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SYSTEMATIC ERRORS IN PHOTOGRAMMETRIC POINT DETERMINATION

Wolfgang FÖRSTNER
Stuttgart, Federal Republic of Germany

ABSTRACT

The refinement of the functional model used for photogrammetric point determination has lead to a significant increase of the accuracy, being about 3-8 µm at photoscale. It is discussed how the functional or the stochastical model may be further refined to compensate for varying, systematic effects and for local distortions which are caused by time-dependent changes of the flight and measuring conditions.

1. Introduction

1.1 During the last 15 years aerial triangulation has increasingly been applied for point determination as a basis for subsequent mapping and for network densification. This is mainly due to the use of rigorous adjustment procedures exploiting the potential of the highly developed hardware components especially films, cameras and measuring equipment. If a number of prerequisits are fullfilled (4-fold overlap, selfcalibration, targeted points, statistical test procedures) one can reach a precision of about 3-8 µm at image scale in all three coordinates and also with reliability values $\overline{\delta} \leq 5$ is able to guarantee for the quality of the result.

1.2 Photogrammetric and geodetic point determination have comparable characteristica. As an image represents a bundle of rays photogrammetry is a 3-dimensional and purely geometrical method using angular information only. Thus there is no separation of planimetry and height (as long as the bundle method is used) and no assumptions about the geoid are necessary. As a method for the densification of point fields it essentially depends on control points or at least some scale information, especially in close range applications. The feature of the image conserving the metrical information may be a reason to prefer photogrammetry in deformation analysis, provided the relative precision of 3-10 ppm. is sufficient.

1.3 The underlying mathematical model in most cases simply is the perspective relation between the image and the terrain points. The introduction of additional unknowns, i.e. the application of the socalled selfcalibration technique, for the compensation of systematic errors is widely applied and has proven to be effective. It has lead to a rather good coincidence between empirical results and theoretical prediction. This is astonishing, as the stochastical model is still oversimplified: In most cases the photogrammetric observations are assumed to be uncorrelated and of equal precision.

1.4 Though further refinements of the functional model by using different groups of parameters for each strip have lead to an increase of the precision, this approach is not satisfying, primarily because it is an ad hoc solution, which just argues analytically and does not reflect reality. But also the control of the stability of the system have caused serious problems demonstrating the imperfection of the attempt.

On the other hand any refinement of the stochastical model has to cope with numerical difficulties, which however can only be a short termed argument considering the future computer facilities.

But in both cases the justification of further refinements require comprehensive empirical tests which themselves have to be justified by the theoretically founded formulation of a group of competing hypothesis.

1.5 This paper is supposed to discuss the possibilities of refining the mathematical model of photogrammetric point determination. Section 2 gives a motivation, classifying the error sources within the photogrammetric measuring process with respect to the treatment in the mathematical model. Based on the theoretical influence of systematic errors onto the adjustment result section 3 deals with limitations of the estimation procedures in order to check the necessity of establishing certain types of hypothesis. In section 4 some empirical results about the effect of refined models onto the accuracy are compiled, especially considering the duality of functional and stochastical models. These results are used in section 5 for the formulation of two equivalent refinements of the mathematical model.

2. Error sources in photogrammetric point determination

2.1 The photogrammetric measuring process consists of several distinct steps each being influenced by physical effects which disturb the ideal geometry of the "perspective" model.

Table 1 lists the different stages and gives a classification of the type of error with respect to a possible subsequent refinement of the mathematical model discriminating

- constant systematic errors
- variable systematic errors

- correlations between different points
- correlations between the coordinates of one point
- variations of variance.

Table 1 Classification of types of error sources

	Systematic errors		correlation		
	const	var.	global	local	Variance
Object (point definition, illumination)		x		x	x
athmosphere (refraction)		x	x	x	
aeroplane (turbulence of athmosphere)		x	x	x	
objective (lens distortion)	x	(x)			
pressure plate (moving part)	x	x			
film (emulsion)			x	x	x
image motion	x			x	x
film development		x	x	x	x
copy		x	x	x	
measuring (comparator,contrast)	x			x	x
corrections	x	x			

Without going into detail table 1 demonstrates that most of the errors are varying with time, at least cannot be treated as constant, lead to correlations and to variations of the variance. Main effects are caused by

- the difference between calibration and real disposition
- the influence of the athmosphere and the film development
- the instability of the instruments, especially the pressure plate, which is the moving part in the camera.

