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ON THE GEOMETRIC PRECISION OF DIGITAL CORRELATION

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ABSTRACT

The geometric precision of digital correlation can be described by the standard deviation of the estimated shift. The paper shows how the precision depends on the signal to noise ratio, the number of pixels involved and the texture of the object and discusses the choice of a low pass filter which minimizes the variance of the estimated location in order to obtain an optimal sampling frequency.

INTRODUCTION

Digital correlation is the basic technique for geometric processing of digitized images. It can be used for the measurement of passpoints for rectification, for point transfer in aerotriangulation, for automatic profiling or for line following procedures. There exist several algorithms for correlation, maximizing the cross-correlation coefficient being the most common one. In a classical paper Helava (1976,|3|) discussed the different aspects of correlation with respect to an automatic procedure, and provided clear suggestions for the design of a correlator, regarding the effects of noise, the general texture of an image and the influence of filtering onto the convergency (the pull in range). However, very little discussion is found in the literature concerning the geometric precision of correlation; i. e., the standard deviation of the estimated location of the correlation maximum.

The performance of the usual procedure, which maximizes the cross-correlation coefficient, is evaluated using the correlation coefficient ρ itself. There appears to be no commonly accepted opinion on how large ρ should be to indicate an acceptable correlation. It seems that each investigator tends to use his own criteria depending on the data he has learned from. This paper will show that evaluating the correlation coefficient is equivalent to evaluating the average signal to noise ratio. But clearly the precision of the correlation will also depend on the sharpness of the correlated object, i. e. only the presence of sufficient high frequence signals will guarantee a precise correlation. Thus the texture of the object has to be taken into account.

One straight forward approach for estimating the geometric precision would be to make use of the curvature (2nd derivative) at the maximum of the cross correlation function. The empirical solution, given by Wiesel (1981,|6|), verifies the consistency of the correlation by correlating 9 adjacent points and determining the root mean square error of the similarity transformation of the original onto the transferred point pattern. This paper adapts a different approach and uses the standard deviation of the estimated shift resulting from a least squares solution, which minimizes the sum of the square gray-level differences of the two images and which leads to the same location for the point of maximum.

The paper will show that in addition to the correlation coefficient or the signal to noise ratio geometric precision will also depend on the variance of the first derivative, i. e. the gradient of the correlated image. As can be shown in the frequency domain, high frequency signals of moderate amplitude have a high influence on the precision. The variance of the gradient can be calculated in the spatial domain, being useful in a real correlation procedure, or in the frequency domain using Parseval's identity. This is extremely useful for planning purposes, in case one has proper information about the power spectra of the object and the contaminating noise.

In part 2 a derivation of the basic formulas for evaluating the precision of template matching will be presented and in part 3 this algorithm will be extended for the case of correlating two images. In part 4, the results will be compared with the usual procedure of cross correlation, and a practical example will be presented. In part 5 the results will be generalized to the correlation of images which have been preprocessed with a linear filter. This can be used for an optimal correlation in case of coloured noise and for the determination of an upper frequency of an ideal low pass filter which leads to an optimal precision. This part is intended to show the applicability of the developed formulas. Hence simplified assumptions were made, although more realistic ones could have been chosen.

2. TEMPLATE MATCHING BY LEAST SQUARES

Let the known object be described by the one dimensional gray level function $g(x)$, $x \in [0,L]$. The image $g_1(x)$ of the object is assumed to be

$$g_1(x) = g(x + x_o) + n(x) \quad , \qquad (1)$$

where x_o is an unknown shift and $n(x)$ is the additive noise.

With the observations $\Delta g(x) = g_1(x) - g(x)$, the corrections $v(x) = -\hat{n}(x)$ and the derivative $dg(x)/dx = g'(x)$ the following linearized error equations can be developed for each position x:

$$\Delta g(x) + v(x) = g'(x)\hat{x}_o \qquad (2)$$

(2) can be solved according to least squares techniques. Let the gray levels of $g_1(x)$ and $g(x)$ be observed at N regularly distributed points x_i within the interval $[0,L]$ and have equal weight; thus $n(x)$ is assumed to be white noise, i. e. normally distributed with a mean of zero. The normal equations

$$B\,\hat{x}_o = c \qquad (3)$$

then contain

$$B = \sum_{i=1}^{N} g'^2(x_i) \quad ; \qquad c = \sum_{i=1}^{N} g'(x_i)\,\Delta g(x_i) \; . \qquad (4a)\ (5a)$$

If the observations are sufficiently dense, the sums can be replaced by integrals, i. e.

$$B = \frac{N}{L} \int_{o}^{L} g'^2(x)\ dx = N\ \sigma^2_{g'} \tag{4b}$$

and

$$c = \frac{N}{L} \int_{o}^{L} g'(x)\ \Delta g(x)\ dx = N\ \sigma_{g'\Delta g}\ . \tag{5b}$$

Thus $\sigma^2_{g'}$ is the variance of the gradient of $g(x)$.

