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ON THE MORPHOLOGICAL QUALITY OF DIGITAL ELEVATION MODELS

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Summary:
The paper discusses the morphological quality of digital elevation models (DEM). Quality is understood as the precision and the reliability of the height, the slope and the curvature at interpolated points. Whereas precision is described by the standard deviation, reliability - according to Baarda - describes the effect of incorrect heights or incorrect assumptions about the type of the terrain onto the interpolated DEM.

First the influence of the sampling intervall onto the representation of the morphology of profiles with different spectra is discussed. It is shown that the sampling intervall leading to a preset relative height fidelity is not sufficient to reach an acceptable representation of the slope or even the curvature of the terrain, provided all frequencies are of equal interest.

Therefore the effect of additional form measurements (slopes and curvatures) onto the quality of the interpolated DEM is investigated. Using the method of finite elements it is shown, that additional measurements of slopes lead to an increase of precision and reliability of appr. a factor 1.4, thus the maximum influence of nondetectable errors is decreased by a factor 2.

It is shown that in addition to the power spectrum the distribution of the modelling stochastic process is decisive for the average sampling density, at the same time suggesting to sample the terrain by data compression using form elements.

1. Introduction

1.1 Digital elevation models (DEM) have recently become subject of intensive research. This is caused by the increasing need of digital information about the topography in cartography, civil engineering or remote sensing. DEMs are used for generating contour maps, for determining volumes or for rectifying aerial photos or scanner images. There exist quite a number of powerful computer programs which are able to produce a DEM from irregularly distributed terrain data including breaklines,spot heights or structure lines.

A DEM consists of a data base containing discrete information about the height (or any other feature) of a surface and a specification how to interpolate between the stored points. The quality of the DEM depends on 1. the representation of the true surface (i. e. the one that was meant when it was measured) and 2. the ability of the interpolation method to approximate the surface.Representation and interpolation interrelate; this interrelation however is not yet investigated.

1.2 The generation of a contour map, or any other product, using a DEM can be interpreted as an information transfer from the surface to the map (cf. Makarovic, 1972). Using the terminology of information theory (cf. Shannon, Weaver, 1949) the terrain surface is the information source (cf.

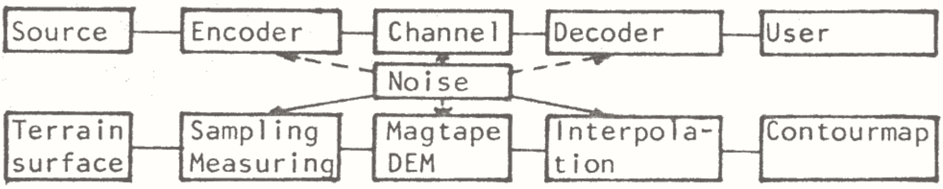


Fig. 1 Information transfer using a DEM

fig. 1) which is encoded by selecting and measuring certain points or features. The measuring errors and the nonideal interpolation correspond to additional noise in the channel, while the decoding consists in interpolation possibly with filtering to suppress the noise.

Obviously the interpolation (the decoding) cannot be done indepentently of the sampling. But what is more important, the sampling (the encoding) is decisive for the information which reaches the user. This already was stated by Makarovic in 1972, without referring to information theory.

Nevertheless, geodetic research has nearly exclusively concentrated on the problem of interpolation and on filtering. Again Makarovic was the first who treated the problem of an adequate sampling (1973), well known as progressive sampling which is combined with linear interpolation. His idea is to sequentially densify a regular grid in dependency of the local roughness of the terrain determined from the already measured heights.

1.3 The evaluation of both, the sampling and the interpolation methods, up to now has been based on the height precision of the reconstructed terrain surface (Clerici, Kubik 1975; Rüdenauer, 1980; Ackermann, 1980; Tempfli, 1982 e. g.). Several applications especially in cartography, hydrology of geomorphology however require morphologically accurate information. This paper wants to investigate the morphological quality of DEMs including the reliability of the interpolated points, especially discuss the value of additional form measurements and wants to stimulate the discussion on sampling methods which are not based on a regular point pattern and on height measurements alone.

The paper consists of three sections. Section 1 deals with the theoretical dependency of the morphological fidelity on the sampling intervall. It is based on the power spectra of the height, the slope and the curvature of profiles and on the transferfunction of the reconstruction procedure following the approach of Tempfli (1982). As an increase of the point density does have only little influence on to the fidelity of the morphology, section 2 investigates the influence of additional form measurements onto the precision and the reliability of the interpolation. Section 3 shows that for terrains with identical power spectra the type of the distribution of the underlying stochastic process is decisive for the average point density suggesting to sample the terrain by data compression.

2. Morphological fidelity of profiles

Fidelity can be measured by the standard error between the true and the reconstructed profile. If one can assume homogeneity the fidelity can be determined from the power spectrum and the transferfunction of the reconstruction procedure, provided it is linear (cf. Tempfli, 1982, Frederikson, Jacobi, Justesen, 1978). This approach does not necessarily need a stochastical model for the description of the terrain surface, it is even purely deterministic if measuring errors are not taken into account, as will be done in this section.

As the phase information of the terrain, contained in the amplitude spectrum, is lost in the power spectrum or - what is equivalent - the distribution of the modelling stochastic process alternatively describing the surface does not influence the power spectrum, the approach does not use the entire information about the terrain. This will be demonstrated in section 4. This section, however will stricty follow the linear filter approach, based on a stochastical model for the terrain surface.

2.1 Power spectra of terrain profiles

Extensive studies by Frederikson, Jacobi and Justesen (1978) have shown that power spectra of terrain profiles are linear in a wide range if the power $P(u)$ and the frequency u are measured in a logarithmic scale, i.e.

$$\log P(u) = \alpha \log(u) + \beta , \qquad u_1 < u < u_2, \tag{1}$$

where the parameter α describes the distribution of the energy between the high and low frequencies and β determines the general roughness of the profile. The slope of the power spectra has been found to range between -2 and -8. Of course eq.(1) has to be restricted to a frequency band, at least for empirical spectra, as $P(u)$ will be limited for low frequencies and will not tend to zero for large frequencies due to measuring errors. This is in full agreement with the empirical findings.

Jacobi and Kubik 1982 have proposed the model of fractional noise for a description of terrain profiles, explaining the linear behaviour of the power spectra and allowing the parameter α to vary continuously. We will use a different model in order not only to explain the phenomen terrain surface but also have an interpolation method at hand. There surely will also be an interpolation method based on the model of fractional noise but it seems that it will be more complicated.

