cdot \sigma_x$.

The main results can be summarized as follows:

a) The reliability is very homogeneous in the interior of the blocks thus being independent on block size and shape.
b) The border parts are less reliable, especially in single blocks (cf. Fig. 1). This disadvantage can be overcome by extending the block by half a baselength at the perimeter. This is at the same time a remedy to increase the precision in this area (cf. 2.2).
c) The control points can not be checked by photogrammetric means ($\bar{\delta}_{oi} > 10$) unless the control points distance is very small. The targeting and the identification can be checked by photogrammetric means using groups of control points which occur in at least three photos in order to be able to distinguish between identification and coordinate errors. Moreover the block should be planned in a way which allows a free blockadjustment. The quality of the control point coordinates has to be documented by the geodesists.

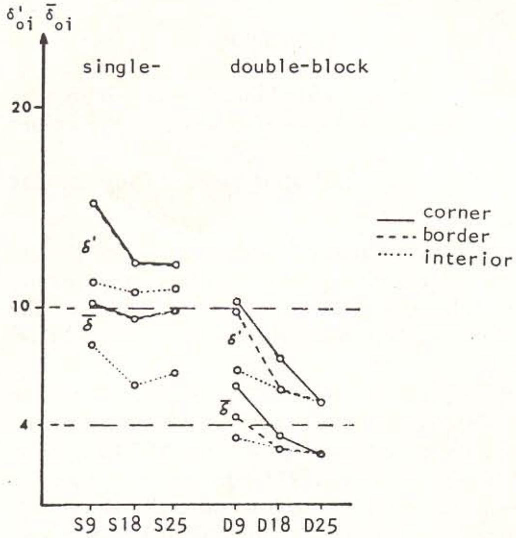

Fig. 1 Internal and external reliability of bundle blocks (from FÖRSTNER, 1982)
Controllability factors δ'_{oi} for the standard errors σ_0 lead to lower bounds $\delta'_{oi} \cdot \sigma_0$ for detectable gross errors. Sensitivity factors $\bar{\delta}_{oi}$ for the standard errors σ_x lead to maximum influence $\bar{\delta}_{oi} \cdot \sigma_x$ of nondetectable gross errors onto coordinates x. Sm = Single blocks, Dm = Double blocks, m = number of points per image

d) The reliability of bundle blocks cannot be increased by using more tie points but only by increasing overlap, i.e. by using fourfold overlap (cf. Fig. 1). With respect to reliability two blocks with 20% sidelap flewn crosswise are superior to blocks with 60% sidelap (cf. LI, 1985).
e) The absolute values of external reliability are comparable if not superior to those in geodetic networks. Gross errors which are not detectable with a statistical test do not deteriorate the adjusted coordinates more than the 4 times their standard deviation if fourfold coverage is used ($\bar{\delta}_{oi} \leq 4$).

Due to the high homogeneity of the geometry of photogrammetric blocks, the quality in terms of precision and reliability of photogrammetrically determined coordinates is well predictable if the measuring equipment, determining the measuring accuracy, and some parameters of the block geometry, namely overlap, control and tie point density, are specified.

4. Digital correlation for aerial triangulation

4.1 The quality measures all refer to the precision of the measuring process, namely the standard deviation σ_0 of the image coordinates. Empirical values for σ_0 range between 2 and 6 μm (cf. ACKERMANN, 1983). However, there is no possibility to come to an instant and objective estimate for the precision, if the points are measured manually. Redundant observations are necessary to obtain a reliable estimate for σ_0 from a least squares adjustment, possibly during intermediate steps. Transferring the measuring process to the computer changes this situation.
4.2 Computational stereo is based on images which are in or have been converted into digital form, like TV images, with the difference that the brightness at a given position is represented by a number, the graylevel. The process of finding corresponding points in two or more images can be achieved in the same manner as e.g. corresponding parts of a radio signal in very long base line interferometry or in other phase angle techniques for distance measurements are found, namely by searching for the maximum of the crosscorrelation function of the two signals.
4.3 There are several properties in common with digital image correlation. The most important one is, that the value of the maximum correlation gives a figure of merit for the measured distance or parallel resp. In addition to the correlation coefficient or the signal to noise ratio $SNR = \sigma_s/\sigma_n$,

the size of the signal (length, number n of picture elements) and the effective bandwidth b or the sharpness of the signal are decisive for the precision of the measured value in terms of a standard deviation: $\sigma_x = (2\pi \cdot \sqrt{n} \cdot \mathrm{SNR} \cdot b)^{-1}$ (cf. FÖRSTNER, 1984). From the autocorrelationfunction of the signal of photogrammetric images one can derive a theoretical precision for locating targeted points or for measuring parallaxes of natural points. They lie in a range between 1 and 2 µm for good imagery. Thus, they are fully comparable to the precision of human stereopsis.