Of course one must keep in mind the absolute size of the variations being only a few micron, but significantly larger than the pure measuring error.

2.2 Though most of the effects have been investigated (cf. the comprehensive report by Schilcher, 1980) no general physical model is available. This is due to the difficulty to study the interaction of the different effects under realistic conditions and even if this would be possible the calibration had to be performed for each project, requiring a reduction of the number of free parameters, which then scarcely would be separable.

2.3 This is the reason why test field calibration only aims at the combined constant part of all possible effects. Up to a great extent this also holds for the selfcalibration thechnique. But this attempt without much additional field work can be extended towards a more general analytical model for use in practice. The refinement does not necessarily have reflect the physics of the photogrammetric process completely, but rather can be set up considering aspects of performance as numerical stability, ability to estimate parameters or to evaluate the result in a simple manner. Of course the actual reasons for the deviations from the idealized model have to be investigated, e.g. in order to improve the instruments. But the development of practical procedures has to be done independently.

3. Theoretical considerations

The possibilities of refining the mathematical model cover a great range of special alternatives. In order to get information, which extensions can be checked, e. g. by statistical means, and which type of hypothesis are not discernable we investigate the influence of errors in the mathematical model onto the result. Only those systematic errors are necessary to be modeled which really may distort the result.

Notation: Small letters designate scalars and vectors, capital letters matrices, stochastical variables are underscored. Model errors are designated with a ∇ (nabla) in front of the variable.

3.1 Let the linearized model be given

$$\underline{l} = A\, x + \underline{\varepsilon}, \quad \underline{\varepsilon} \sim N(0,\, C_{\varepsilon\varepsilon}) \tag{1}$$

with the observation vector $\underline{l} = (\underline{l}_i)$, the design matrix A, the u unknowns x and the model errors $\underline{\varepsilon}$. Their covariance matrix $C_{\varepsilon\varepsilon}$ is assumed to be known. In order to compensate for systematic errors this model can be extended to the mixed model

$$\begin{aligned} &\underline{l} = A\, x + H\, \underline{s} + \underline{\varepsilon}, \\ &\underline{\varepsilon} \sim N(0,\, C_{\varepsilon\varepsilon}),\ \underline{s} \sim N(s_o,\, C_{ss}). \end{aligned} \tag{2}$$

The additional parameters $\underline{s}$ are treated as stochastical variables with an unknown mean s_o and a known covariance matrix C_{ss}. It is well known that this model has two equivalent formulations which can be used for practical parameter estimation.

The first one is useful, if $C_{\varepsilon\varepsilon}$ and C_{ss} are diagonal matrices:

$$\begin{aligned} &\underline{l} = A\, x + H\, s_o + H\, t + \underline{\varepsilon}, && \underline{\varepsilon} \sim N(0,\, C_{\varepsilon\varepsilon}) \\ &\underline{0} = \qquad\qquad\qquad\ \ t + \underline{\varepsilon}_t, && \underline{\varepsilon}_t \sim N(0,\, C_{ss}). \end{aligned} \tag{3}$$

The second one in general leads to a full covariance matrix of the observations:

$$\underline{l} = A\, x + H\, s_o + \underline{n}, \quad \underline{n} \sim N(0,\, C_{\varepsilon\varepsilon} + H\, C_{ss}\, H') \,. \tag{4}$$

As all three formulations are statistically identical thus lead to the same results, it is possible to replace the extended functional model eq. (3) by an extended stochastical model eq. (4) and vice versa.

Model eq. (3) is frequently used in photogrammetric block adjustment with $s_o = 0$; often the second group of observation equations is omitted treating the additional parameters as free unknowns. In some cases at least very low weights are given to these fictitious observations just in order to achieve a stable solution (cf. Kilpelä, 1980).

We will now discuss the influence of not detected errors in the mathematical model onto the estimated unknowns $\underline{\hat{x}}$, starting from the original model eq. (1).