We assume the terrain to be adequately describable by a markov process, especially an autoregressive (AR) process of order p. The i-th height g_i of a regularly sampled profile then depends on the p previous heights

$$g_i = -\sum_1^p a_k\, g_{i-k} + \varepsilon_i , \qquad i = 0, \pm 1, \pm 2, \ldots \tag{2}$$

and deviates from the prediction, the sum in eq. (2), by the generating process $\{\varepsilon_i\}$, which usually is assumed to be white, i. e. consisting of identically distributed uncorrelated random variables with variance σ^2. The power spectrum of the autoregressive process is given by

$$P(u) = \frac{\sigma^2}{\left|1 + \sum_1^p a_k \exp(-j2\pi u\Delta x)\right|^2}, \tag{3}$$

where Δx is the sampling intervall.

Eq. (2) contains several special cases which will be used furtheron. The processes

$$\begin{aligned} &AR(1): \quad a_1 = \gamma_1 \\ &AR(2): \quad a_1 = 2\,\gamma_2,\ a_2 = -\,\gamma_2^2 \\ &AR(3): \quad a_1 = 3\,\gamma_3,\ a_2 = -\,3\,\gamma_3^2,\ a_3 = \gamma_3^3\ , \end{aligned} \tag{4}$$

are p-fold integrated white processes if $\gamma_p = 1$. They are nonstationary. The second order process AR(2) with $\gamma_2 = 1^p$ is the basis for Ebners interpolation method with finite elements (1979). The ε_i can be interpreted as curvature of the profile.

For the investigation in this section we use the continuous version of eq. (2) with the coefficients of eq. (4) leading to the power spectrum (cf. Gelb, 1974) ($a=\gamma$)

$$P_g(u) = \frac{P_o}{(1 + (au)^2)^p}\ . \tag{5}$$

The reason is twofold. Eq. (5) is easier to handle than eq. (3) and is a spectrum of type eq. (1) with slope $\alpha = -2\,p$. Thus at least for even values of α empirical spectra can be explained by an autoregressive process of order $-\alpha/2$.

In case the power spectrum is composed of linear parts with different slopes and positive curvature it can be thought of being the sum of AR-processes of different order, similar to the idea of Kubik and Botman (1976) who composed the covariance function (the inverse fourier transform of the power spectrum) by additive components.

2.2 Power spectra of slope and curvature

From the representation of a continuous profile $g(x)$ by its amplitude spectrum $G(u)$

$$g(x) = \int G(u)\ \exp(j2\pi ux)\ dx \tag{6}$$

one can derive the amplitude and the power spectra of the 1st and 2nd derivative $g'(x)$ and $g''(x)$ resp.. With $P(u) = |G(u)|^2$ one obtains

$$\begin{aligned} P_{g'}(u) &= 4\,\pi^2\,u^2\,P_g(u) \\ P_{g''}(u) &= 16\,\pi^4\,u^4\,P_g(u). \end{aligned} \tag{7}$$

For a continuous markov process of order 2 with the coefficients of eq.(4) one then obtains the power spectra for the height, the slope and the curvature

$$P_g(u) = \frac{P_o}{(1+(au)^2)^2}\ ,\quad P_{g'}(u) = \frac{u^2\,P_o'}{(1+(au)^2)^2}\ ,\quad P_{g''}(u) = \frac{u^4\,P_o''}{(1+(au)^2)^2}\ . \tag{8}$$

The power spectrum $P_{g''}(u)$ of the curvature is not integrable and therefore no fourier spectrum (cf. fig. 2). We cannot use it for the following analysis. For demonstration purposes we therefore apply the power spectra

$$P_g(u) = P_o\ e^{-au}\ ,\quad P_{g'}(u) = P_o'\ u^2\ e^{-au},\quad P_{g''}(u) = P_o''\ u^4\ e^{-au} \tag{9}$$

which possess an exponential term, thus all being integrable (cf. fig 2b). This type of power spectrum is used to describe digitized images, but is not found for terrain profiles.

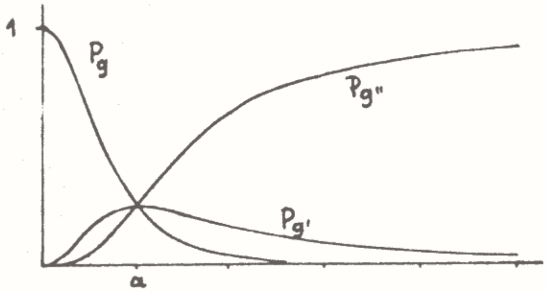


Fig. 2a
Power spectra of AR(2)-process and its derivatives eq. (8)

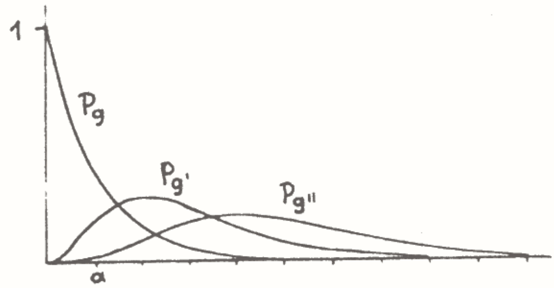


Fig. 2b
Exponential power spectra eq. (9)

2.3 Transfer function of reconstruction

Any linear reconstruction method can be described by its transfer function $H(u)$. For its construction one has to consider several steps (cf. fig 3).

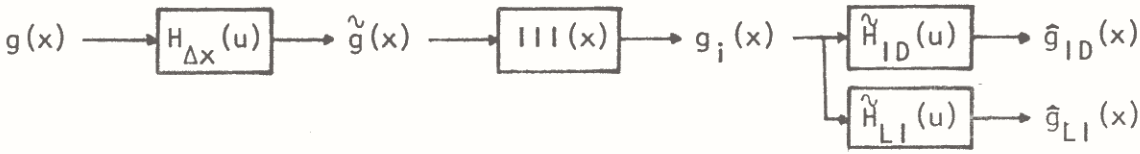


Fig. 3 Reconstruction method, ID = ideal interpolation, LI = linear interpolation, $H_{\Delta x}(u)$ ideal low pass filter, $III(x)$ ideal sampling

The terrain is not bandlimited. This causes aliasing effects during sampling which can be circumvented by previous low pass filtering, i. e. by smoothing. We assume an ideal low pass filter with transfer function

$$H_{\Delta x}(u) = \begin{cases} 1, & |u| < u_o = 1/2\Delta x \\ 0, & \text{elsewhere} \end{cases} \tag{10}$$

The sampling is assumed to be ideal. Thus the smoothed profile $\tilde{g}(x)$ is measured at evenly distributed point with an intervall of Δx and without measuring errors, leading to the sampled profile $g_i(x)$. For comparison reasons we apply two different interpolation methods, ideal interpolation with the sinc-function and linear interpolation (ID and LI resp.). Whereas the ideal interpolation is restricted to frequencies $< u_o$, linear interpolation is not. In order to keep the analysis simple we only consider the transfer function within the range of $-u_o$ and u_o.