The solution of the normal equations leads to

$$\hat{x}_o = \frac{\sum_{i=1}^{N} g'(x_i)\ \Delta g(x_i)}{\sum_{i=1}^{N} g'^2(x_i)} \quad \text{or} \quad \hat{x}_o = \frac{\sigma_{g'\Delta g}}{\sigma^2_{g'}} \tag{6a,b}$$

An estimate of the variance σ^2_n of the noise can now be computed as follows:

$$\hat{\sigma}^2_o = \frac{\sum_{i=1}^{N} v^2(x_i)}{N - u} \quad \text{or} \quad \hat{\sigma}^2_o = \frac{1}{N-u} \frac{N}{L} \int_{o}^{L} v^2(x)\ dx = \frac{N}{N-u}\ \sigma^2_v \tag{7a,b}$$

where u is the number of unknowns (u=1 in this case). An estimate of the variance $\hat{\sigma}^2_x$ of the shift $\hat{x}_o$ can then be computed by the following expression:

$$\hat{\sigma}^2_x = \frac{1}{N-u} \frac{\sum_{i=1}^{N} v^2(x_i)}{\sum_{i=1}^{N} g'^2(x_i)} \quad \text{or} \quad \hat{\sigma}^2_x = \frac{1}{N-u} \frac{\sigma^2_v}{\sigma^2_{g'}}\ . \tag{8a,b}$$

If the variance of the noise n(x) is known, the theoretical precision, i. e. the <u>expectation</u> of $\hat{\sigma}^2_x$ is given by

$$\sigma^2_x = \frac{1}{N} \frac{\sigma^2_n}{\sigma^2_{g'}}\ . \tag{9a}$$

If instead of σ^2_n the <u>signal to noise ratio</u>

$$SNR = \sigma_g / \sigma_n\ , \tag{10}$$

is known, equ. (9a) can be written in the following form:

$$\sigma^2_x = \frac{1}{N \cdot SNR^2} \frac{\sigma^2_g}{\sigma^2_{g'}} \tag{9b}$$

Eqs. (6a), (7a) and (8a) can be used in a real correlation procedure, whereas eqs. (9a) or (9b) can also be used for planning purposes, expecially in case where the power spectrum of the object is known.

3. CORRELATION OF TWO IMAGES

The correlation process and its evaluation can easily be generalized to the matching of two images of an object. Instead of correlating $g_1(x)$ and $g(x)$, the process is changed to correlating $g_1(x)$ and $g_2(x)$, where

$$g_1(x) = g(x + x_o) + n_1(x) \quad , \tag{11}$$

and

$$g_2(x) = g(x) + n_2(x) \quad . \tag{12}$$

Taking the difference and linearizing lead to the same error equations expressed by eq. (2), i. e.

$$\Delta g(x) + v(x) = g'(x)\, \hat{x}_o$$

where now $\Delta g(x) = g_1(x) - g_2(x)$ and $v(x) = v_1(x) - v_2(x) =$ $= \hat{n}_2(x) - \hat{n}_1(x)$.

Eqs. (6) and (7) can again be used to solve for $\hat{x}_o$ and σ_o^2 respectively.

If the power spectra of $n_1(x)$ and $n_2(x)$ are equal and constant, the theoretical precision of $\hat{x}_o$ is given by

$$\boxed{\sigma_x^2 = \frac{2 \cdot \sigma_n^2}{N \cdot \sigma_{g'}^2} = \frac{2}{N \cdot SNR^2} \cdot \frac{\sigma_g^2}{\sigma_{g'}^2}} \tag{13}$$

Remark:

All formulas for the variance of $\hat{x}_o$ given above depend on the texture of the true object, namely the variance $\sigma_{g'}^2$, of the gradient. If the form of the object is not known, as it is often the case when correlating two images, the object has to be estimated, i. e. restored, from one of the images (or both). The gradient $g'(x)$ must then be replaced by an estimate $\hat{g}'(x)$. Using $g_1(x) + v_1(x)$, for example, is not optimal, as the correlation procedure is not designed to restore an image but rather only to estimate the shift x_o (cf. discussion in Castleman, 1979, pp. 217, |1|). Any restoration techniques may be used to restore $g'(x)$, which may not be optimal for a restoration of $g(x)$. Of course correlating two restored images can lead to quite different estimates for precision. Evaluation of the precision should then take into account the correct power spectra of the filtered noise, which will no longer be white (cf. ch. 5).