3.2 Systematic errors $\nabla l = H\, \nabla s$ in the observations cause changes of the unknown $\underline{\hat{x}}$

$$\nabla x = (A'\, P\, A)^{-1}\, A'\, P\, H\, \nabla s \tag{5}$$

with $P = \sigma^{-2}\, C_{\varepsilon\varepsilon}^{-1}$. A scalar measure is

$$\bar{\delta}^2 = \nabla x'\, C_{xx}^{-1}\, \nabla x = \nabla s'\, H'\, P\, A\, (A'\, P\, A)^{-1}\, A'\, P\, H\, \nabla s \,/\, \sigma^2 \tag{6}$$

which describes the total deformation of the network (including orientation parameters). This deformation is zero only if $A'\, P\, H = 0$:

$$\boxed{\bar{\delta}(\nabla l) = 0, \quad \nabla l = H\, \nabla s \;\leftrightarrow\; A'\, P\, H = 0} \tag{7}$$

thus if the parameters are orthogonal to the unknowns.

3.3 Errors in the stochastical model do not influence the unbiasedness of the estimate $\underline{\hat{x}}$. But a wrong weight coefficient matrix

$$\bar{Q} = \sigma^{-2}\, \bar{C} = Q + \nabla Q = Q + H\, \nabla Q_{ss}\, H' \tag{8}$$

leads to a change ($\nabla Q \ll Q$, cf. Koch, 1980, p. 167)

$$\underline{\nabla x} = (A'\, P\, A)^{-1}\, A'\, P\, \nabla Q\, P\, (I - A\, (A'\, P\, A)^{-1}\, A'\, P)\, \underline{l} \tag{9}$$

in the estimates $\underline{\hat{x}}$. The expectation $E(\underline{\nabla x}'\, C_{xx}^{-1}\, \underline{\nabla x})$ of the total deformation then results in

$$\bar{\delta}^2 = E(\underline{\nabla x}'\, C_{xx}^{-1}\, \underline{\nabla x}) = \mathrm{trace}(P\, \nabla Q\, P\, Q_{\bar{l}\bar{l}}\, P\, \nabla Q\, P\, Q_{vv}) \tag{10}$$

with the weight coefficient matrices $Q_{\bar{l}\bar{l}} = A\, (A'\, P\, A)^{-1}\, A'$ and $Q_{vv} = Q - Q_{\bar{l}\bar{l}}$ of the adjusted observations $\underline{\bar{l}} = A\, \underline{\hat{x}}$ and the residuals $\underline{v} = \underline{\bar{l}} - \underline{l}$ resp..

With $\nabla P = P \nabla Q P$, the vec operator, which maps a matrix into a vector and the Kronecker product $\otimes$, eq. (10) can be written as a quadratic form

$$\overline{\delta}^2 = (\mathrm{vec}\nabla P)' \; (Q_{\overline{l}\overline{l}} \otimes Q_{vv}) \; \mathrm{vec}\nabla P \qquad (10a)$$

If furthermore ∇P is a diagonal matrix this expression can be simplified using the Hadamard product $A * B = (a_{ij} \; b_{ij})$ (cf. Pukelsheim, 1977)

$$\overline{\delta}^2 = (\mathrm{diag}\nabla P)' \; (Q_{\overline{l}\overline{l}} * Q_{vv}) \; \mathrm{diag}\nabla P \qquad (10b)$$

where diag ∇P is a vector only containing the diagonal elements of ∇P.

Example: Starting from eq. (10b) with $Q = I$ the low influence of single weight errors ∇p_i onto the coordinates is proved, as in this case $\overline{\delta}_i = \sqrt{r_i \; (1-r_i)} \; \nabla p_i < \nabla p_i/2$, $r_i = (Q_{vv} \; P)_{ii}$ being the redundancy number of the observation $\underline{l}_i$.

From eq. (10) we now again derive conditions for $\overline{\delta} = 0$ using the decomposition $\nabla Q = H \; \nabla Q_{ss} \; H'$, which may be interpreted as a neglected set of additional stochastical parameters $H \; \underline{s}$. The influence of errors in the covariance matrix onto the coordinates is zero if $\nabla Q \; P \; Q_{\overline{l}\overline{l}} = 0$ or $\nabla Q \; P \; Q_{vv} = 0$. The interpretaion of the second condition is simplified if we write the functional model in terms of condition equations, i. e. according to standard problem I in the terminology of Tienstra: $U' \; \underline{l} = \underline{w}$. Then $U' \; A = 0$ and $Q_{vv} = Q \; U \; (U' \; Q \; U)^{-1} \; U'Q$. The conditions for $\overline{\delta} = 0$ then read as

$$\boxed{\overline{\delta}(\nabla Q) = 0, \quad \nabla Q = H \; \nabla Q_{ss} \; H' \;\leftrightarrow\; \begin{array}{ll} 1. & A' \; P \; H = 0 \quad \text{or} \\ 2. & U' \; H \quad = 0 \end{array}} \qquad (11)$$

This result already has been found by Rao (1967), here however it is derived from the general expression eq. (10).