Altogether this leads to the transfer functions

$$H_{ID}(u) = H_{\Delta x}(u)$$

and (11)

$$H_{LI}(u) = si^2 \frac{\pi u}{2u_o}, \qquad |u| < u_o$$

with si x = sin x / x.

Using Parsevals theorem the fidelity of the reconstruction can be determined by

$$\sigma_r^2 = \int_{-\infty}^{\infty} (g(x)-\hat{g}(x))^2 dx = \int_{-\infty}^{\infty} (1-H(u))^2 P(u) du \qquad (12)$$

(cf. Tempfli (1982), eq. (15)). For the ideal interpolation eq. (12) leads to

$$\sigma^2_{r,ID} = 2 \int_{u_o}^{\infty} P(u)\ du\ . \qquad (13)$$

In order to be able to solve the integrals analytically the complement $1-si^2(\pi u/2u_o)$ of the transfer function $H_{LI}(u)$ is approximated by $(u/au_o)^2$ with $a=1.3$ (from $si^2(\pi/2)=(1/a)^2=0.41$). One then obtains

$$\sigma^2_{r,LI} = 2 \int_0^{u_o} (u/au_o)^2\ P(u)\ du + \sigma^2_{r,ID} \qquad (14)$$

The actual used transfer functions are shown in fig.(4).

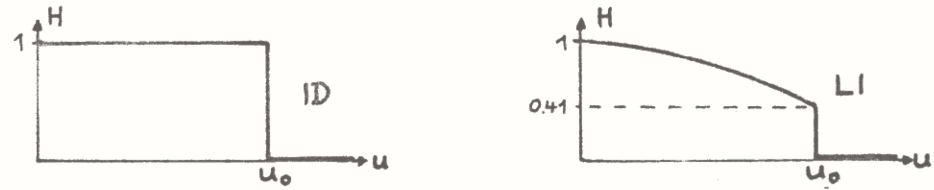


Fig. 4 Used transfer functions of reconstruction method

2.4 Fidelity of reconstructed profiles

The fidelity of the reconstructed profiles will be determined using the power spectra eq. (8) and (9) and will be given in terms of relative rmse. With the standard deviations σ_g, $\sigma_{g'}$ and $\sigma_{g''}$ of the heights, the slopes and curvatures we determine the relative fidelities or fidelity ratios

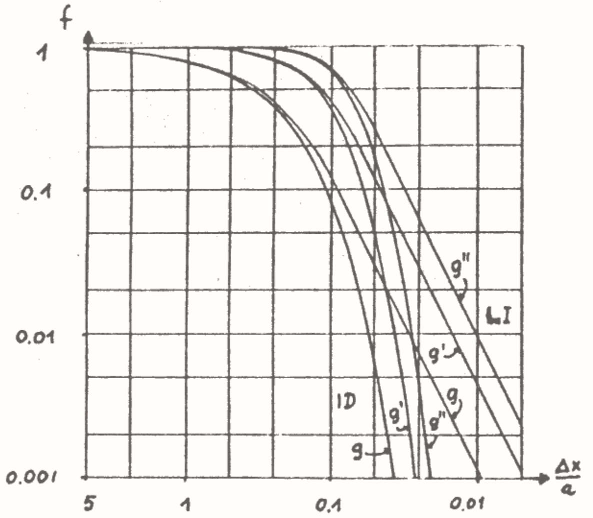
$$f_g = \sigma_r/\sigma_g\ , \quad f_{g'} = \sigma_{r'}/\sigma_{g'}\ , \quad f_{g''} = \sigma_{r''}/\ \sigma_{g''} \qquad (15)$$

where $\sigma_{r'}$ and $\sigma_{r''}$ are derived from eq. (12) using the power spectra of the derivatives.

We first discuss the results of the exponential power spectra. Fig. 5a shows the fidelity ratios f_g, $f_{g'}$ and $f_{g''}$ for ideal and linear interpolation in dependency of the sampling intervall Δx in a doubly logarithmic scale.

A qualitative interpretation can be summarizes as follows:

1.) Linear interpolation leads to worse results than ideal interpolation, the difference increasing with the sampling density.
2.) The relative fidelity of the slope is much less than the relative fidelity of the height by a factor 4 - 10 depending on the type of inrepolation. The fidelity of the curvature again is worse than that of the slope by a factor 2 - 5.
3.) The differences between the relative fidelity measures are nearly independent of the sampling intervall, except for very large Δx.
4.) The relative fidelity increases exponentially if ideal interpolation is used, whereas it increases with the square of the sampling intervall, if linear interpolation is applied.



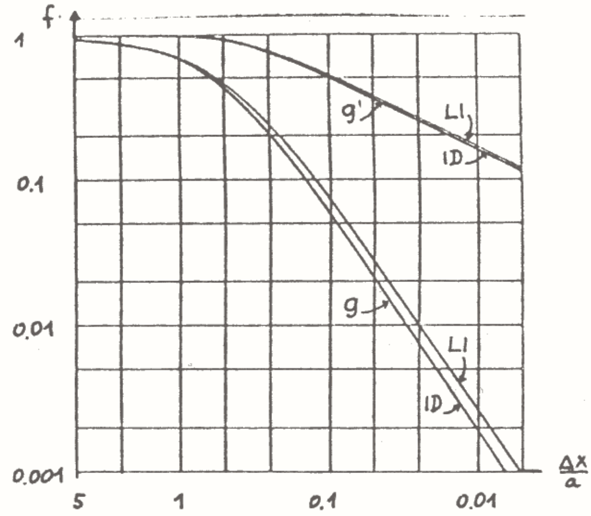


Fig. 5a Fidelity of height, slope and curvature for profiles with power spectrum eq. (9)

Fig. 5b Fidelity of height and slope for profiles with power spectrum eq. (8)

The interpretation of the relative fidelity for the 2nd order markov process, shown in fig. 5b yields the following results:

1.) Linear interpolation is slightly worse than ideal interpolation, differing by 25 % for the height and only by 10 % for the slope. The difference between the fidelities of the two reconstruction methods however does not depend on the sampling intervall, at least for smaller Δx.
The results referring to the height already have been found by Kubik and Botman (1976), there based on processes with exponentially decreasing covariance functions, being characteristic for markov processes. One of them corresponds to a 1st order markov process.
2.) The relative fidelity of the slope is much less than the fidelity of the height. The factor ranges up to 100 in the investigated band. The curvature could not be analysed as already mentioned (but see remarks below)
3.) The dependency of the fidelity reflects the type of power spectum, the fidelity of the height increasing with $\sqrt{\Delta x^3}$, the fidelity of the slope increasing with $\sqrt{\Delta x}$. Thus the qualitatively different result obtained for the exponential power spectrum is specific for that case.

