Photogrammetry & Robotics Lab

Orthophotos

Cyrill Stachniss

Partial slides courtesy by Konrad Schindler, ETH Zurich

5 Minute Preparation for Today

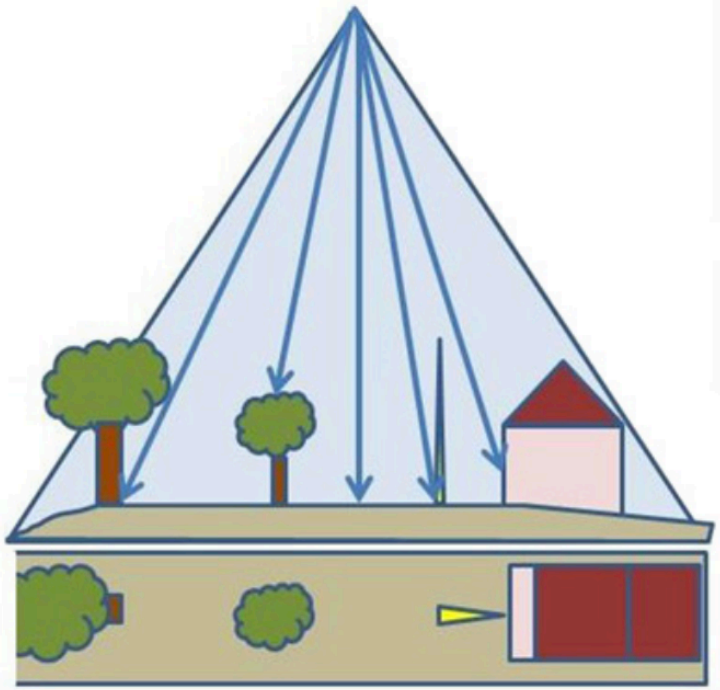
https://www.ipb.uni-bonn.de/5min/

Motivation

[Courtesy: Google Maps]

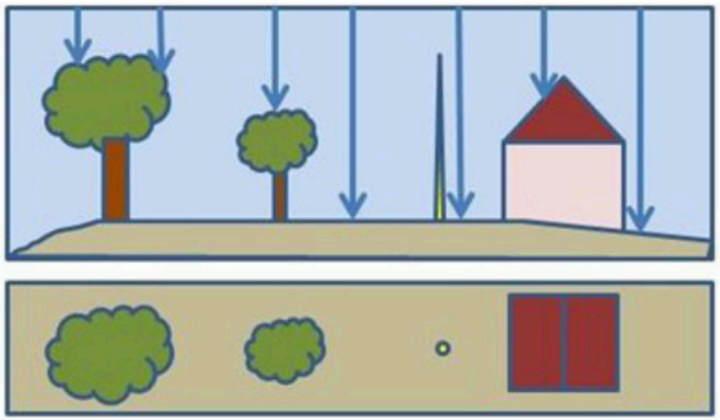
Different Projections Result in Different Images

perspective projection



perspective image

orthogonal parallel projection



orthophoto

Image courtesy: Geavis

An Example from Cologne

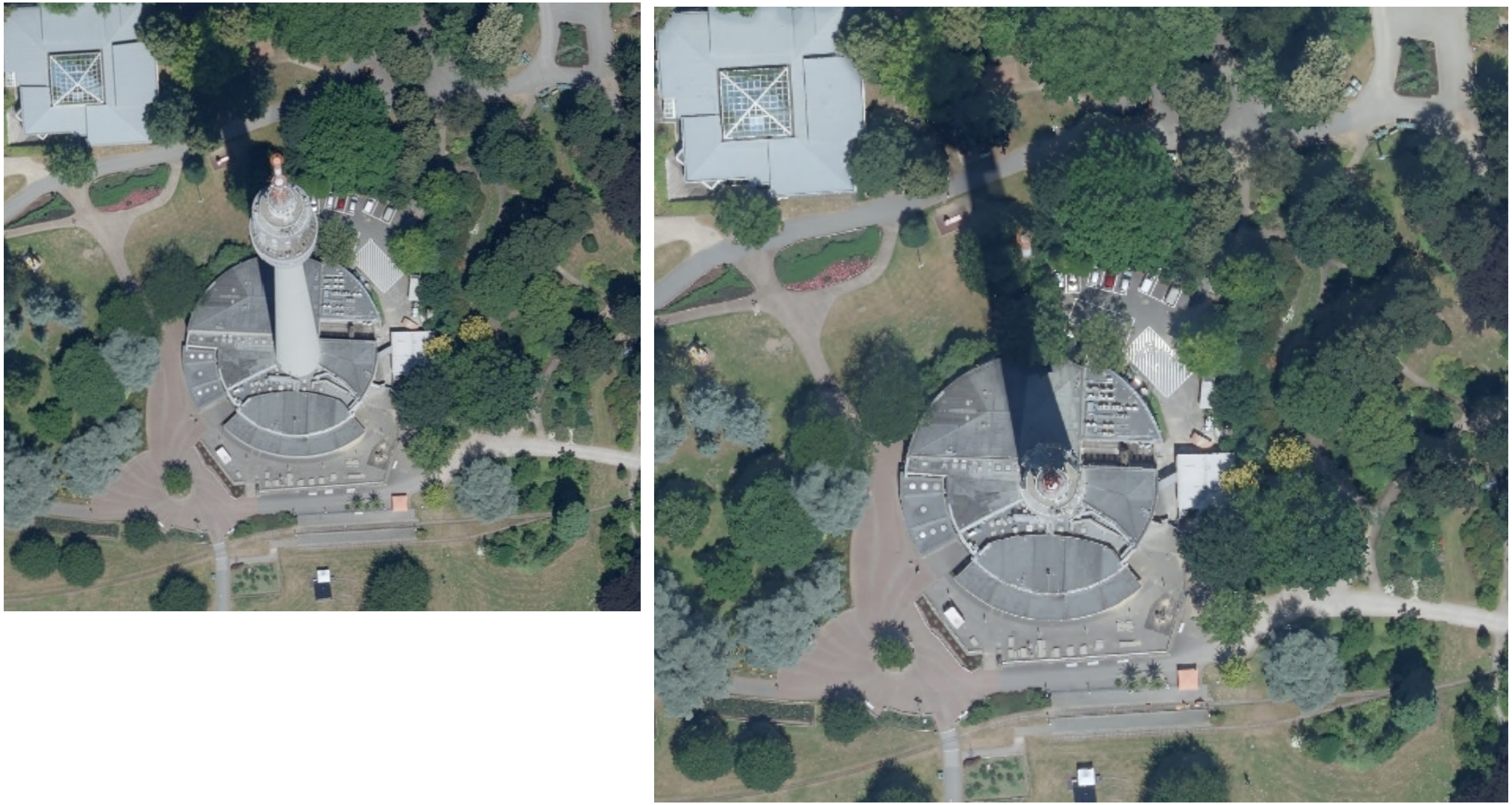


Image courtesy: Bezirksregierung Köln

Motivation

Image courtesy: Förstner

Orthophoto

- Orthophoto is an image of a surface in **orthogonal parallel projection**
- Satellite images can be seen as a good approximation of orthophotos
- Orthophotos are usually **generated synthetically** (actually recording an orthogonal parallel projection is hard)

Input

- Image of the surface of interest
- Interior and exterior orientation of the image
- 3D information about the scene