4. COMPARISON WITH CROSS CORRELATION

It is well known that minimizing the norm of $\Delta g(x)$ and maximizing the covariance of $g_1(x)$ and $g_2(x)$ lead to the same location for the extrema. If moreover g_1 and g_2 have the same variance σ_g^2, the correlation coefficient $\rho_{12} = \sigma_{12}/\sigma_g^2$ is directly related to the signal to noise ratio SNR by

$$SNR^2 = 1/(1-\rho_{12}) \quad \text{or} \quad \rho_{12} = 1 - 1/SNR^2 \tag{14}$$

Substituting eq. (14) into eq. (8) yields the following expression for the estimated standard error of $\hat{x}_o$:

$$\hat{\sigma}_x^2 = \frac{2(1 - \hat{\rho}_{12})}{N - u} \frac{\sigma_g^2}{\sigma_{g'}^2} \tag{15}$$

The usual correlation processes only need to be extended by the calculation of the ratio $(\sigma_g/\sigma_{g'})^2$, which takes into account the texture of the object.

Eq. (14) shows that evaluating the quality of matching by using the correlation coefficient gives the same information as σ_o^2 or the signal to noise ratio SNR. But it does not give any information on how precise the location of the maximum really is. Two comparable correlation coefficients may belong to matches with quite different precision.

The formulation of the correlation process as an adjustment procedure allows the introduction of additional unknowns to compensate for geometric or radiometric distortions of the images; e. g. an affinity in two dimensional correlation to take into account the geometry of the projection, or two parameters for a relative gray scale transformation.

Example:

Figures 1 and 2 show perspective sketchs of the gray level functions $g(x,y)$ of a garden bed and a traffic island respectively. The two stereo pairs were correlated using the algorithm described in ch. 2 in a two dimensional version using linear transformations for geometry and radiometry. The pixel size was $(20\ \mu m)^2$. The correlation coefficients were found to be 0.90 and 0.91 for image pairs 1 and 2 respectively, which indicate a high quality of the match. The standard deviations (σ_x) were 0.6 µm and 0.4 µm for image pairs 1 and 2 respectively, which show that the point transfer using image 2 is better than using image pair 1. On the other hand, both standard deviations are clearly better than the standard deviation of manual point transfer using natural points, which is about σ_p = 5 µm. This results from the high quality of the image and the large number of pixels used for correlation. The smoothed images are sketched to make the unsmoothed image readable. The correlation was performed on the unsmoothed images. ■