The results have shown that the type of dependency of the fidelity on the sampling distance is governed by the relation between power spectrum and reconstruction method. The square term in the transfer function of the linear interpolation prohibits an exponential increase of the fidelity. On the other hand profiles with exponential power spectrum allow a rather good reconstruction also of form elements. Though this type of spectrum has not been found for terrain surfaces, the results are interesting for a thouroughinterpretation of the quality of different interpolation methods.

Example:
Assume, a profile with power spectrum eq. (8) has to be reconstructed with a relative fidelity of f_g=1 % or 1 ‰. Then the average sampling distance has to be $\Delta x = a/32$ or $\Delta x = a/150$, where a is the charakteristic length of the terrain. With this spacing one finds $f_{g'} = 0.38$ and $f_{g''} = 0.13$ resp..Thus the standard deviation of the slope, due to recon-

struction is 38 % or 13 % of the standard deviation of the total profile. This immediately shows the weakness of the reconstuction of form elements. Of course the reason is not the chosen reconstruction method but the type of profile.Moreover, increasing the sampling rate by a factor 5 to yield a 10 times better height representation only leads to a poor increase of the slope fidelity. ■

Curvature cannot be reconstructed at all, if all frequencies are taken into account. This is to be expected for 2nd order markov processes, which in the limiting case $\gamma_2 = 1$ are doubly integrated white noise processes, which have no bounded energy and can be interpreted as the curvature of the integrated process.

If however the user specifies an upper frequency u_o (or a lower bound for the sampling distance) then it is possible to reconstruct the curvature. But then it is necessary to measure with the given sampling density, as the fidelity $f_{g''}$ of the curvature proportionally depends on the deviation from u_o.

The results can easily transferred to processes with order 1 or 3. First order processes possess a nonintegrable power spectrum for the slope and therefore describe locally rough terrain. Thus the results found for the curvature of second order processes above are then valid for the slope. On the other hand , third order processes which are useful for smooth terrain have an integrable curvature spectrum. Therefore the findings for the slope of secondorder processes are then valid for the curvature.

Generally the fidelity values decrease with the sampling intervall dependent on the order p of the process as follows:

$$f_g \sim \Delta x^{(2p-1)/2}$$

$$f_{g'} \sim \Delta x^{(2p-3)/2}$$

$$f_{g''} \sim \Delta x^{(2p-5)/2}$$

The figures given by Ackermann (1980) suggest that the investigated terrains , with the exception of the difficult terrains, can be approximated by markov processes with an order of at least 2. This probably will also be caused by the fact, that breaklines were never neglected during interpolation.

Main conclusions of this section are:
- It is difficult to reach a heigh fidelity for the morphological reconstruction of terrain profiles by increasing the sampling density. Especially a large increase of the sampling density leads only to slight improvements of the slope fidelity.
- The curvature of profiles only can be reconstructed in smooth surfaces, describable by marcov processes of order 3 or larger, or if the user specifies an upper frequency of interest, i. e. the size of the smallest terrain feature to be reconstructed. The consequences, a sampling density which generally is too high, are not encouraging to follow this line of thought.

The next section therefore investigates whether additional form measurements increase the morphological quality of profile reconstruction.

3. Quality of interpolation

The quality of interpolation usually is given in terms of the precision of interpolated heights. We will extend the notion and - according to Baarda - also discuss the reliability of the interpolated points. Reliability is described by the controllability of the observations (internal reliability) and the sensitivity of the result with respect to errors in the observations or the mathematical model (external reliability). The evaluation of the reliability thus requires redundant information and the interpolation process to be an adjustment procedure. Therefore we use the finite element method according to Ebner (1979).

3.1 Mathematical model

The interpolation is based on observations of different type which determine the u unknown heights $\hat{g}_i$ of a regular pattern of grid points corresponding to a constant sampling distance.

a) The heights g_i in the given points, also referred as nodes, in this investigation always coincide with a grid point. They lead to the error equations

$$g_i + v_{g_i} = \hat{g}_i , \qquad i = 1, \dots , n_g \; ; \sigma_g \qquad (16)$$

b) The ficticious curvature observations c_i connect three adjacent grid points

$$\underset{\equiv 0}{c_i} + v_{c_i} = \hat{g}_{i-1} - 2\,\hat{g}_i + \hat{g}_{i+1} \qquad i = 1, \dots , n_c \, , \sigma_c \qquad (17)$$

The value of c_i is zero. (17) reflects the underlying AR(2) process eq. (2) also used in sect. 2.4, with the expectation of the generating process to be zero. The results have to be compared with those of sect. 2.4..

We now introduce two types of additional observations, extending the original approach of Ebner.

c) Additional slope measurements s_i lead to

$$s_i + v_{s_i} = -\hat{g}_{i-1} + \hat{g}_{i\;1} \qquad i = 1, \dots , n_s \, , \sigma_s \qquad (18)$$

d) Additional real curvature measurements, designated with k_i, yield

$$k_i + v_{k_i} = \hat{g}_{i-1} - 2\,\hat{g}_i + \hat{g}_{i+1} \qquad i = 1, \dots , n_k \, , \sigma_k \qquad (19)$$

The standard deviations σ_g, σ_s and σ_k have to be chosen according to the measuring accuracy. The standard deviations σ_s and σ_k also can be used to impose constraints onto the profile by setting $\sigma_s = \sigma_k = 10^{-6}$ (zero is not possible for numerical reasons). The standard deviation σ_c represents the roughness of the terrain with respect to the chosen intervall between the grid points. For the real and the fictitious observations the standard deviations may also be determined using variance estimation procedures.

The error equation system eq. (13)-(19) has redundancy $r=n_g+n_c+n_s+n_k-u$ thus can be solved using least squares technique.

The <u>precision</u> of the interpolated heights $\hat{g}_i$ can be determined from $\sigma_{\hat{g}_i} = \sigma_o \sqrt{q_{ii}}$. Similarily on can obtain thestandard errors $\sigma_{\hat{g}'_i}$ and $\sigma_{\hat{g}''_i}$ of the slope and the curvature at the grid points by error propagation.