4.4 On the other hand there are properties of digital image correlation which differ from correlation techniques in distance measurements. First, the signal is twodimensional (in x- and y-direction) which leads to a heavy computational load. It is payed off by the discreteness of the signal, which allows flexible adaption to varying situations. This immediately leads to the third difference, which is crucial for computational stereo: the necessity to have a guidance by a human operator, due to its interpretation capabilities. Thus aerial triangulation with digital correlation for supporting the measuring process will, at least for the next future, stay operator controlled. The correlator thus solely replaces the human parallax measuring process, leading to controlled, hence reliable image coordinates and providing the following adjustment procedure with the necessary statistical information, namely the precision of the measurements.

5. Empirical results

5.1 Numerous controlled tests have been performed parallel to the theoretical studies to prove the validity of the accuracy models. The evaluation in all cases was based on the comparison of photogrammetrically and at the same time geodetically determined check points, which were not used as control points in the block adjustment. The differences lead to average and maximum values $\hat{\mu}$ and ε_{max} giving an estimate for the absolute accuracy. The following examples demonstrate the necessity of compensating for systematic errors and of using fourfold coverage, and the high accuracy which also can be obtained in practical application.

5.2 The test field Appenweier (ACKERMANN, 1975) was established in 1973 in connection with the densification of a given third-order trigonometric network. The test was designed to get a planimetric accuracy of 3 cm. The photo scale of the 4 single blocks was 1:7800. This allowed to investigate the influence of overlap onto the absolute accuracy.

Table 2 gives the estimated precision $\hat{\sigma}_0 = \sqrt{v'Pv/r}$ of the image coordinates and the r.m.s. and the maximum errors at the 85 check points, both for single and double blocks, i.e. twofold and fourfold coverage.

Table 2 Testblock Appenweier, empirical accuracy of bundle block adjustment (from KLEIN, 1980)
Image scale 1:7800, area 9.1 × 10.4 km², estimated precision of terrestrial control and check point coordinates 1.2 cm
$\hat{\mu}_{xy}^2 = (\hat{\mu}_x^2 + \hat{\mu}_y^2)/2$, $\hat{\mu}_x$, $\hat{\mu}_y$ = r.m.s. residuals at 85 check points; ε_{max} = maximum residual at check points

		coverage/sidlap							
		2-fold/20%				4-fold/60%			
	self-calibration	$\hat{\sigma}_0$ µm	$\hat{\mu}_{xy}$ cm	ε_{max} cm	$\frac{\hat{\mu}_{xy}}{\hat{\sigma}_0}$	$\hat{\sigma}_0$ µm	$\hat{\mu}_{xy}$ cm	ε_{max} cm	$\frac{\hat{\mu}_{xy}}{\hat{\sigma}_0}$
1	no parameters	3.0	5.7	42.9	2.4	3.6	3.4	10.4	1.2
2	12 parameters blockwise	2.4	3.8	18.6	2.0	2.7	2.6	9.3	1.2
3	12 parameters stripwise	2.3	3.4	13.8	1.9	2.6	2.0	7.0	1.0

The adjustment was performed 1. without any compensation of systematic errors, 2. with 12 additional parameters, common for all image of each block and 3. with 12 parameters for each strip in order to consider possible changes of the image deformation between the strips. Not determinable parameters were excluded to obtain a stable solution.

The table shows clearly:

- The accuracy increases with increasing refinement of the model. This proves that the additional parameters really compensate for varying systematic errors.
- The maximum errors significantly decrease in the single blocks, which is of utmost importance for practical applications. On the other side the maximum errors of double blocks in all cases are smaller than those in single blocks.
- The results, though extremely good, are not quite in accordance with theory as the ratio $\hat{\mu}_{xy}(\mu m)/\hat{\sigma}_0(\mu m)$ should be 0.9 for single and 0.6 for double blocks. This discrepancy may be explained by neglections in the mathematical model, neglected correlations between the image coordinates and certainly also unrealistic assumptions about the precision of the control and check points, which have an average precision of 1.2 cm.