Output

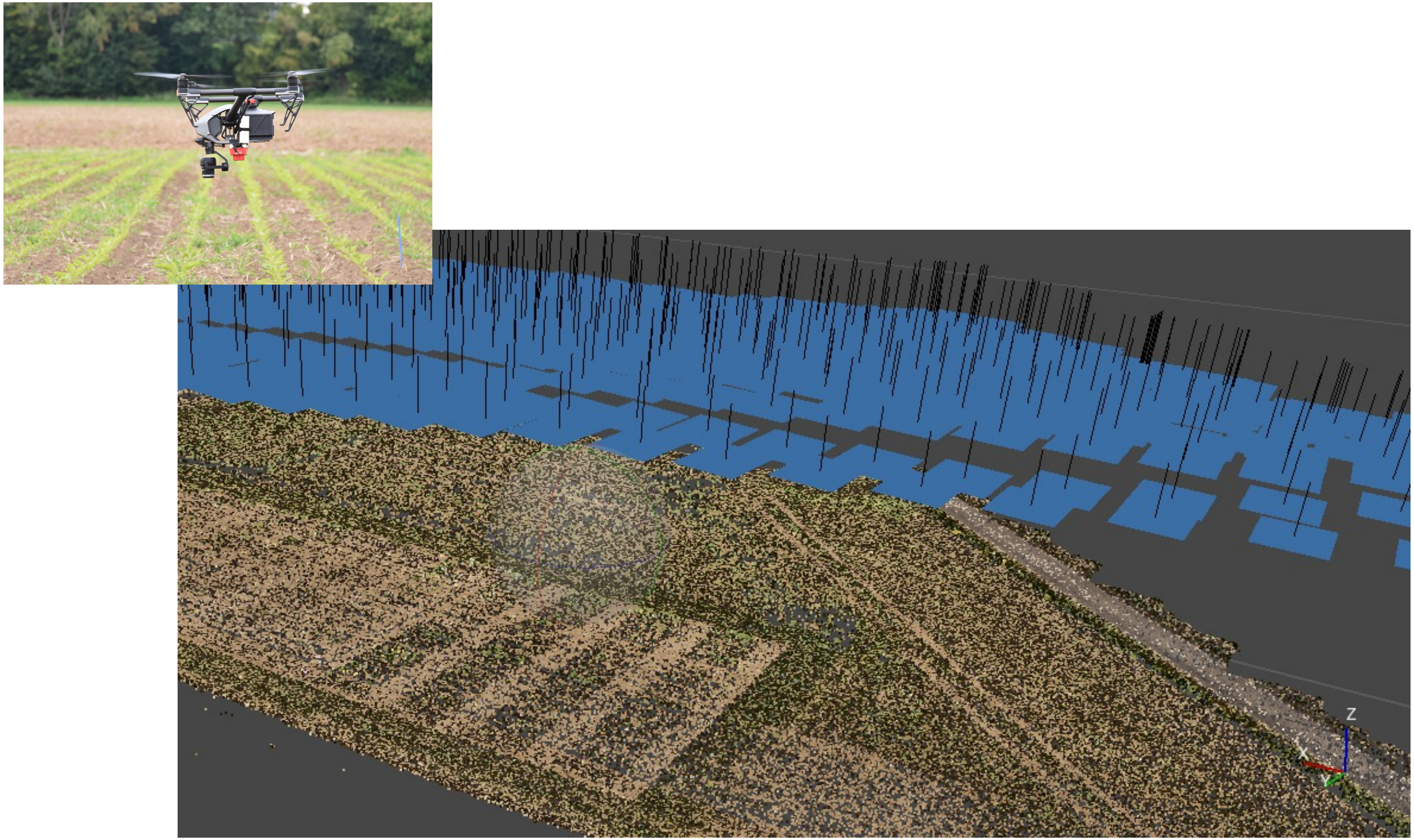
- Input image as acquired with an orthogonal parallel projection

Orthophoto from Aerial Images

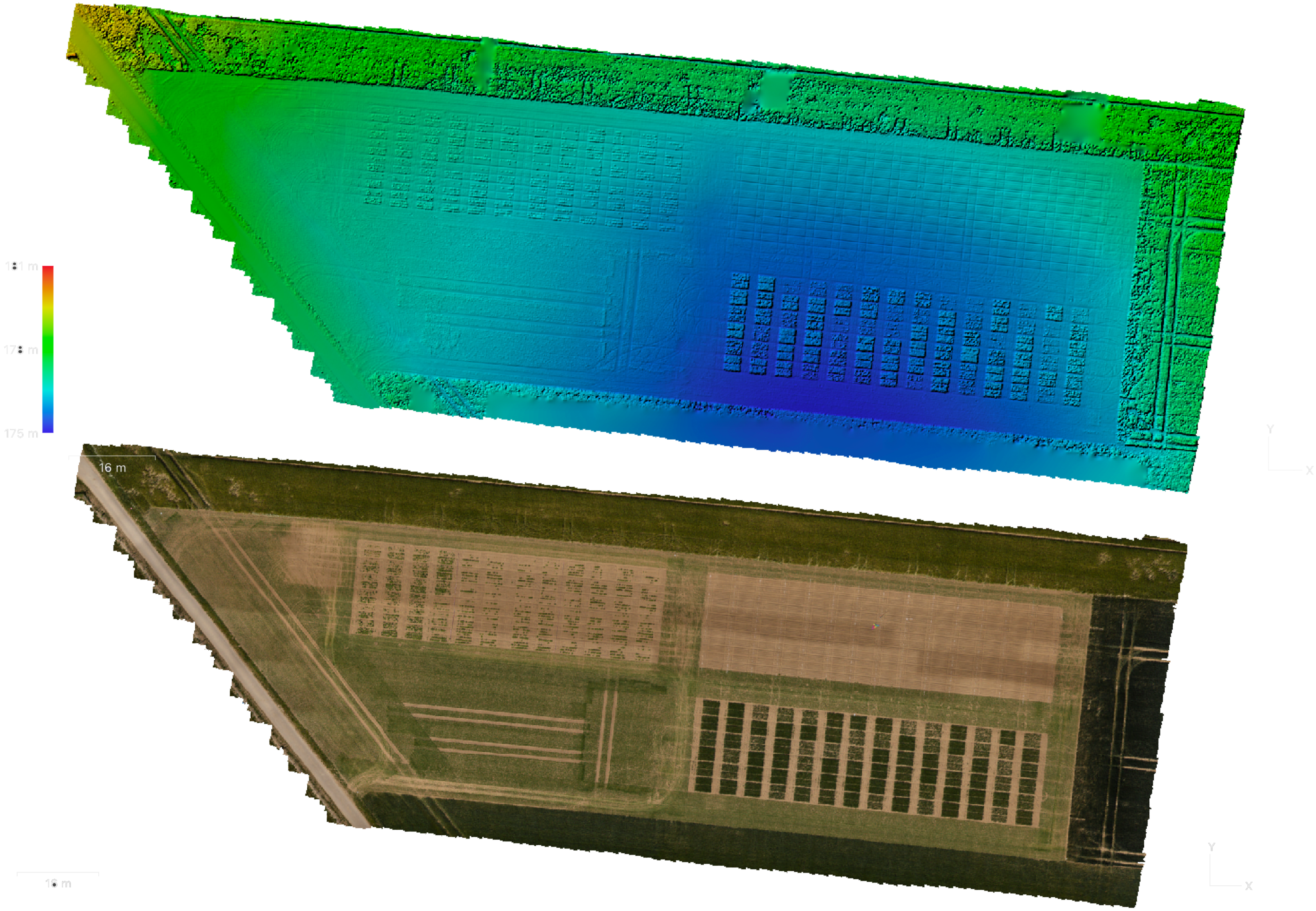
3D Scene Information

- DTM: Digital Terrain Model
  (DE: Digitales Höhenmodell, DHM)
- DSM: Digital Surface Model
  (DE: Digitales Oberflächenmodell, DOM)
- Airborne laser scanning
- Dense 3D reconstruction (using BA) from aerial images
- …