The conditions eq. (11) have a geometric meaning: Neglected stochastic parameters have no influence onto the estimation of x, if they are already contained in the functional model as a linear combination either of the condition equations or of the parameters. This is because $A' \; P \; H = 0$ is equivalent to $H \in \mathrm{col}(U)$ and $U' \; H$ is equivalent to $H \in \mathrm{col}(A)$, as $A' \; U = 0$ and the matrix $(A \; U)$ has full rank. col(A) designates the column space of A.

Example: The arithmetic mean is invariant with respect to (equal) correlations between the observations. Here $A' = (1,1,\ldots,1) = H'$, $Q = I$ and $\overline{Q} = I + \rho/(1-\rho) H \; H'$ and $H \in \mathrm{col}\;(A)$.

The second condition eq. (11.2) could not be found for errors in the functional model (cf. eq. (6)) as they would have lead to a singular normal equation matrix. But practically additional parameters which are very similar to already existing ones do not deteriorate the result, if the solution is stable enough to avoid rounding errors.

Thus eq. (11) gives complete conditions for additional parameters, fix and stochastical ones, to have no influence on the result. Additional parameters meeting these conditions then are not estimable in an extended model. The normal equation matrix (A H)' P (A' H') for x and s_o is singular. Moreover, if the covariance matrix C_{ss} is to be estimated, the equation system for the unknown variance components describing C_{ss} will be singular showing that the variance components are either not estimable or not discernable.

The common conditions may form a basis for a joint evaluation of simultaneous refinements of the functional and the stochastical model.

4. Empirical results

The last section has provided some tools to evaluate possible extensions of the mathematical model. The following results of practical investigations want to show how far a mathematical model is able to represent reality and which further increase of the final accuracy one might expect.

4.1 The first example deals with the bundle block adjustment of the test block Appenweier (Klein, 1980). Table 2 gives the estimated precision $\hat{\sigma}_o = \sqrt{v'Pv/r}$ of the image coordinates and the r.m.s. and the maximum errors at the 85 check points which were not used in the adjustment, both for single blocks with 20 % sidelap and for double blocks with 60 % sidelap. The adjustment has been performed 1. without any refinement of the model, 2. with 12 additional parameters common for all images of each block to compensate for systematic image errors and 3. with 12 parameters for each strip in order to consider possible differences of the deformations between the strips. Not determinable parameters were excluded to obtain a stable solution. We are only concerned with the planimetric results here.

The table shows clearly:

- The accuracy increases with increasing refinement of the model. This proves that the additional parameters really compensate for varying systematic errors.
- The maximum errors significantly decrease in the single blocks, which is of utmost importance for practical applications.

Table 2   Accuracy of bundle block adjustment Testblock Appenweier scale 1 : 7 800, area 9.1 x 10.4 km^2, estimated precision of terrestrial control and check point coordinates 1.2 cm

	version	single blocks (sidelap 20 %) $\hat{\sigma}_o$ µm	μ_{xy} - cm -	ε_{max}	$\frac{\mu_{xy}}{\hat{\sigma}_o}$	double blocks (sidelap 60 %) $\hat{\sigma}_o$ µm	μ_{xy} - cm -	ε_{max}	$\frac{\mu_{xy}}{\hat{\sigma}_o}$
1	no parameters	3.0	5.7	42.9	2.4	3.6	3.4	10.4	1.2
2	12 parameters blockwise	2.4	3.8	18.6	2.0	2.7	2.6	9.3	1.2
3	12 parameters stripwise	2.3	3.4	13.8	1.9	2.6	2.0	7.0	1.0

$\mu_{xy} = \sqrt{\mu_x^2 + \mu_y^2}$; μ_x, μ_y = r.m.s. residuals at 85 check points

ε_{max} = maximum residuals at check points

- The results, though extremely good, are not quite in accordance with theory as the ratio μ_{xy} (µm)/$\hat{\sigma}_o$ (µm) should be 0.9 for single and 0.6 for double blocks. This discrepancy may be explained by neglections in the mathematical model, neglected correlations between the image coordinates and certainly also unrealistic assumptions about the precision of the control and check points, which have an average precision of 1.2 cm.