The reliability of the interpolation mainly depends on the distribution of the redundancy r onto the observations, namely the redundancy numbers $r_i = 1 - \sigma^2_{\hat{g}_i} / \sigma^2_{g_i}$, with $\Sigma r_i = r$. Small redundancy numbers (< 0.1) indicate a low controllability of the observations. The complement $u_i = 1 - r_i$ is the contribution of the ith observation to the unknowns, with $\Sigma u_i = u$. Therefore small redundancy numbers, thus large u_i-values indicate, that this observation is decisive for the interpolation. The external reliability of the result can be evaluated using the sensitivity factors $\overline{\delta}_{oi} = \delta_o \sqrt{u_i / r_i}$ ($\delta_o = 4$). The influence ∇x of nondetectable errors onto the unknowns $\hat{g}_i$ is bounded by $\nabla x \le \overline{\delta}_{oi}\, \sigma_x$. The sensitivity values should be less 10 to guarantee a reliable result which is not distorted by errors in the observations or incorrect assumptions in the mathematical model.

The idea behind this notion of reliability is to check the residuals with respect to gross errors in the observations. A preset lower bound β_o for the power of the test leads to lower bounds for just detectable gross errors. The maximum influence of nondetectable gross errors onto the result is a measure for the sensitivity of the result. The noncentrality parameter δ_o (=4) depends on the critical value and the power β_o of the test (k=3.3, β_o=80 % used, cf. Förstner (1982)).

It has to be pointed out that also the fictitious observations c_i can be evaluated. Thus it can be checked whether the assumption $c_i = 0$ is justified.

For the following investigation the standard deviations

$$\sigma_g = 0.3 \text{ m}, \quad \sigma_c = 0.5, \quad \sigma_s = 0.21 \text{ m and } \sigma_k = 0.33 \text{ m}$$

are used. σ_g and σ_s are the resùlt of a variance estimation (Lindlohr, 1982) using a photogrammetric profile sampled with an intervall of 10 m in a scale of 1 : 10 000. The variances for the additional measurements are assumed to be appr. a factor 1.4 better. As has been tested, the results are not much effected by using a slightly different choice. The The sampling of the profile, simulated in the study, varies from i = 2 Δx = 20 m to i = 8 Δx = 80 m.

3.2 The influence of additional form measurements onto the precision at interpolated points

Fig. 6 shows the precision of the heights, the slopes and the curvatures for a profile with 6 nodes having a distance of i = 8 Δx, thus with 49 points. For symmetry reasons only the left half of the profile is shown. The 4 precision curves correspond to the following 4 cases

0:	only height measurements	(g)
1:	additional slope measurements	(g, g')
2:	additional curvature measurements	(g, g'')
3:	additional slope and curvature measurements	(g, g', g'').

The measurements are assumed to be performed in the nodes only.

The figure allows the following statements:

- The height precision of the interpolated points increases rapidely with their distance from the nodes. The maximum standard deviation lies in the middle between the nodes. The influence of additional slope measurements (case 1 and 3) increase the height precision significantly appr. by a factor 1.5. On the otner side curvature measurements have only little influence onto the height precision.

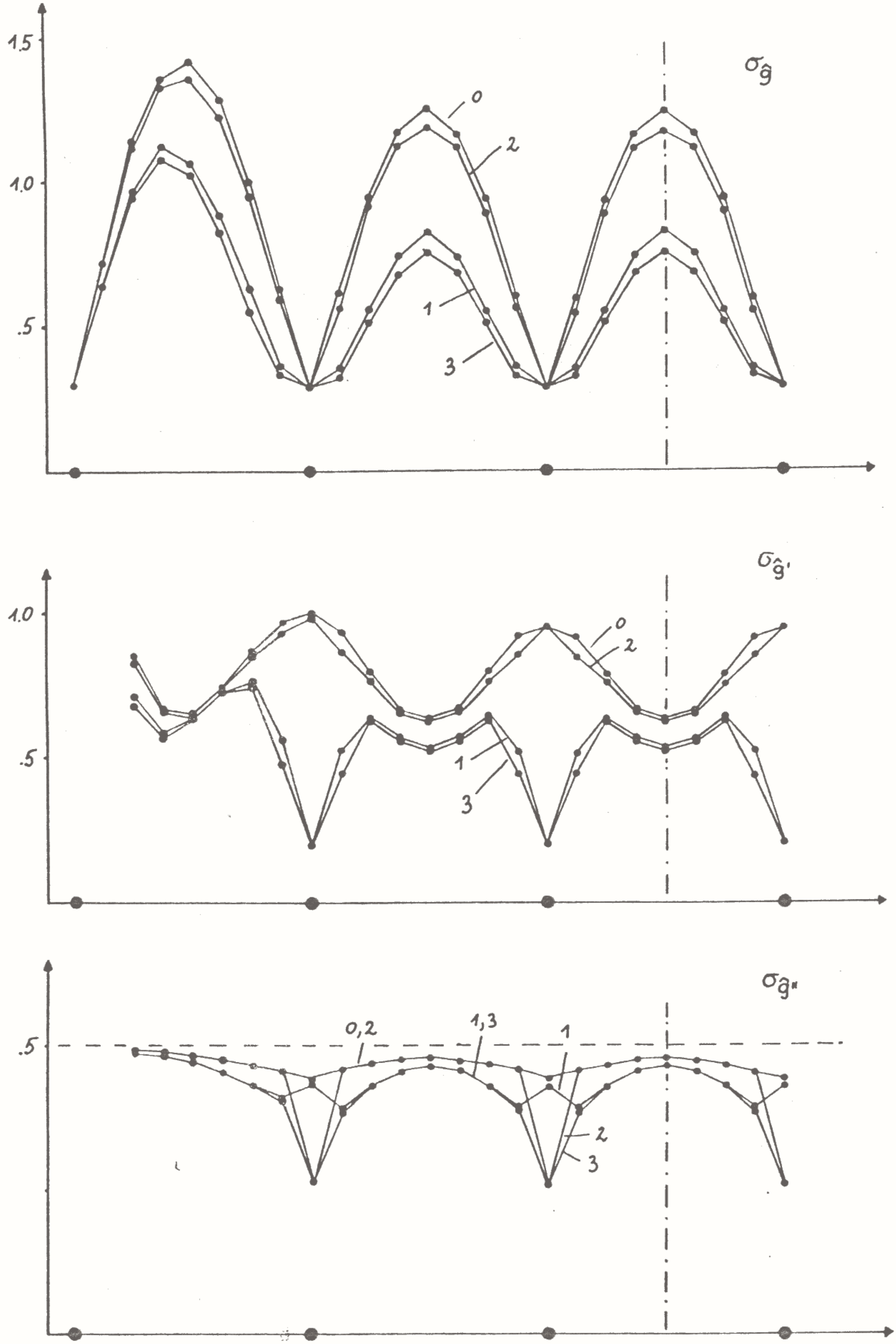


Fig. 6 Precision of height slope and curvature at interpolated points sampling intervall $i = 8\ \Delta x$, influence of additional form measurements (slopes:1 and 3, curvatures: 2 and 3)