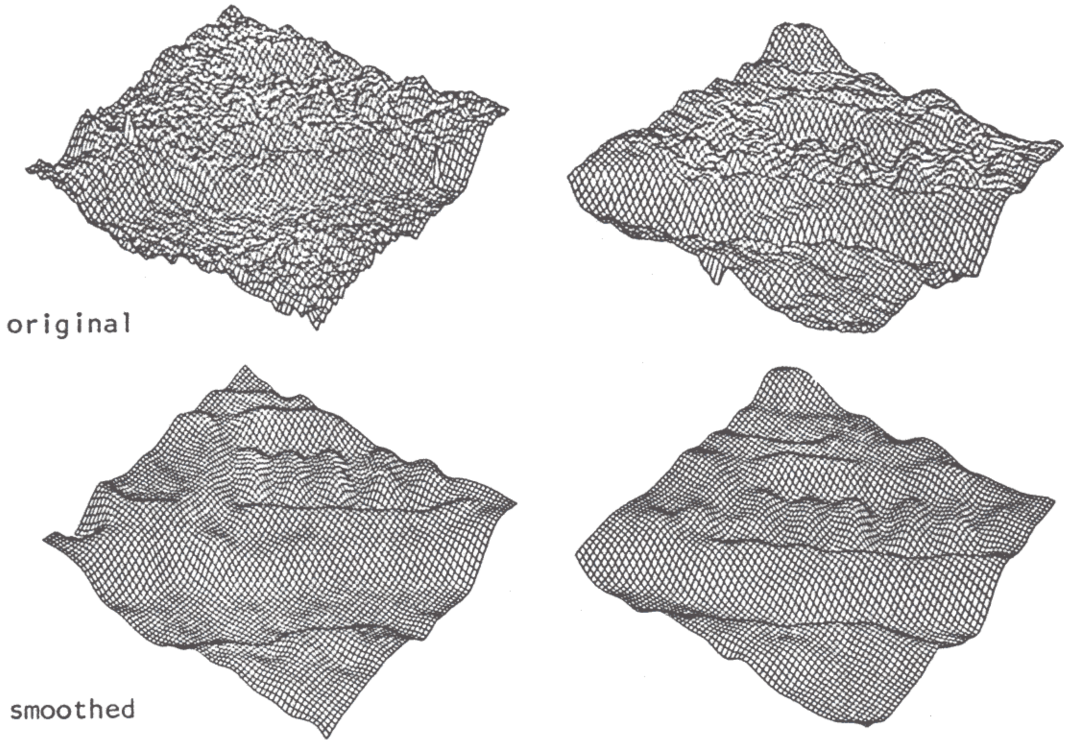

Fig.1 Garden bed, Δx = 20 µm, ρ = 0.90, N = 3265, $\hat{\sigma}_x$ = 0.6 µm

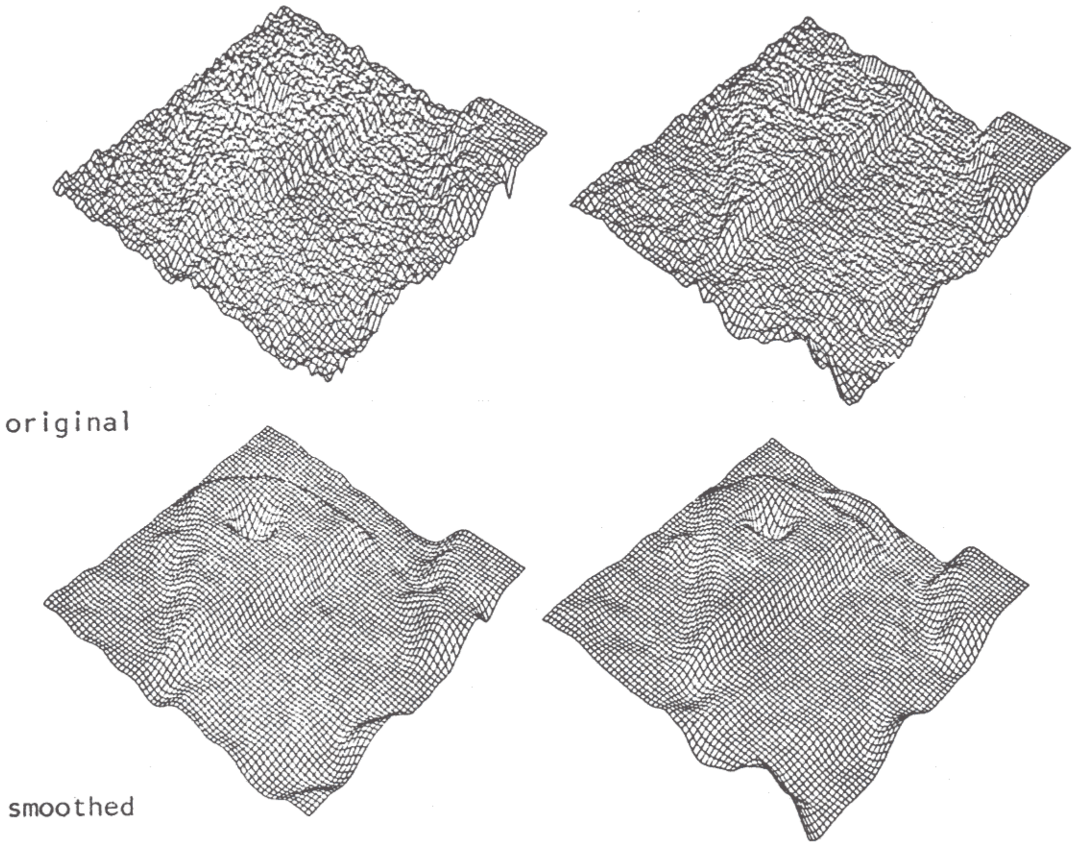

Fig. 2 Traffic island, Δx = 20 µm, ρ = 0.91, N = 3497, $\hat{\sigma}_x$ = 0.4 µm

5. CORRELATION OF FILTERED IMAGES

The procedure will now be extended to the correlation of images which have been preprocessed with a linear filter. The development uses the possibility to express the amplitude and the power spectrum of the derivative of a function in terms of the amplitude and the power spectrum of the original function. Let $g(x)$ have amplitude spectrum $G(s) = \int g(x) \exp(-j2\pi xs)dx$ and thus power spectrum $P_g(s) = |G(s)|^2$ then the derivative $g'(x)$ has amplitude spectrum $G'(s) = -j2\pi s\ G(s)$ and thus power spectrum $P_{g'}(s) = 4\pi^2 s^2\ P_g(s)$, , where both x and s range from $-\infty$ to $+\infty$. Also Parseval's identity is used to determine the variance σ_g^2 of a function $g(x)$ as $\int g^2(x)\ dx = \int P_g(s)\ ds = \sigma_g^2$.