Example UAV Mapping



DSM and Orthophoto

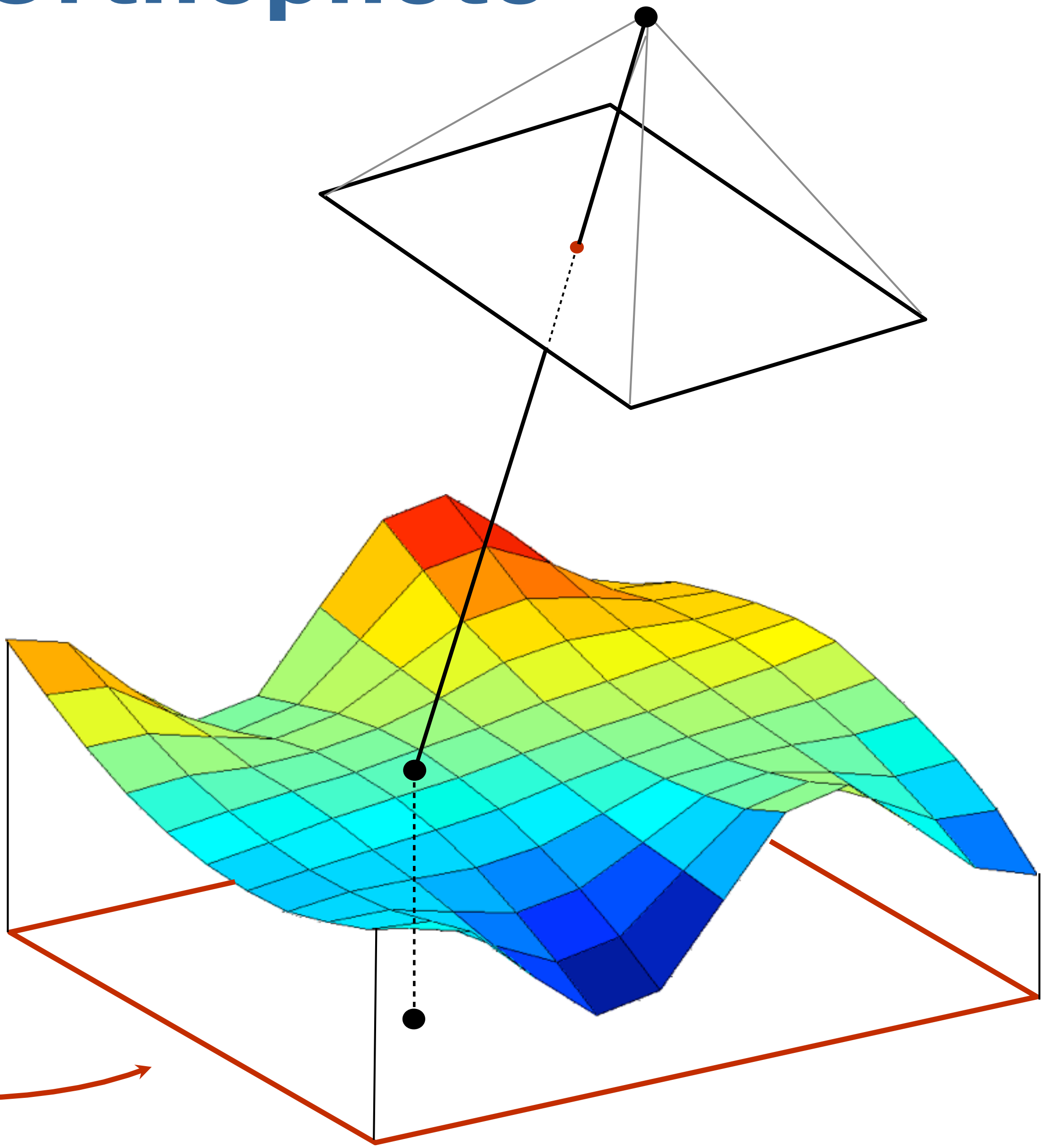


DTM vs. DSM

- Digital terrain models contain the terrain only and not buildings, vegetation, etc.
- Digital surface models also contain the objects such as building or vegetation
- Creating orthophotos requires **DSMs**