- The slope precision at the interpolated points increases with their distance from the nodes, the minimum standard deviation lying in the middle between the nodes. Additional slope measurements improve the slope precision only in the vicinity of the nodes, the improvement of the precision in the middle between the nodes is only 15 %. Again the curvature measurements have practically no influence.
- The precision of the curvature at the interpolated points shows only slight variations, in case only heights are measured (case 0). Additional slope measurements have only little, additional curvature measurements have only local influence onto the curvature precision. The overall precision is mainly determined by the fictitious measurments c_i.

3.3 The influence of the sampling distance onto the quality of interpolation

Fig. 7 shows the standard deviations and the sensitivity factors in dependency of the sampling intervall. The standard deviations refer to the points in the middle of the investigated profile, thus represent the maximum[1] standard error of interpolation. The sensitivity values refer to the three types of observations at the nodes.

The results obtained for $i = 8\ \Delta x$ are obviously also valid for the other sampling distances.

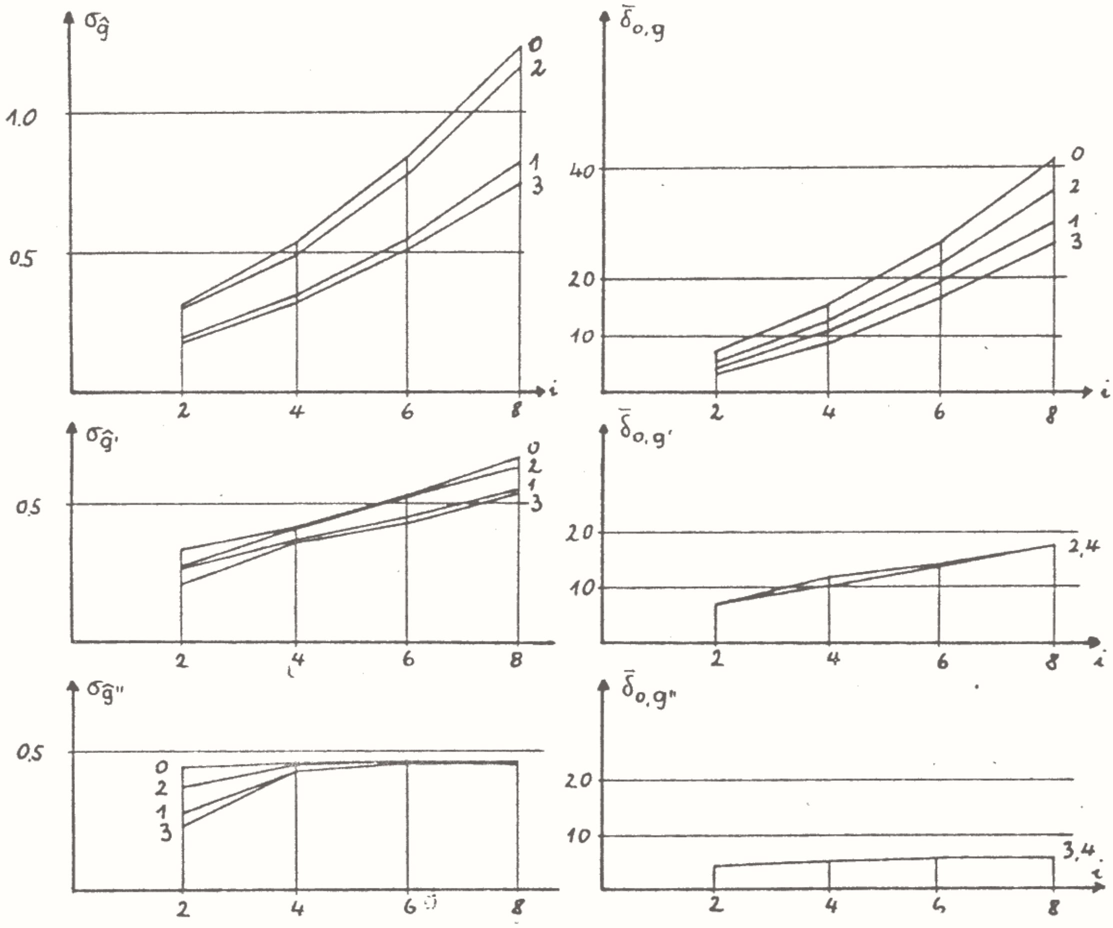


Fig. 7 Quality of interpolated profiles
Maximum[1] standard deviations and sensitivity factors for heights, slopes and curvatures in dependency of the sampling interval i (Δx)

1) and minimum resp.

Both quality measures of the heights increase with $i^{3/2}$ as expected (cf. Ackermann, 1965 and sect. 2.3). The theoretical dependency $i^{1/2}$ of the slope quality measures on the samling intervall does not show in the investigated range . The quality measures of the curvature are practically constant with the exception of the standard deviation for $i = 2$.

The standard deviations of the heights increase from 0.3 m for $i = 2$ to 1.3 m for $i = 8$. Compared with the given standard deviations of the observations this could be expected. The sensitivity values however are rather high. Already for moderate sampling distances ($i > 2$) they are not acceptable (> 10). Only for $i = 2$, i. e. one new point between two nodes the reliability is good ($\overline{\delta}_{oi} = 5$).

Example:
In order to visualize the quality improvement by additional form measurements, let us assume a sampling intervall $i = 4\ \Delta x = 40$ m. If only heights are measured the maximum standard deviation is 0.52 m. As the sensitivity factor is 15 the influence of nontetectable height measurements is less than 15 . 0.52 m = 8 m. With additional slope measurements we obtain a maximum standard deviation of 0.35 m and a sensitivity factor of 11. Thus nondetectable errors have an influence less than 3.8 m onto the profile. The additional slope measurements significantly improve the quality of the interpolated profile.

Of course increasing the density of the nodes by a factor two leads to an even better result with $\sigma = 0.31$, $\overline{\delta} = 7$ thus a maximum influence less than 2.1 m. ■

The investigation showed that additional slope and curvature measurements can improve the quality of the morphology and that slope measurements also increase the quality of the heights. The effects mainly are of local nature because of the poor quality transfer of the fictitious curvature observations. The findings are in full agreement with the results of sect. 2, where it was stated that for 2nd order processes the curvature of the terrain cannot be reconstructed from the height measurements.