A plot of the parameter values (not shown here) reveals them to vary significantly from strip to strip. As there is no justification for this type of splitting the block with respect to the setup of the additional parameters the systematic errors of the individual images were investigated.

4.2   The systematic errors of time series of up to 76 images were derived from flights with reseau cameras (Schroth, 1982). The deviations from the ideal reseau reflect the deformation of the film caused by film transport, pressure plate, film development, temperature, humidity during the measurement etc.. The time dependency of the deformations is desribed by the time series of 18 orthogonal parameters, namely the 6 parameters usually needed for the orientation and 12 additional parameters. Fig. 1 - 3 give some representative examples.

The parameters significantly differ from zero. The mean value is given by a straight line. The dotted lines indicate the 3-fold standard deviation of the estimated parameter. Obviously the variation of the paramters cannot be explained by random errors only.

Also the type of the time series varies. Most of the time series show no correlation between the images (cf. fig. 1). Some time series seem to have

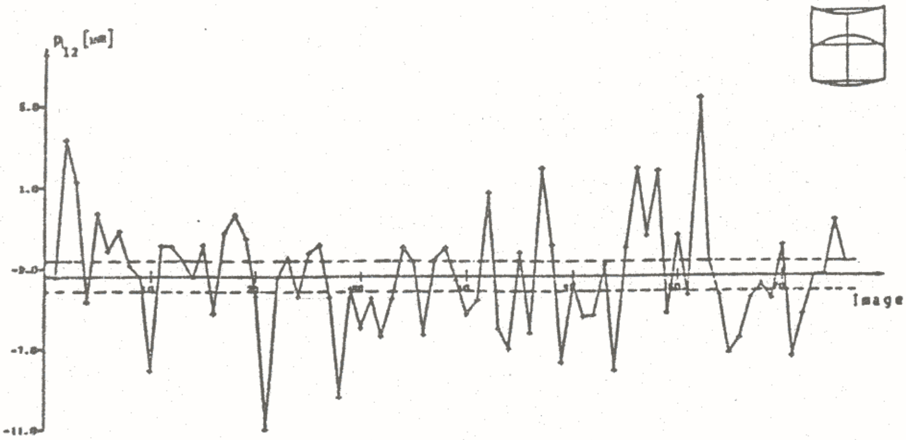

Fig. 1 Standardized values of parameter p_{12}

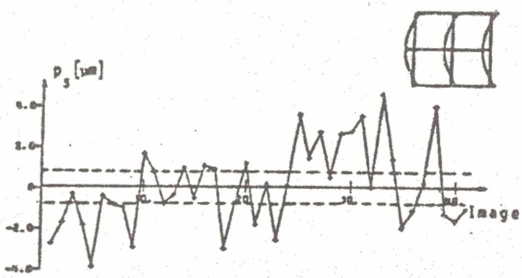

Fig. 2 Standardized values of parameter p_5

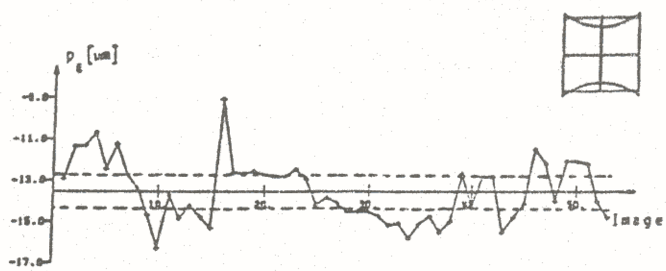

Fig. 3 Standardized values of of parameter p_8

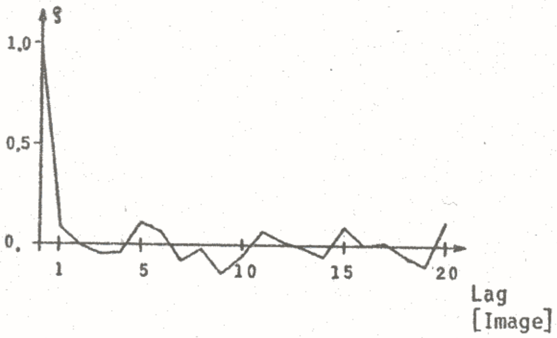

Fig. 4 Autocorrelation function of time series fig. 1

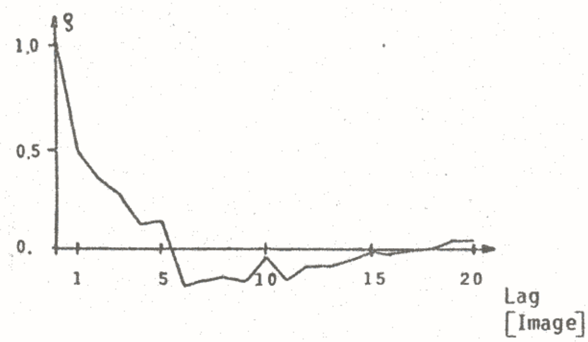

Fig. 5 Autocorrelation function of time series fig. 3

(from Schroth, 1980)

a trend which cannot be described by a constant value(cf. fig. 2). Some time series show rather large correlations between the parameters of adjacent images (cf. fig.3). The autocorrelation functions of the time series fig. 1 and 3 are given in fig. 4 and 5. The autocorrelation function fig. 5 can be approximated by an exponential function exp(-c•Lag), being characteristic for a discrete 1st order Markov-process.

This investigations shows that the additional parameters may be modelled as stochastic variables.

Also time series for the variation of the scale of the images have been obtained showing high variations, which mainly are caused by the humidity during the measuring process. But this parameter only has to be modelled if geodetic measurements (e. g. with inertial systems) are used to get information about the position of the camera platform. Otherwise scale variations are absorbed by the z-coordinate of the projection centre. Thus the scale parameter of an image is fullfilling one of the conditions eq. (11), namely (11.2).

4.3 In section 3 it was shown, that the refinement of the mathematical model can be achieved by either extending the functional or the stochastical model. Of course one can also think of mixtures. Schilcher (1980) has proved this empirically.

He analysed 120 images taken with two different cameras, a wideangle and a superwide angle camera, flown over the test field Rheidt. He distinguished 3 functional models of different quality shown in table 3.