Image courtesy: Schindler

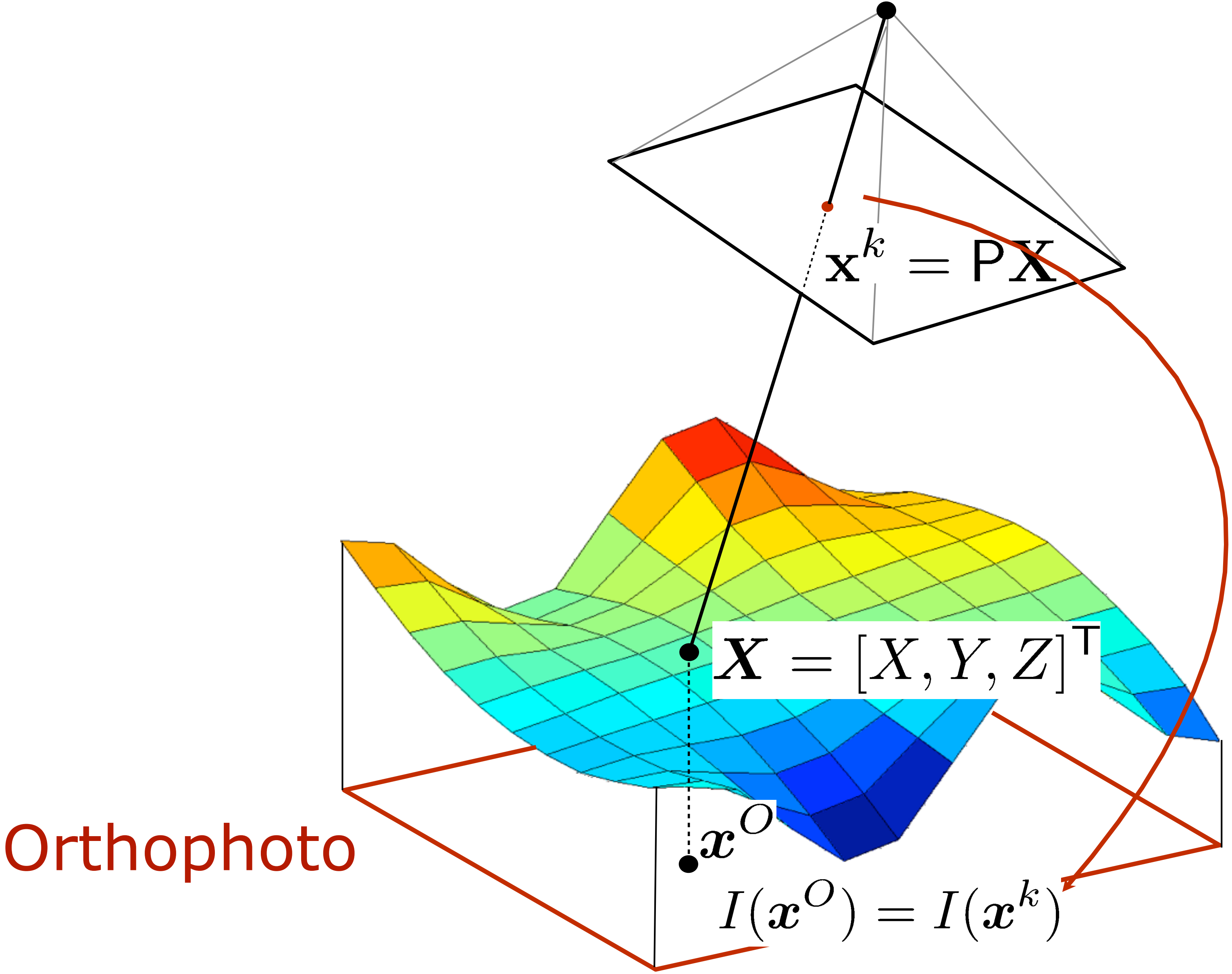
Creating an Orthophoto



Orthophoto

Image courtesy: Schindler

Rectification Illustrated



$\mathbf{x}^k = \mathsf{P}\boldsymbol{X}$

$\boldsymbol{X} = [X, Y, Z]^\mathsf{T}$

$\boldsymbol{x}^O$

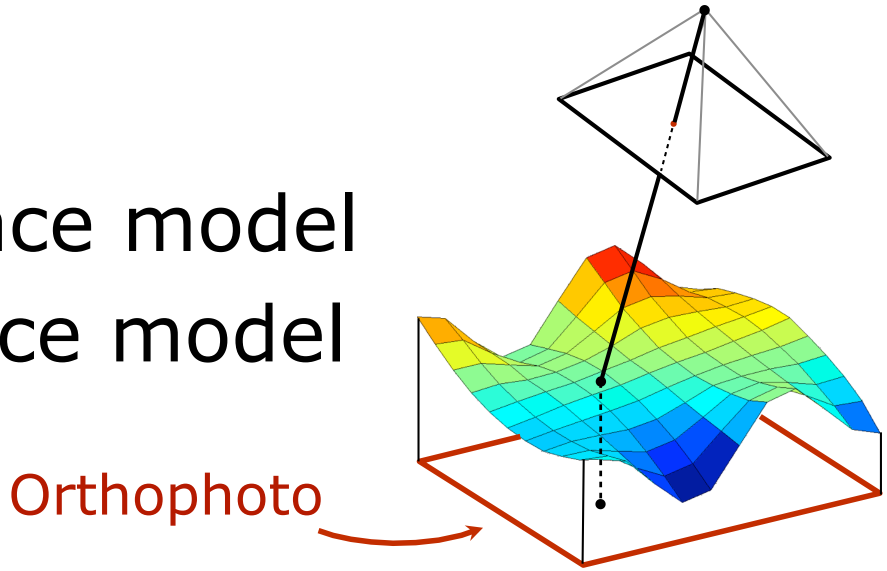
Orthophoto

$I(\boldsymbol{x}^O) = I(\boldsymbol{x}^k)$

Rectification

Idea

- Project image to surface model
- Project textured surface model onto (X,Y)-plane



Process (simplified)

- For each point $\boldsymbol{x}^O$ of the orthophoto
  - Determine the surface point $\boldsymbol{X} = [X, Y, Z]^\mathsf{T}$
  - Map the surface point to the original camera image $\mathbf{x}^k = \mathsf{P}\mathbf{X}$
  - Use image intensity value for the orthophoto: $I(\boldsymbol{x}^O) = I(\boldsymbol{x}^k)$

Image courtesy: Schindler

Rectification For a Pixel Raster That Does Not Match the Surface Model or Image

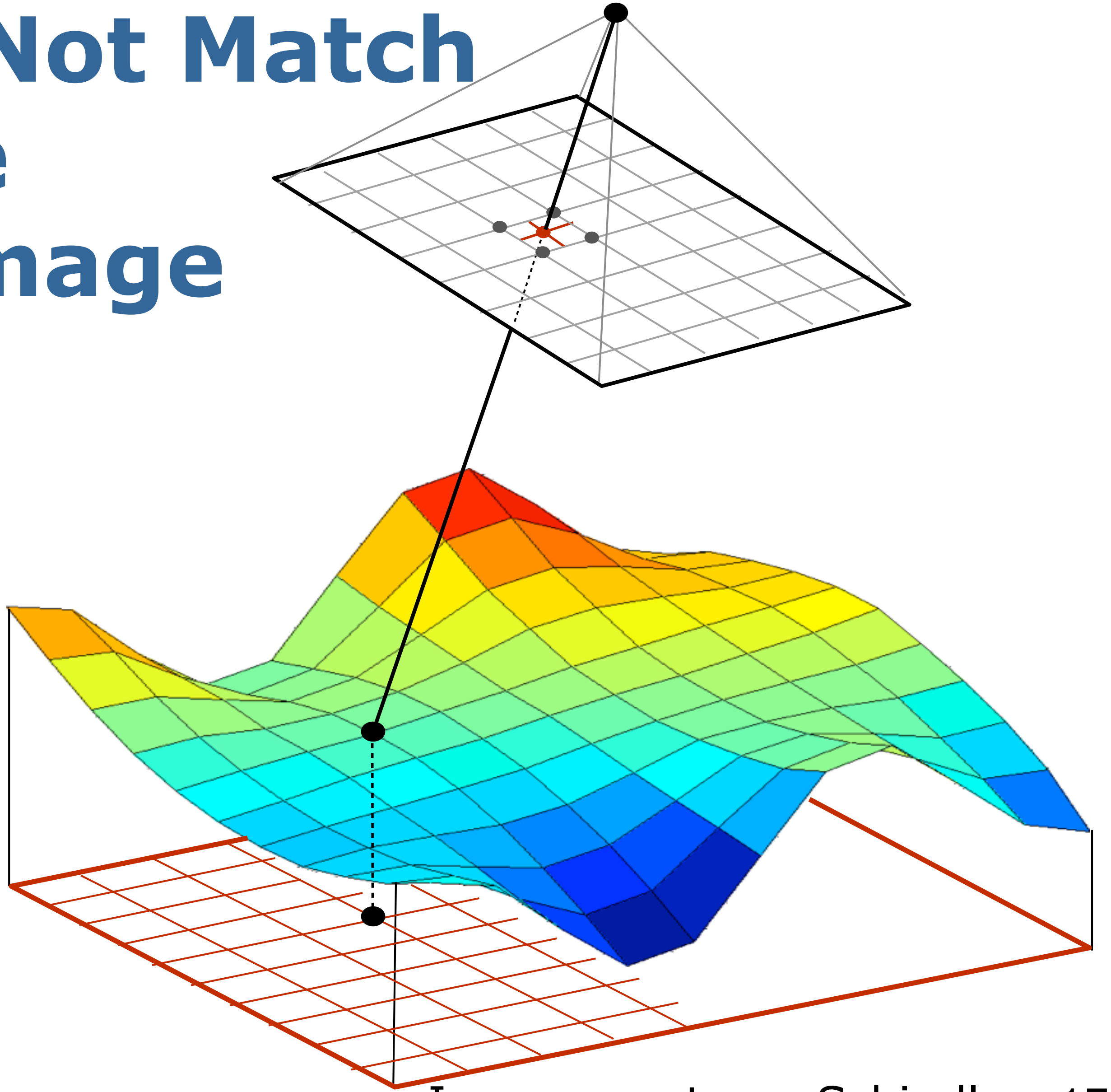


Image courtesy: Schindler

Rectification For a Pixel Raster

- Define pixels in the orthophoto plane
- Interpolate surface height at those positions
- Project 3D surface points to image points
- Interpolate intensity value for the orthophoto from the neighborhood in the original image

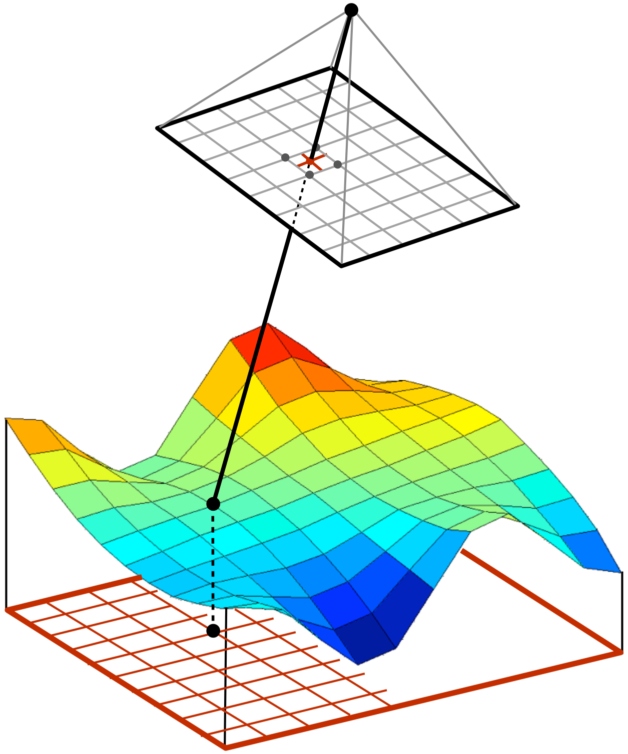


Image courtesy: Schindler

DSM Interpolation

- Interpolation of surface heights
- At every pixel of the orthophoto, determine the height via interpolation