5.3 The second example is taken from a practical application in open cast mining (REICHENBACH, 1982). The comparison is based on 24, later 59 check points which were determined with an accuracy of about 0.5 cm, distributed over an area of 3 km^2.

Table 3 gives the result of 6 projects performed with 8-fold coverage in a scale of 1:7000, i.e. a flying height of 1050 m. The high coverage allows an increase of accuracy and a save of control points without increasing the photogrammetric effort compared to a block with double scale (1:3500). The projects 4–6 include an additional strip flown in a slightly larger scale 1:5800, i.e. with a flying height of about 900 m.

Table 3 Test area "Opencast Mining", empirical accuracy of bundle blockadjustment (from REICHENBACH, 1982)
Area 3 km^2, estimated precision of terrestrial control and check point coordinates 0.5 cm, $\hat{\mu}_{xy}$ and ε_{max} cf. Table 2

project	scale	instrument	selfcalibration[1]	estimated measuring accuracy $\hat{\sigma}_0$ µm	cm	no. of check points	absolute accuracy $\hat{\mu}_{xy}$ cm	ε_{max} cm	$\frac{\hat{\mu}_{xy}}{\hat{\sigma}_0}$
1	1:7000	PSK 2	–	5.4	3.7	24	3.0	9.9	0.81
			+	4.0	2.8		1.5	3.2	0.56
2	1:7000	PSK 2	–	5.0	3.5	24	1.8	4.6	0.53
			+	4.4	3.1		1.5	4.7	0.51
3	1:7000	PK 1	+	3.8	2.6	24	1.2	2.5	0.46
4	1:7000 1:5800	PK 1	+	3.4	2.2	59	1.5	3.8	0.72
5	1:7000 1:5800	PK 1	+	3.7	2.4	59	1.5	4.6	0.67
6	1:7000 1:5800	PK 1	+	3.0	1.9	59	0.9	2.6	0.46

[1] 12 parameters per block

The homogeneous measuring accuracy $\hat{\sigma}_0$ between 3 and 4.5 μm indicates the high reliability of the whole procedure. The absolute accuracy is fully sufficient for network densification and in good agreement with the theoretical expectation ($\hat{\mu}_{xy}/\hat{\sigma}_0 \approx 0.6$). The maximum errors are <5 cm, i.e. about 3 times the standard deviation μ_{xy} of the coordinates, thus fully satisfactory.

The blocks 1 and 2 were first adjusted without additional parameters. The effect of the selfcalibration onto the absolute accuracy is quite different in both projects. In project 1 the absolute accuracy is increased by a factor 2 in the average (from 3.0 cm to 1.5 cm) and a factor 3 in the maximum errors (from 9.9 cm to 3.2 cm). Here obviously strong systematic errors were in the data which had to be compensated. The effect of the selfcalibration in the second project is negligible.

5.4 The following example is to show the precision, especially of distances, obtainable with colour film, which should be used for reliable identification. The single block consisting of 35 images covering an area of 8 km^2 was flown for the point determination in the land consolidation "Allmendingen". The image scale is 1:4000 corresponding to a flying height of appr. 600 m. The image coordinates were measured with a Zeiss comparator PK 1. The measuring accuracy estimated in the blockadjustment was $\hat{\sigma}_0 = 2.5\ \mu m \mathrel{\hat{=}} 1$ cm. 57 check points were available based on a geodetic traverse net, the standard deviation of the coordinates being appr. $\hat{\sigma}_k = 0.9$ cm.

The r.m.s. deviation between photogrammetric and geodetic coordinates is $\hat{\mu}_{xy} = 1.3$ cm, thus both methods have about the same precision. The deviations are plotted in fig. 2a. Nearly all deviations lie within a circle of 3 cm radius.