This leads us to the meaning of the fictitious observations c_i. Their redundancy numbers range between 0.25 for $i = 2$, which is fully acceptable, to 0.08 for $i = 8$, which is not acceptable. As the precision of the adjusted profile curvature with appr. 0.45 m is in the order of the given standard deviation 0.5 features with slope changes smaller than 4-6 m over a distance of 10 m will be filtered away.

Besides the poor quality transfer this is a reason to measure form elements and integrate them into the interpolation proces (vis. Wild, 1980). In other words, the sampling has directly to reflect the terrain properties and should not be based on a preset sampling pattern. This is demonstrated in the next section.

4. On sampling of profiles

The last two sections have treated the dependency of the quality on the sampling density. In both cases a constant sampling spacing was assumed, which made the analysis tractable especially when using the power spectra of the profile and its derivatives. As already mentioned at the beginning of sect. 2 the power spectra do not contain the entire information about the terrain surface. More precisely, the findings of section 2 can only be used if the stochastical variables used to model the terrain are known to have a distribution which is identical for different terrains.

This section wants to give two counter examples to the theory of sect. 2 demonstrating the influence of the probability distribution of the generating process. They immediately lead to the problem of an adequate sampling.

4.1 Counterexamples to theory of section 2

The examples consist of two profiles each which have the same power spectrum but need a different number of sampling points for reconstruction.

Example 1:

The delta-function $\delta(x-x_o)$ and white noise with variance 1 both have power spectrum $P(u) = 1$. Obviously the delta-function needs only one value (x_o) to be defined, whereas white noise needs infinitely many values to be described.

The result of this example can be generalized, visualizing the reason for the difference of the two profiles with respect to the sampling density or what is equivalent the needed storage.

Given profile $g_1(x)$ with power spectrum $P_1(u) = |G_1(u)|^2$. $g_1(x)$ may be deterministic. Filtering white noise $n(x)$ with transfer function $H(u) = \sqrt{P_1(u)}$ yields a signal $g_2(x) = n(x) * h(x)$ with power spectrum $P_2(u) = P_1(u)$. $g_2(x)$ is also called couloured noise.

The difference between the two profiles in both cases is hidden in the phase of $G(u)$, which is decisive for the number of necessary values for reconstruction. Whereas the phase of the delta function is deterministic the phase of white or couloured noise is stochastic and evenly distributed in the intervall $|0, 2\pi|$.■

The construction principle is similar in

Example 2:

Fig. 8 shows two profiles A and B with 128 points each. Both are AR(2) processes with the same coefficients $a_1 = 1.6$ and $a_2 = -0.8$. Both processes are generated using sequences $\{\varepsilon_i\}$ of uncorrelated random variables with variance 0.5. According to equ. (3) they theoretically have the same power spectrum. This is shown in fig. 9. A closer look at the profiles reveils that profile A is rougher than B whereas B has more sudden changes of slope.

The difference lies in the distribution of the ε_i. They are given by

A:		B:	
$P(\varepsilon_i = 0)$	$= 0.5$	$P(\varepsilon_i = 0)$	$= 0.875$
$P(\varepsilon_i = 1)$	$= 0.25$	$P(\varepsilon_i = 2)$	$= 0.0625$
$P(\varepsilon_i = -1)$	$= 0.25$	$P(\varepsilon_i = -2)$	$= 0.0625$

The generating processes are given also in fig.8 below the profiles. Obviously the effort to describe profile A is higher than to describe B, i. e. profile A contains more information than B which can be proved using the results of information theory. Without going into details here it can be shown that on an average one needs less than half the storage to store profiles of type B compared with profiles of type A. This may be decisive if one thinks of storing DEMs for a whole country. ■

The essential quantity, besides the power spectrum, is the entropy of the generating process thus giving a clear guideline for an efficient sampling method. If a profile adequately can be described by an autoregressive process, whose parameters may be estimated from the data, only the genera-