Example

- Surface model in raster form (coarser than orthophoto)
- Bilinear interpolation

$$\begin{aligned} z \;=\; & (1-\Delta x) \cdot (1-\Delta y) \cdot z_{00} \;+ \\ & \Delta x \cdot (1-\Delta y) \cdot z_{01} \;+ \\ & (1-\Delta x) \cdot \Delta y \cdot z_{10} \;+ \\ & \Delta x \cdot \Delta y \cdot z_{11} \end{aligned}$$

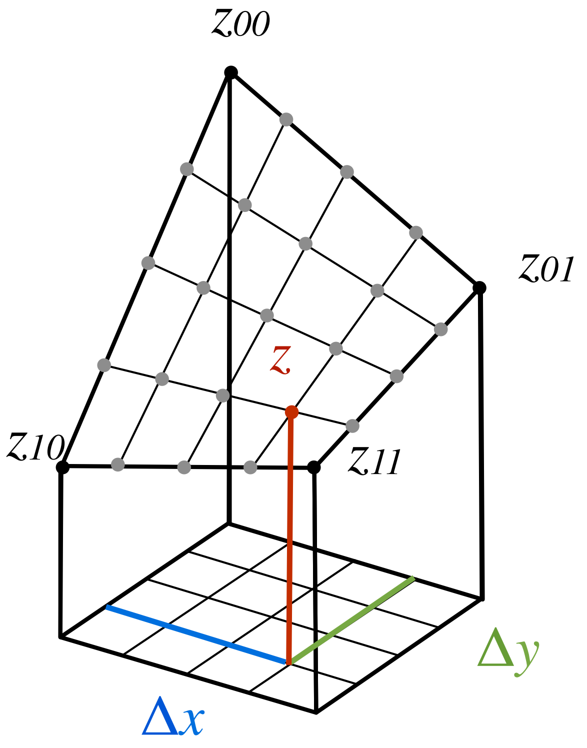


Image courtesy: Schindler

Resampling

- Estimate intensity values for the orthophoto
- For each raster point $\mathbf{X} = [X^O, Y^O, Z]^\mathsf{T}$ on the surface of the DSM
  - Project to camera image with $\mathbf{x}^k = \mathsf{P}\mathbf{X}$
  - Compute intensity $I(\mathbf{x}^k)$ from surrounding intensity values
  - Invert projection, i.e. assign $I(\mathbf{x}^O) = I(\mathbf{x}^k)$

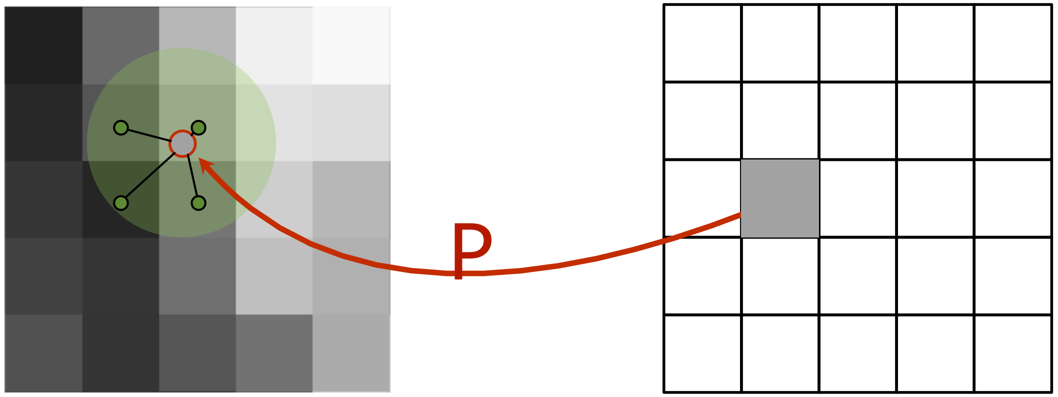


Image courtesy: Schindler

Multi-Channel Resampling

In multi-channel images compute intensities for each channel separately

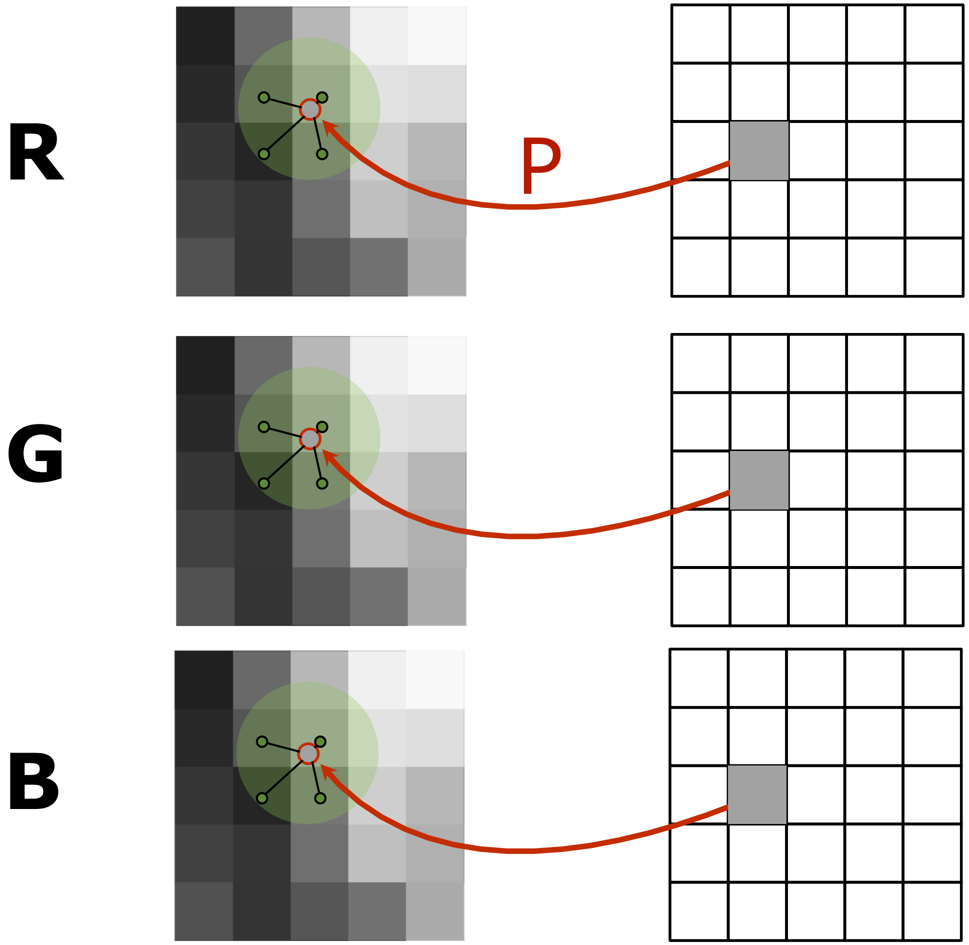
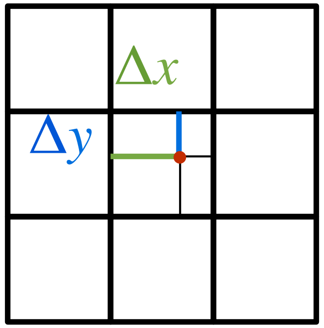