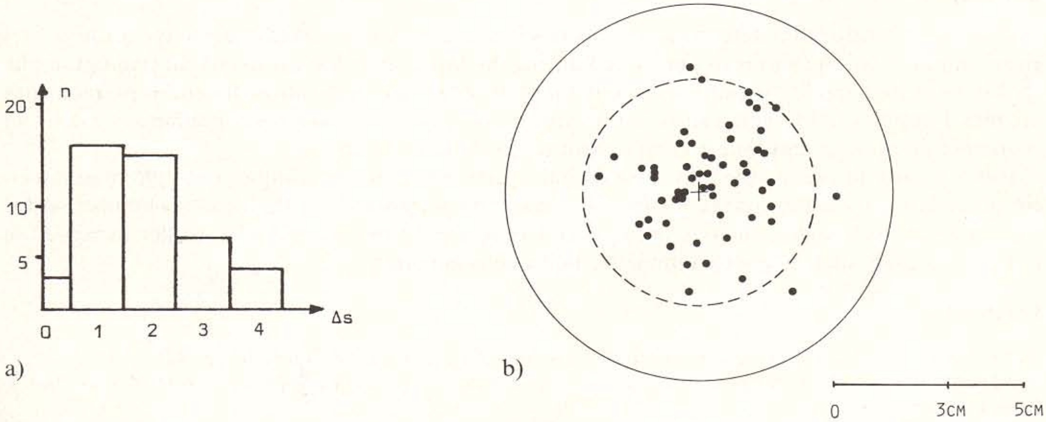

Fig. 2 Testblock Allmendingen
Photo scale 1:4000, single block, accuracy of terrestrial control and check points 0.8 cm
a) Residuals at 57 check points $\hat{\mu}_{xy} = 1.3$ cm, $\varepsilon_{max} = 3.7$ cm
b) Differences of 45 distances r.m.s. difference = 1.5 cm, maximum error 4.1 cm

Fig. 2b shows the histogram of 45 differences between geodetic and photogrammetric distances, ranging between 100 and 300 m. The average distance error is 1.5 cm, the maximum is 4.1 cm. As the precision of the geodetic distances is in the order of $\sigma_s = 0.4$ cm, the average distance difference of 1.5 cm reflects the errors of the photogrammetric measurements only. Thus also very small distances will be determined photogrammetrically with a precision of 1.4 cm or better. This corresponds to a relative precision of 5 ppm for distances of about 3 km. As the absolute precision is nearly independent on the blocksize, a relative precision of 3–5 ppm in large blocks easily can be achieved.

5.5 The last example is concerned with the empirical precision of digital image correlation which in future will replace human stereopsis. In contrast to the previous examples it compares the human and the computational stereo based on the rest parallaxes after relative orientation of image-pairs containing natural and targeted points. The same points were measured by an experienced operator and by using the digital correlator at the Zeiss Planicomp C 100 (cf. PERTL, 1984; ACKERMANN, 1984). The internal, i.e. theoretical standard deviations for the parallaxes derived from the correlation were less than 1 μm for targeted and less than 1.4 μm for natural points. The estimated standard deviations σ_0 ($\sigma_0^2 = \sum vvp/(n-5)/2$) from relative orientations are shown in Table 4, they also contain the errors of the instrument and the film. The values are in full accordance with the results shown in the previous examples and demonstrate digital correlation to be equivalent to human stereopsis.

Table 4 Standard deviations σ_0 derived from y-Parallaxes after relative orientation

σ_0 [μm]	natural points	targeted points
Operator	3.0	2.2
Digital Correlator	2.7	2.6

6. Conclusions

Photogrammetric point determination has reached a level which makes it attractive for high precision application. Various tests performed during the last decade have proved that photogrammetry can meet high quality requirements, e.g. those of network densification, if certain prerequisites are met. For point fields with a sufficiently large number of new points photogrammetry can be an economic alternative and thus a complement to geodetic methods.

It is too early to predict the impact of digital image processing techniques onto photogrammetric procedures, including image analysis for mapping purposes. But the results obtained so far with classical techniques, such as image correlation, are promising and will further increase the potential of high quality photogrammetric point determination.

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Some Remarks on Duality between Fourier Series and Discrete Fourier Transforms

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Summary

There are close connections between the common used Fourier series and discrete Fourier transforms, which can be used to substitute each other. Besides the analytical description of both procedures two examples demonstrate the applicability and effectiveness of the discrete Fourier transform.