5.1 Let it be assumed that the power spectra of $n_1(x)$ and $n_2(x)$ are identical to $P_n(s)$, with s being the frequency expressed in lp/mm, and that $P_n(s)$ does not have any zero elements. With the linear filter $Q_n(s) = 1/\sqrt{P_n(s)}$ and the corresponding impulse response $q_n(x)$, eq. (11) can be transformed by convolving it with $q_n(x)$:

$$g_1(x) * q_n(x) =: \bar{g}_1(x) = \bar{g}(x + x_o) + \bar{n}_1(x) \qquad (16a)$$

$$g_2(x) * q_n(x) =: \bar{g}_2(x) = \bar{g}(x) + \bar{n}_2(x) \qquad (16b)$$

The power spectra of $\bar{n}_1(x)$ and $\bar{n}_2(x)$ are now $P_{\bar{n}}(s) = 1$. The adjustment procedure of ch. 2 can be applied to yield the following expression for the precision of $\hat{x}_o$:

$$\sigma_x^2 = \frac{2}{N} \frac{\sigma_{\bar{n}}^2}{\sigma_{\bar{g}'}^2}, \qquad (17)$$

with

$$\sigma_{\bar{g}'}^2 = 4\pi^2 \sum_{-s_o}^{+s_o} s^2 \cdot |G(s)|^2 \cdot |Q(s)|^2 \quad , \qquad (18)$$

where $G(s)$ is the amplitude spectrum of $g(x)$ ($g(x)$ is assumed to be periodic here) and

$$\sigma_{\bar{n}}^2 = \sum_{-s_o}^{+s_o} P_n(s) \cdot |Q_n(s)|^2 \qquad (19)$$

This procedure for decorrelating the noise retains the optimality properties of the correlation process (cf. Castleman, 1979, p. 214,|1|). The upper and lower bounds for the sums are necessary in evaluating eqs. (18) and (19),not for eq. (17).

For practical applications only a computer program for the filtering of the images with $Q_n(s)$ is needed. Of course, if another filter $q_n(x)$ is used, eq. (17) still is valid.

5.2 This fact can be used to design a filter which is optimal with respect to the geometric precision of correlation. In order to keep the formulas simple we restrict the discussion to ideal low pass filters with boundary frequency s_c applied to images with exponential and continuous power spectra, i. e.

$$P_g(s) = P_g(0) \cdot e^{-a|s|} \quad (20)$$

which are contaminated by white noise having a power spectrum expressed by

$$P_n(s) = N_o^2 \quad . \quad (21)$$

This is a realistic assumption as far as photogrammetric images are concerned. (cf. Helava, 1976,|3|).

An ideal low pass filter with boundary frequency s_c can be used as an approximation for the sampling with a pixel size $\Delta x = 1/2s_c$.

In the following sections, the theoretical case where all frequencies are used will first be discussed. Then, an algorithm will be developed to determine the frequency s_c which leads to the smallest variance of $\hat{x}_o$, assuming that the number of pixels used and the signal to noise ratio are given. Finally the dependency of the precision on the sampling, represented by s_c, will be determined by fixing the size of the correlated patch and the signal to noise ratio.

5.2.1 If all frequencies of the power spectrum in eq. (20) are used, the standard deviation eq. (13) can be calculated from the simple relation

$$\sigma_x = \frac{1}{2\pi\sqrt{N}} \frac{a}{SNR} \quad . \quad (22)$$

This directly follows from eq. (13) with $\sigma_g^2 = 2\, P_g(0) \cdot \int_0^\infty e^{-as} ds = 2\, P_g(0)/a$ and $\sigma_{g'}^2 = 2\, P_g(0) \cdot 4\pi^2 \int_0^\infty s^2 e^{-as} ds = 2\, P_g(0) \cdot 4\pi^2 \cdot 2/a^3$.

Example 1:

a) Photogrammetric image of high quality (cf. Helava, 1976,|3|)

$a = 0.2$, $SNR = 5$, $N = 100$ $\rightarrow \sigma_x = 0.64$ µm.

b) Photogrammetric image of low contrast, tidal lands (cf. Ehlers,1980,|2|)

$a = 3.8$, $SNR = 1.5$, $N = 100$ $\rightarrow \sigma_x = 40$ µm.

The relative magnitude of the two values of σ_x seems to be realistic. But the absolute precision in both cases is somewhat higher than expected. The reason seems to be that ideal assumptions concerning object and noise were used. In reality, noise will be correlated and not all frequencies can be represented by only 100 observed gray level differences. Therefore at least low pass filtering has to be taken into consideration.