Image courtesy: Schindler

Intensities Off The Pixel Raster

Bicubic interpolation with (4x4) neighbors

- Cubic polynomial in each coordinate
- Very good results in practice



$$I(x,y) = \sum_{i\le 3}\sum_{j\le 3} c_{ij}\,\Delta x^i\,\Delta y^j$$

Image courtesy: Schindler

Image Resampling Issues: Aliasing

Downsampling may lead to aliasing

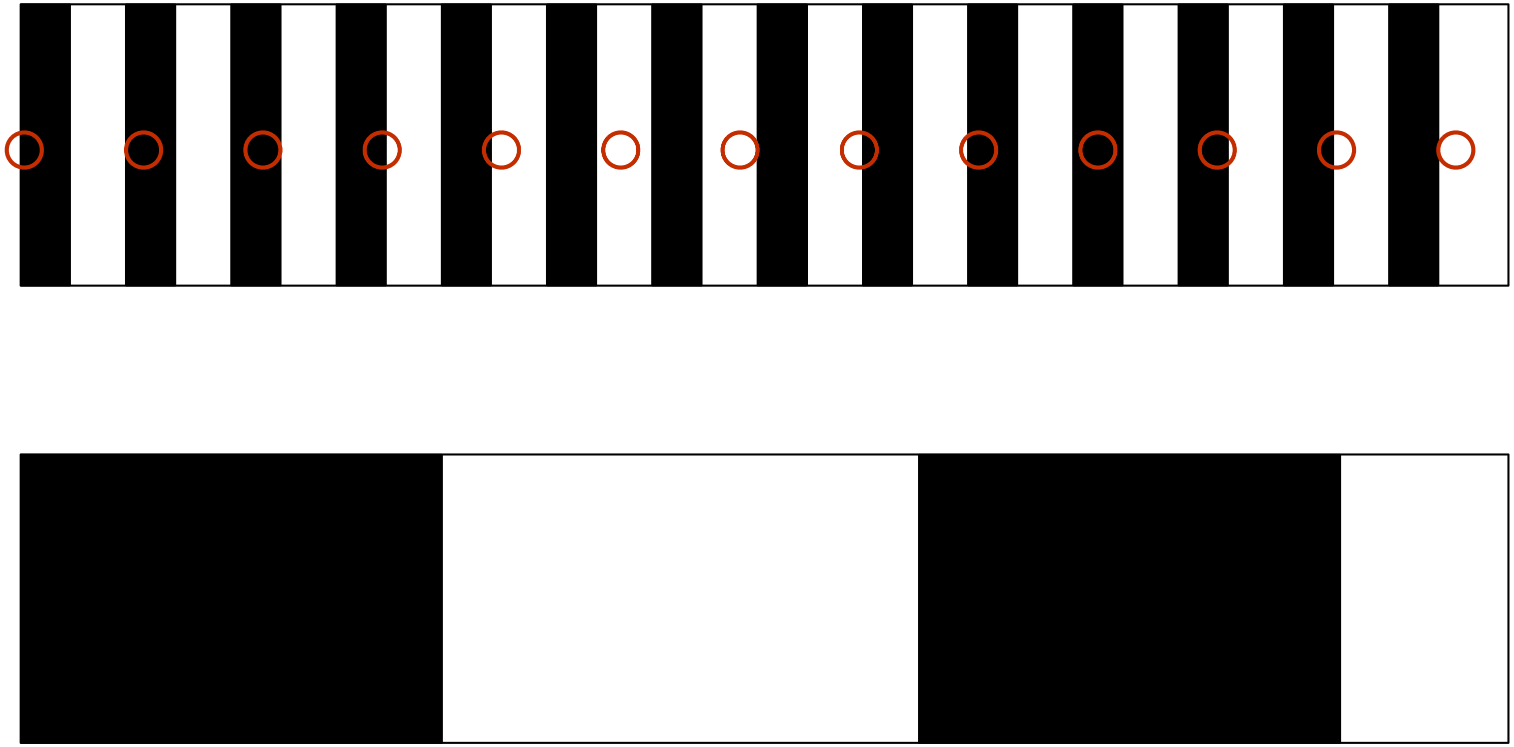
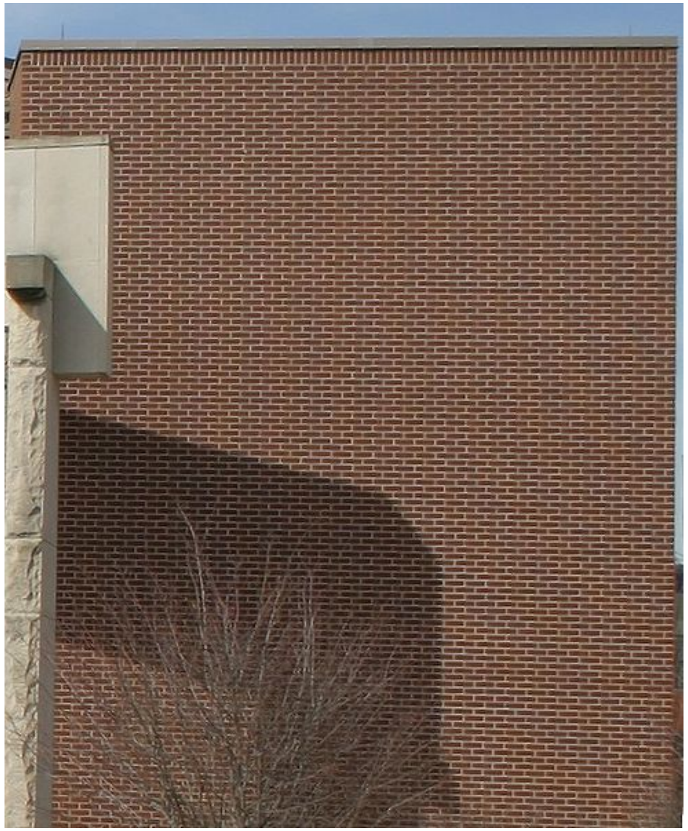


Image courtesy: Schindler

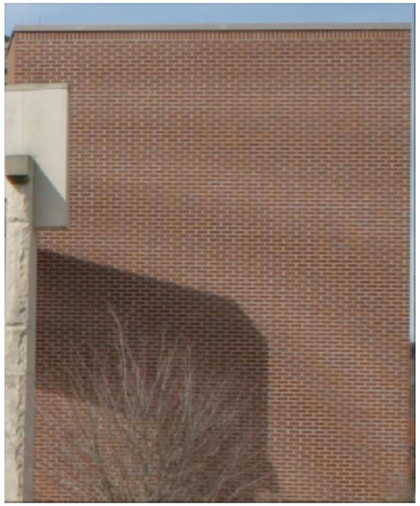
Smoothing to Reduce Aliasing

60%



input image

no smoothing



$\sigma = 0.5 \cdot 1 \,/\, 0.6$

$\sigma = 1.0 \cdot 1 \,/\, 0.6$

$\sigma = 2.0 \cdot 1 \,/\, 0.6$

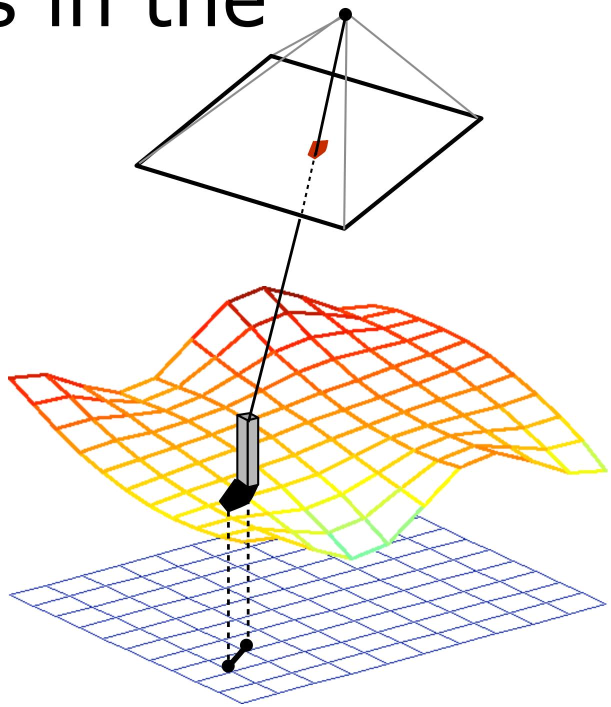
Image courtesy: Schindler

Orthophoto Errors

Two Main Error Sources

Inaccurate DSM

- Height errors in the surface model are mapped to planimetric errors in the orthophoto
- Points are displaced in radial direction w.r.t. the nadir