A is the most simple, C the most refined functional model.

Table 3 Theoretical precision of checkpoints for different functional and stochastical models (after Schilcher (1980)).

	model				
	A	B	C	C'	
a priori corrections: lens distortion, earth curvature, refraction	no	yes	yes	yes	
systematic errors common to all images	no	no	yes	yes	
covariance matrix	$\hat{C}_A$	$\hat{C}_B$	$\hat{C}_C$	$I \cdot \hat{\sigma}_o^2$	
precision wide angle	3.7	3.8	3.7	4.3	µm
precision superwide angle	6.7	5.5	5.1	5.8	µm

From the residuals after a spatial resection he estimates covariance matrices $\hat{C}_A$, $\hat{C}_B$ and $\hat{C}_C$. The theoretical precision of check points in terms of standard errors are derived by error propagation for the different levels of the functional model and the corresponding estimated covariance matrices. For the most refined functional model C also the results using a diagonal matrix $C = I\ \hat{\sigma}_o^2$ are given.

The standard errors for the first three models confirm that the neglections caused by a very simple functional model can be compensated by a refined stochastical model and vice verca. The coincidence of the standard deviations for the wideangle camera can be said to be excellent. The moderate deviations of the values for the superwideangle between the models A and B can be explained by the large constant a priori corrections. Their neglection can not be fully compensated by an appropriate covariance matrix.
In this case the assumption for the equivalence of the models eq. (3) and (4), namely the variability of the parameters, is not met.

The model C' using an extended functional model and an oversimplified stochastical model obviously gives worse results, again demonstrating that the introduction of constant common parameters leaves significant correlations between the observations.

The investigation clearly shows that the freedom of choosing between a refined functional and a refined stochastical model is not only a theoretical statement but can be realized leading to a further increase of the final accuracy.

5. A refined mathematical model for photogrammetric point determination

5.1 The structure of the photogrammetric measuring process and the empirical results suggest to treat the images as a time series, whose deformations may be modelled by a Markov-process.

The deformations thus can be described by the following 1st order autoregressive scheme

$$\begin{aligned} \underline{t}_1 &= \underline{n}_1, & \underline{n}_1 &\sim N(0,\ C_{ss}) \\ \underline{t}_k &= a\ \underline{t}_{k-1} + \underline{n}_k, & \underline{n}_k &\sim N(0,\ (1-a^2)\ C_{ss}),\ |a| \leq 1,\ k \geq 2 \end{aligned} \qquad (12)$$

The parameter vector $\underline{t}_1$ is the starting point. The parameter a controls the degree of correlation between the parameters $\underline{t}_k$ and $\underline{t}_{k'}$ of different images k and k', being $a^{|k-k'|}$. Thus if a = 1 all parameters are equal ($\underline{n}_k = 0$), whereas if a = 0 the parameters are independent.