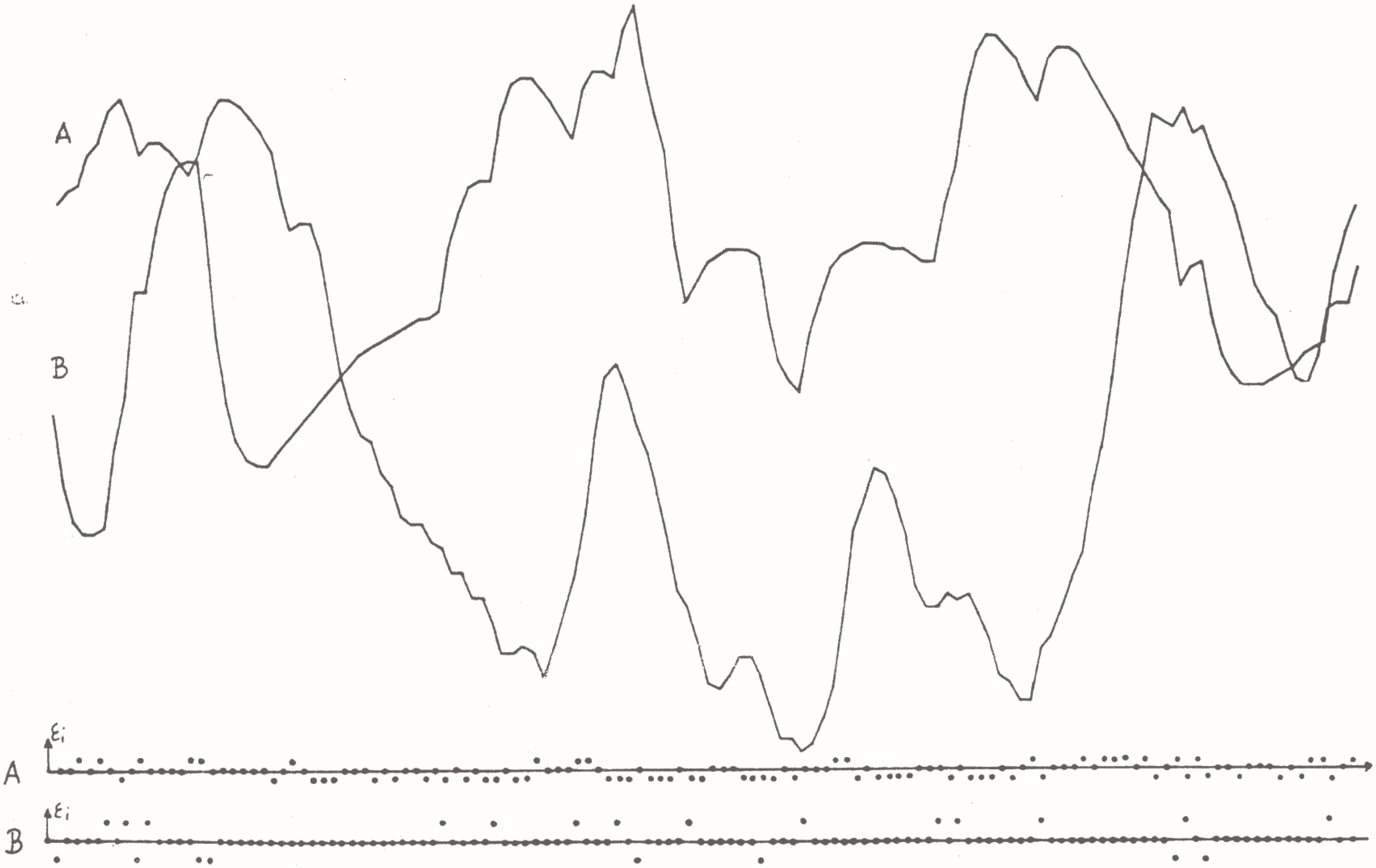


Fig. 8   AR(2) processes with identical coefficients ($a_1 = 1.6$, $a_2 = -0.8$) generated by white processes with variance 0.5, but with different distribution of the ε_i

ing process has to be stored. For processes of order p this essentially is the p-th derivative of the profile being specific form elements and leading back to the ideas of the previous sections.

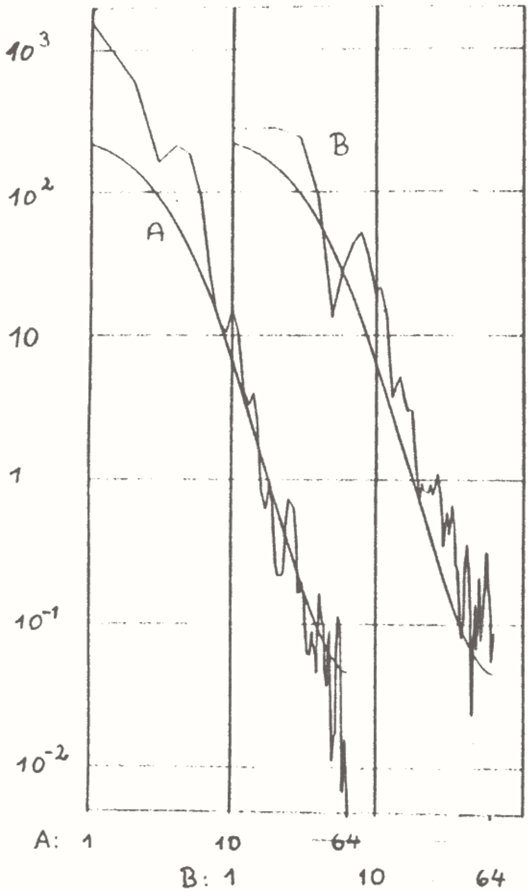


Fig. 9 Power spectra of profile fig. 8 together with the theoretical power spectrum.

The optimal distribution of the ε_i with respect to sampling (assuming the values are bounded) is shown in fig. 10. Values within the peak part of the distribution need not to be stored as they contain no information

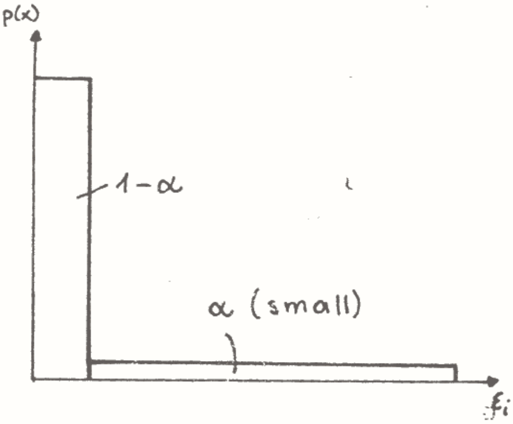


Fig. 10 Optimal distribution of ε_i for data compression

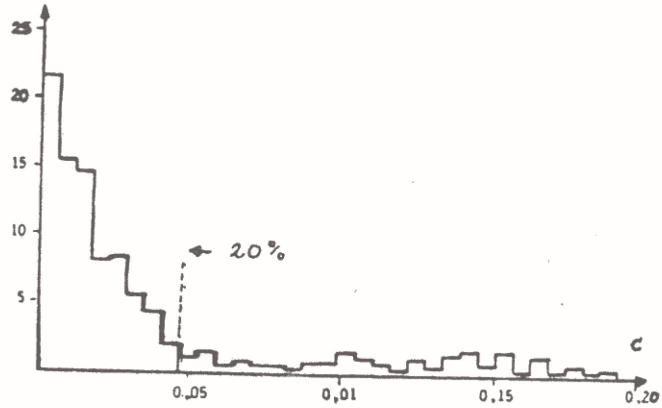


Fig. 11 Distribution of curvature (from Rüdenauer (1980))

as they are to be expected (Förstner, 1982). The same distribution has been independently proposed by Kubik (1982) for the cascade model, being optimal for sampling by data compression. The distribution of curvature elements of terrain profiles shown in fig. 11, which is given by Rüdenauer (1980) suggests the information theoretic approach to be realistic.

4.2 Practical considerations

Makarovic (1973) has classified the sampling method into three groups
1. selective methods, including selective sampling
2. methods using data compression, especially when using correlation devices to generate dense samples and
3. prograssive sampling.

Seen from the standpoint of information theory the selection of points by the operator during sampling and the methods using datacompression qualitatively are not dicernable, the main differenc being the objectivity of the selection. Both methods practically use the entire information of the terrain whereas progressive sampling only uses the information on a coarse grid (cf. Makarovic, 1979). Regarding the present possibilities of image correlation devices sampling by data compression could be an economical alternative to classical sampling procedures.

On the other side it seems to be attractive to use the intelligence of the operator for selective sampling methods which at the same time makes the measuring process more attractive. A loose guidance by the computer, scanning the model patchwise and leaving the final decision of the selection of necessary features to the operator is a compromise between a rigorous dictate by the computer and complete freedom for the operator guaranteeing the completeness of the sampling of the stereo model. It can be expected that in the near future analytical plotters will have correlation devices which will allow the direct measurement of form elements. Until then coding can be restricted to the type of form elements leading to morphologically better DEMs.

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