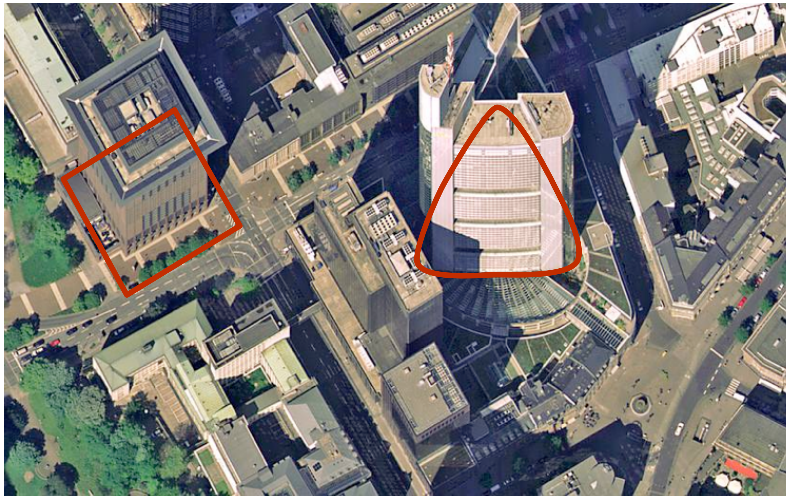


Occlusions

- Strong relief of the surface leads to occlusions
- Intensities required for the orthophoto are not found in the original image

Inaccurate DSM

- Off-the-shelf terrain models do not contain buildings and vegetation
- High-quality surface models and 3D city models improves the situation



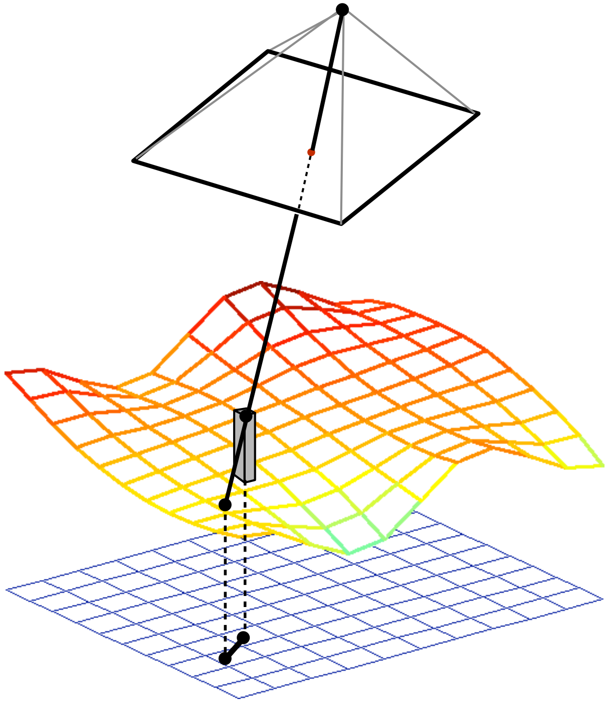
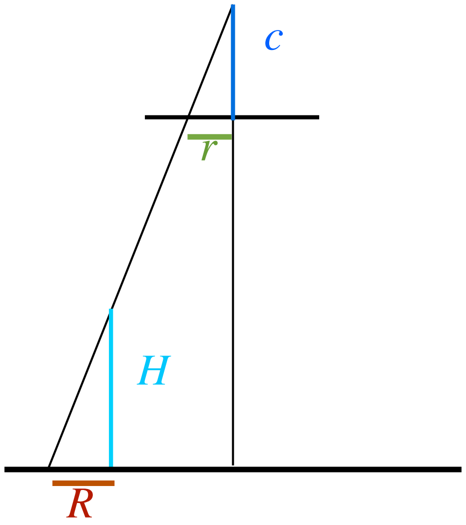


Image courtesy: Schindler

Vertical DSM Errors Lead to a Radial Displacement

- Vertical errors in the DSM are mapped to radial planimetric errors
- Error depends on the
  - Magnitude of the height error
  - Orientation
  - Point's position in the image
  - Surface slope

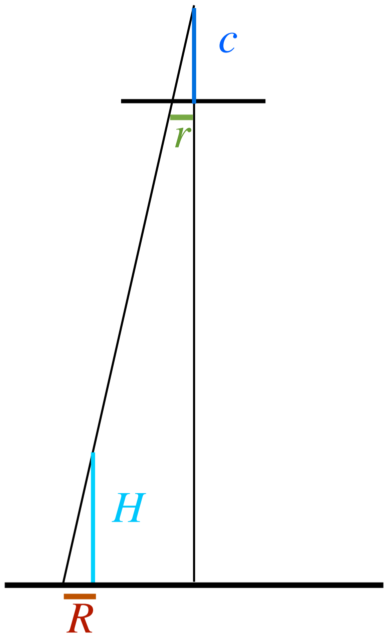


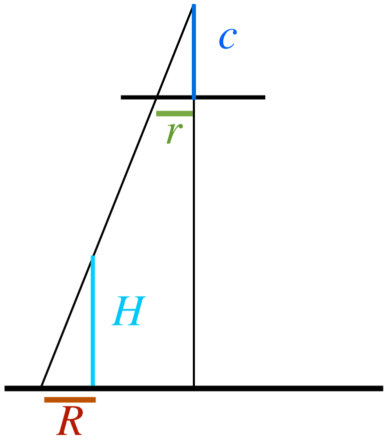
Radial Displacement Errors

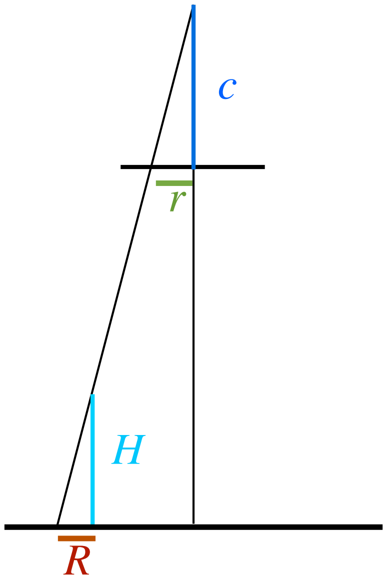
Radial displacement increases with

- increasing distance from the principal point (nadir view)
- Smaller camera constant (or lower flying altitude)

$$R = \frac{r}{c} H$$







Occlusions

- Strong reliefs lead to occlusions
- Regions that are visible in the orthophoto are not observable in the original image
- Perspective foreshortening is too strong to extract usable texture (although visible)