The true reason for the high precision will become obvious in example 3. ■

5.2.2 By applying an ideal low pass filter which cuts off frequencies above s_c, eq. (17) can be written in the following form:

$$\sigma_x^2 = \frac{2}{N} \cdot \frac{\int_{-s_c}^{+s_c} N_o^2 \, ds}{\int_{-s_c}^{+s_c} 4\pi^2 \, P_g(0) \cdot e^{-a|s|} ds} \quad . \tag{23}$$

Reordering of the terms leads to

$$\sigma_x^2 = \frac{1}{N} \cdot \left(\frac{N_o}{\sqrt{P_g(0)}}\right)^2 \cdot \left(\frac{a}{2\pi}\right)^2 \cdot r(s_c) \tag{24}$$

with

$$r(s_c) = 2 \int_0^{s_c} ds \Bigg/ \int_0^{s_c} (as)^2 \, e^{-as} ds \quad .$$

An evaluation of the integrals showed that the function r only depends on the standardized frequency z_c ($z_c = a \cdot s_c$). Therefore,

$$r(z_c) = \frac{2 \, z_c}{2 - e^{-z_c}(z_c^2 + 2z_c + 2)} \quad . \tag{25}$$

The zero amplitude ratio ZAR = $N_o/G(0)$ $\left(G(0) = \sqrt{P_g(0)}\right)$ can first be determined as a function of the signal to noise ratio SNR, restricting noise and signal to frequencies $s < s_{on}$, where $P_g(s) > N_o^2$ (cf. fig. 3). This is equivalent to eliminating all frequencies $s > s_{on}$. The upper frequency

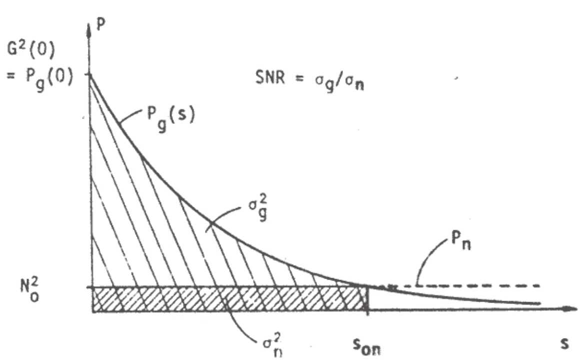

Fig. 3: On the assumed relation between power spectra of noise and signal and the signal to noise ratio SNR

s_{on} of an object with the power spectrum in eq. (20) can be obtained from

$$s_{on} = \frac{z_{on}(SNR)}{a} \tag{26}$$

where z_{on} is the standardized frequency. It depends on SNR and is obtained by solving $e^{z_{on}} - 1 = z_{on}\,SNR^2$.

SNR	1.5	2	3	5	10	20
$z_{on} = a \cdot s_{on}$	1.45	2.33	3.47	4.80	6.48	8.08
s_{on} (a=0.2)	7.5	12	17	24	32	40
ZAR = $N_o/G(0)$	0.48	0.31	0.18	0.091	0.039	0.018

Table 1 Upper frequencies s_{on} of signal and zero amplitude ratio ZAR = $N_o/G(0)$ as a function of SNR and the parameter a of the power spectrum $P_g(s) = P_g(0)\exp(-a|s|)$.

Table 1 lists some values of z_{on} and the corresponding zero amplitude ratio ZAR = $N_o(G(0)$ as a function of SNR. For example, in a good photographic image with a = 0.2 and SNR = 5 (cf. Example 1), there is hardly any information above s_{on} = 24 lp/mm, which agrees very well with experience.

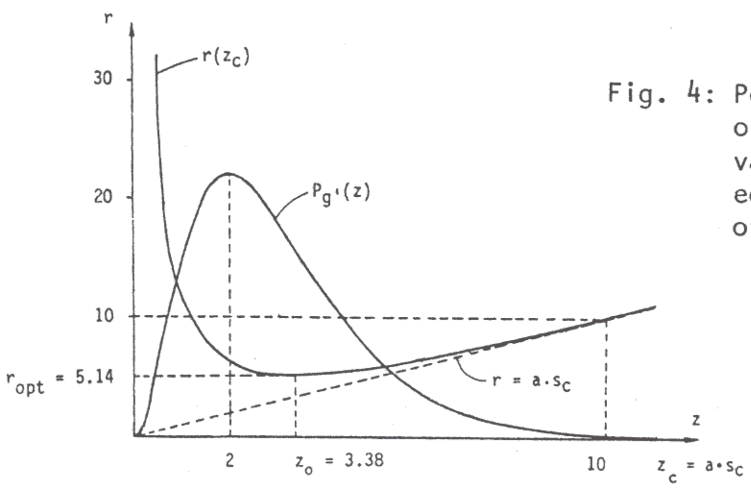

Fig. 4: Power spectrum $P_{g'}(as)$ of gradient $g'(x)$ and variance function $r(as_c)$ eq.(25) for given number of pixels