The stochastic process is observed by measuring the coordinates, contained in the vector $\underline{l}_k$ for each image:

(13)

$$\underline{l}_k = A_k\, x + U_k\, t_o + U_k\, \underline{t}_k + \underline{\varepsilon}_k, \quad \underline{\varepsilon}_k \sim N(0,\, C_{11}^{(k)}). \qquad (13)$$

The unknown parameters x contain the coordinates of the new points and the orientation parameters of each image, t_o is a constant vector of additional parameters desribing the mean deformation of the images.

5.2 This mixed model eqs.(12) and (13) has a similar structure as eq.(2). Thus a first way to estimate the unknowns x and t_k (k = 0,1,...) is solving the equation system (cf. eq. (3))

$$\begin{array}{lll} \underline{l}_k = A_k\, x + U_k\, t_o + U_k\, t_k + \underline{\varepsilon}_k, & \underline{\varepsilon}_k \sim N(0,\, C_{11}^{(k)}) & (14a) \\ \underline{0}_1 = -t_1 + \underline{n}_1, & \underline{n}_1 \sim N(0,\, C_{ss}) & (14b) \\ \underline{0}_k = a\, t_{k-1} - t_k + \underline{n}_k, & \underline{n}_k \sim N(0,\, (1-a^2)\, C_{ss}) & (14c) \end{array}$$

using least squares. Eliminating t_k from eqs. (14b) and (14c) leads to the equivalent model (assuming $Cov(\underline{\varepsilon}_k, \underline{\varepsilon}_{k'}) = 0$, cf. eq. (4))

$$\begin{array}{l} \underline{l}_k = A_k\, x + U_k\, t_o + \underline{\xi}_k, \\ V(\underline{\xi}_k) = C_{11}^{(k)} + U_k\, C_{ss}\, U_k', \quad Cov(\underline{\xi}_k, \underline{\xi}_{k'}) = U_k\, C_{ss}\, U_{k'}'\, a^{|k-k'|} \end{array} \qquad (15)$$

Both forms of the refinement have their advantages:

The extension eq. (14) of the functional model is favourable in large systems, especially in cases with high point density in the images. The number of parameters increases roughly proportional to the number of images. The banded structure of the normal equation system can be preserved. Main advantage is the ease of evaluating the extension, e. g. by testing the $\hat{\underline{n}}_k$. Estimating the parameter a is simple, estimating the covariance matrix C_{ss} is feasible. The reduction of the normal equations onto the orientation and additional parameters easily is possible, if the observations can be treated uncorrelated, at least if no correlations between points can be assumed.

Otherwise an extension of the functional model is unavoidable. This especially holds if local film distortions or similar effects cause correlations between points over short (< 1 cm) distances within an image. These deformations can not be compensated extending the functional model but rather have to be descibed by covariance functions. Thus $C_{11}^{(k)}$ may be split into two additive components representing measuring errors and local film distortions. Refining the stochastical model so has the advantage of allowing all kinds of correlations whithout increasing the numerical effort.

5.3 Though the proposed extension of the mathematical model for photogrammetric point determination could be motivied by the results of quite a number of experiments several problems are open for future investigations:

1. Empirical tests with extended models (eqs. (14) and (15)) have to prove the efficiency of the refinement. Special effort has to be layed upon the question whether the compensation of local distortions really leads to a significant increase of the final accuracy.

2. The numerical effort for the adjustment which is heavily increased by the proposed extensions has to be limited. Therefore it seems necessary

- to find an optimal set of additional parameters which is at the same time as small as poosible,
- to find a strategy for a stepwise refinement of the mathematical model, if that is of any advantage and
- to compare the numerical properties of the two approches eq. (14) and (15) with respect to computing time and stability of the system.

3. The increase of unknown parameters may weaken the whole system if the geometry of the block is not chosen properly. Therefore the quality of the result has at least to be checked. These checks should be as efficient and at the same time simple as possible and may be used to give recommendations for the design of blocks with the aim to reach a high reliability of the result.

It can be hoped that these investigations will lead to a further increase of the accuracy of photogrammetric point determination.

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