The function $r(z_c)$ which is sketched in fig. 4 is now minimized. The optimum frequency s_{on} for the low pass filter is

$$s_{oc} = \frac{z_o}{a} \quad , \quad z_o = 3.38 \tag{27}$$

The optimum value $r_{opt} = 5.14$ is independent of the parameter a, which leads to the following expression for the best standard deviation of x_o that can be obtained when correlating two patches with N pixels and a given ZAR(SNR) = $N_o/G(0)$ and assuming the power spectrum eq.(20):

$$\sigma_x = 0.36 \cdot \frac{a}{\sqrt{N}} \cdot ZAR(SNR) \qquad ZAR\ (SNR) \text{ from table 1} \qquad (28)$$

Fig. 4 demonstrates that the minimum is flat in the direction towards higher frequencies. Using a boundary frequency s_c which is somewhat larger than the optimum would lead to only slightly lower precision, whereas omitting frequencies $\leq s_{oc}$ may deteriorate the precision of correlation.

a	0.2	0.5	1.0	2.0	4.0
s_{oc}	17	6.8	3.4	1.7	0.8

Table 2 Frequency s_{oc} of low pass filter for optimal correlation assuming $P_g(s) = P_g(0) \cdot e^{-a|s|}$ and given number of pixels

Table 2 lists the values of s_{oc} for several values of the parameter a. Correlating two images with a = 0.2, say, gives best precision if only frequencies $\leq z_0/a = 17$ are used. This is within the range where the noise power is smaller than the power of the signal, if the SNR = 5 ($17 < s_{on} = 24$). If however SNR is below 3, say 2, the optimum low pass filter keeps frequencies (between $s_{on} = 2.33/a = 12$ and $z_0/a = 17$) which mainly contain noise; but un upper frequency $s_c = s_{on}$ would lead to inferior results.

This seeming contradiction can be explained using the power spectrum $P_g'(as)$ of the gradient which is shown in fig. 4. The frequency band around 2/a up to z_0/a contains 2/3 of the variance of the gradient. This is an agreement with the findings of Helava (1976, |3|), who stated the optimum band for correlation to be between 5 and 20 lp/mm, assuming a = 0.2. Thus if $z_c = z_{on} < z_0$ parts of this band are not used, which leads to a deterioration of the precision though frequencies between z_{on} and z_0 contain more noise than signal. But in case $z_c = z_{on} > z_0$ the frequencies above z_0 are not powerful enough to increase the precision.

<u>Example 2 (cf. Ex. 1):</u>

a) a = 0.2, SNR = 5, N = 100, $s_{oc} = 17$ $\qquad \sigma_x = 0.66\ \mu m$

b) a = 3.8, SNR = 1.5, N = 100, $s_{oc} = 0.8$ $\qquad \sigma_x = 70\ \mu m$.

Both σ_x are higher than the corresponding values in example 1. Reason is the different value N_0 in both examples: If all frequencies are taken into account, N_0 is practically zero (with $\int_{-\infty}^{+\infty} N_0^2\, ds = \sigma_n^2$), whereas in example 2 N_0 amounts to 9 % in case a) and as much as 50 % in case b) of the zero amplitude of the image. This also explains the different influence onto the precision in cases 2 a) and b). Again, in example 2b) one must be aware, that the <u>given</u> SNR of 1.5 is only needed to determine the ratio $N_0/G(0)$ using frequencies up to 0.36 (= 1,45/a, cf. table 1), whereas the correlation process is assumed to use frequencies up to 0.8 (cf. table 2), which reduces the <u>actual</u> SNR down to 1.1. ■

5.2.3 The precision of $\hat{x}_o$ for a given length d [mm] of the object can next be determined since low pass filtering with upper frequency s_c can approximate sampling with a pixel size $\Delta x = 1/2s_c$. The number $N = d/\Delta x$ in eq. (24) can be combined with the frequency z_c (= $a \cdot s_c$) in eq. (25) to yield

$$\sigma_x^2 = \frac{N_o^2}{P_g(0)} \cdot \frac{a^2}{4\pi^2} \cdot \frac{a}{2d} \cdot \frac{2}{2 - e^{-z_c}(z_c^2 + 2z_c + 2)} \qquad (29)$$

This is a monotonic decreasing funcion of $z_c = a/2\Delta x$ cf. fig. 5).

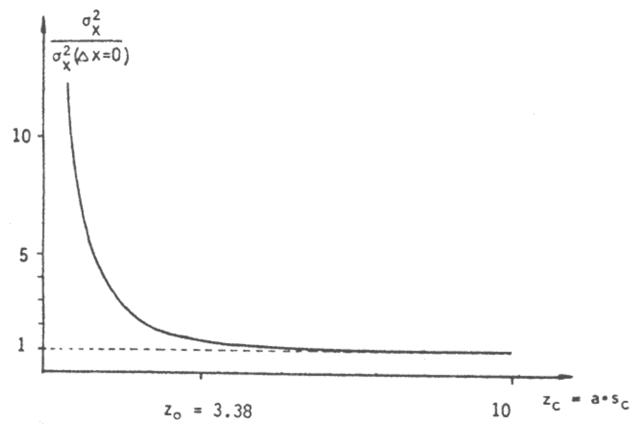

Fig. 5: Variance function $\sigma_x^2(a.s_c)$ eq. (29) for given size d of the object

It describes the influence of the sampling onto the precision of $\hat{x}_o$. For frequencies $z_c > z_{oc}$ the standard deviation can be approximated by

$$\boxed{\sigma_x(d) = 0.11 \cdot ZAR(SNR) \cdot a \sqrt{\frac{a}{d}}} \quad \text{with ZAR(SNR) from table 1} \qquad (30)$$

Thus oversampling does not change the precision of the correlation, as long as a good approximation of g'(x) is used and all frequencies $s < s_c$ are represented.

Example 3:

a) a = 0.2, d = 1 mm, SNR = 5 → $\sigma_x = 0.9$ µm

The precision is lower than in example 2a), since only 34 (2·17 = 34) pixels are necessary to represent the object of length d = 1 mm. 100 pixels (as in example 1) thus represent an object of 3 mm !

b) Correlating an object of length d = 1 mm of which the parameter a = 3.8 does not meet the conditions of the formulas, since the lowest frequency (s_c = 1) is larger than the optimum frequency (s_{oc} = 3.38/a = = 0.85). Thus only frequencies > z_{oc} would be used and the derivation of a low pass filter is meaningless. But correlating an object of length d = 10 mm leads to an optimum precision of σ_x = .12mm. The reason for the lower precision than in example 2b is the same as in 3a).

On the other extreme, if the pixel size is too large, i. e. $s_c << s_{oc}$ eq. (29) leads to

$$\boxed{\sigma_x(\Delta x) = 0.32 \cdot ZAR(SNR)\sqrt{\frac{a}{d}} \cdot \Delta x} \qquad \text{with ZAR (SNR) from table 1} \qquad (31)$$

Thus the standard deviation then increases proportional to the pixel size.

6. DISCUSSION

This paper is intended to show how the geometric precision of digital correlation can be evaluated using the standard deviation of the estimated shift. The basic prerequisite was the use of the least squares approach as correlation procedure.The signal to noise ratio, the estimated variance of the noise and the correlation coefficient are equivalent measures for the quality of the match; but the variance of the first derivative, as a measure of texture, is decisive for the geometric precision.

Generalization of the equations developed in part 2 for preprocessed images is straight forward. It is shown how the images have to be filtered if they have been contaminated by coloured noise. If noise and signal have the same power spectrum (except a constant factor), the adjustment procedure is equivalent to phase correlation (cf. Pearson, 1977,|5|) the precision of which can thus be estimated using eq. (17).

The applicability of the theory is demonstrated in section 5.2. When the number of observations (pixels) and the signal to noise ratio are fixed. minimizing the variance σ_x^2 leads to an optimum boundary frequency s_c=s_{oc} for an ideal low pass filter. Increasing s_c, however, has only small influence on the precision, i. e. a resolution which is too high does not deteriorate the precision of correlation provided that a restored image or the object itself is used for the determination of the gradient g'(x).

On the other hand there exists a lower bound for the standard deviation of the shift which cannot be decreased by using a higher resolution while keeping the size of the object.

In all parts of the paper simplified assumptions had to be made. This was necessary to keep clear the line of thought. But it was also necessary as in many cases no better information was available.

A proper model for describing the noise is urgently necessary. In this context noise includes not only film granularity, sampling or quantization noise but also shadows, reseau crosses or temporal changes of the object or its surrounding etc. These informations have to be used to optimize the correlation procedure as well as the restoration of the image to gain a proper estimate of the gradient of the object, in order to get a reliable information about the precision of the correlation. The ability of an image to produce correlation of high precision may in addition to the signal to noise ratio (cf. Welch, et.al. 1980, |5|) be used as a measure for image quality.

Moreover, any preprocessing of the image has to be taken into account, when determining the variance σ_x^2. In case of template matching, e. g. the measurement of signalized points, the transfer function of the optical system must also be considered. In section 5.2 only an ideal low pass filter was treated. Sampling and bounding an object are linear filters, and thus can be treated in a similar way. This may lead not only to an optimal sampling but also to an optimal size of the correlated part of an object.

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