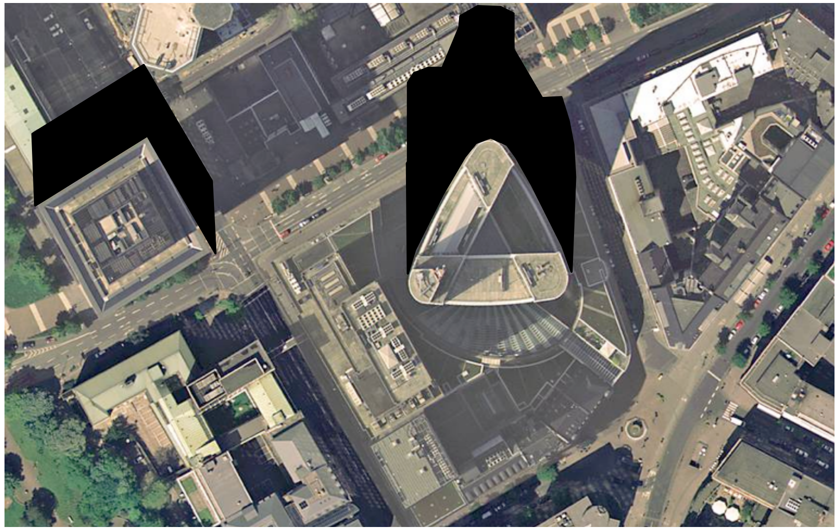
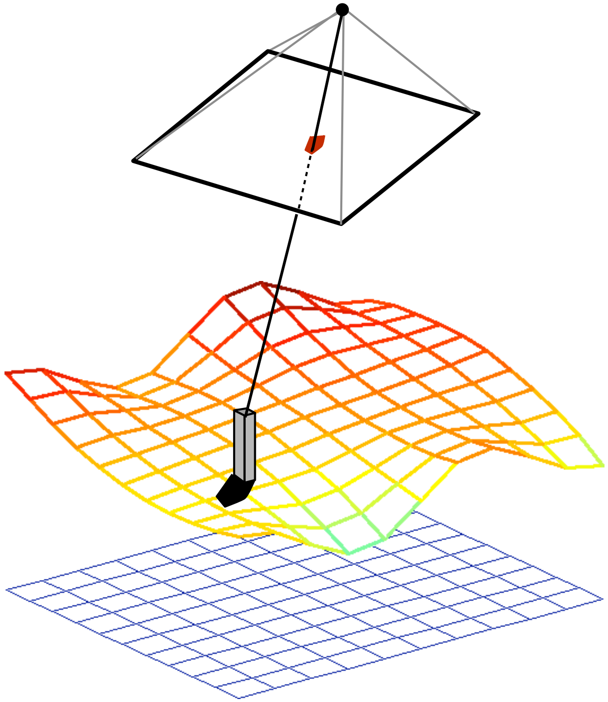


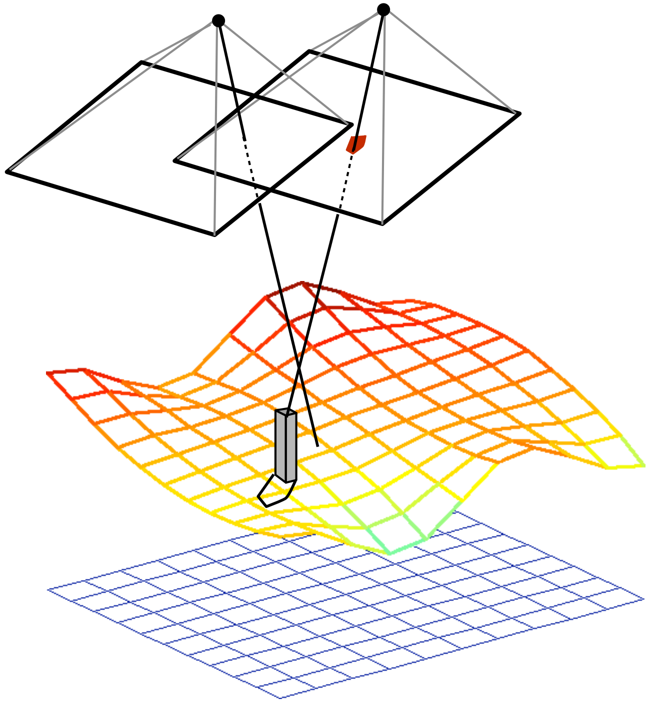
Image courtesy: Schindler



True Orthophoto

- Detection of occlusion using the DSM
- Use of multiple images to obtain an orthophoto without holes
- Often called “true orthophoto”

Image courtesy: Schindler



Orthophotos from Aerial Images

- Given aerial images and 3D data, we can compute an orthophoto
- Precise height information is key
- Combine multiple photos to avoid occlusions (true orthophoto)

Orthophoto Stitching: Combining Multiple Orthophotos

Orthophoto Stitching

Three tasks

- Global lighting normalization
- Blending of values from different images
- Seam-carving

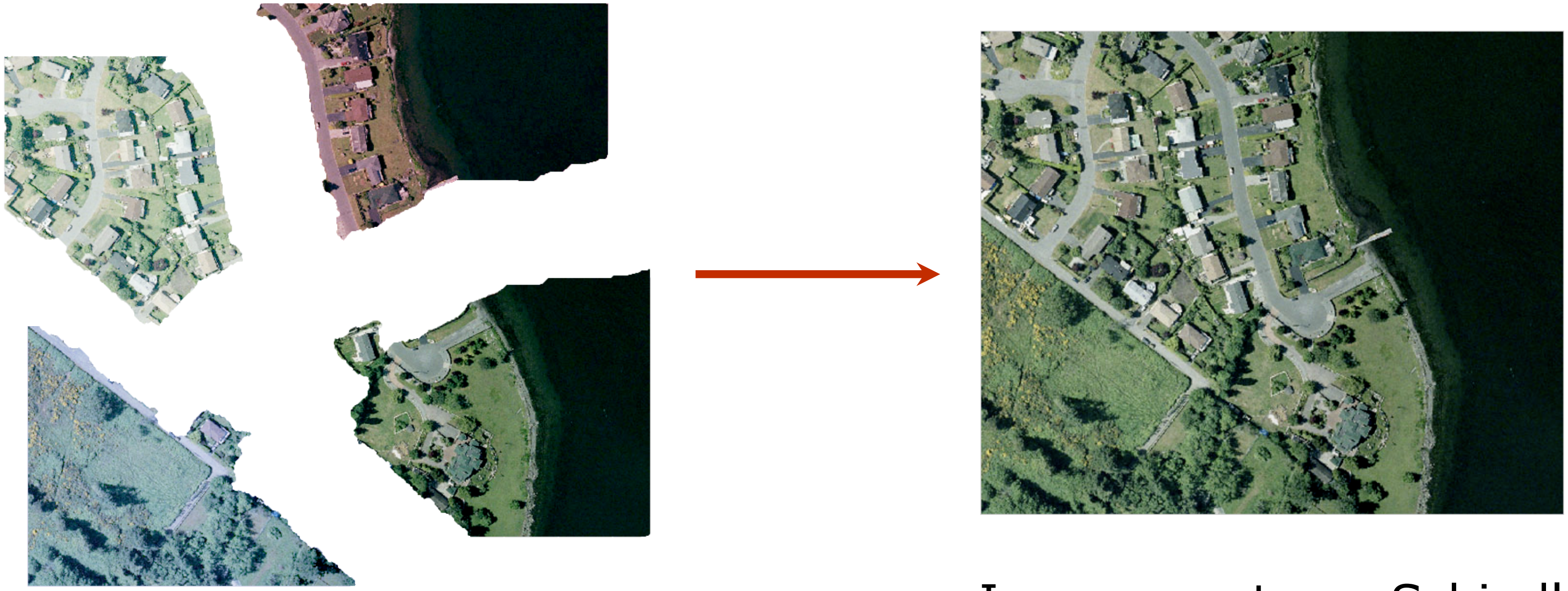
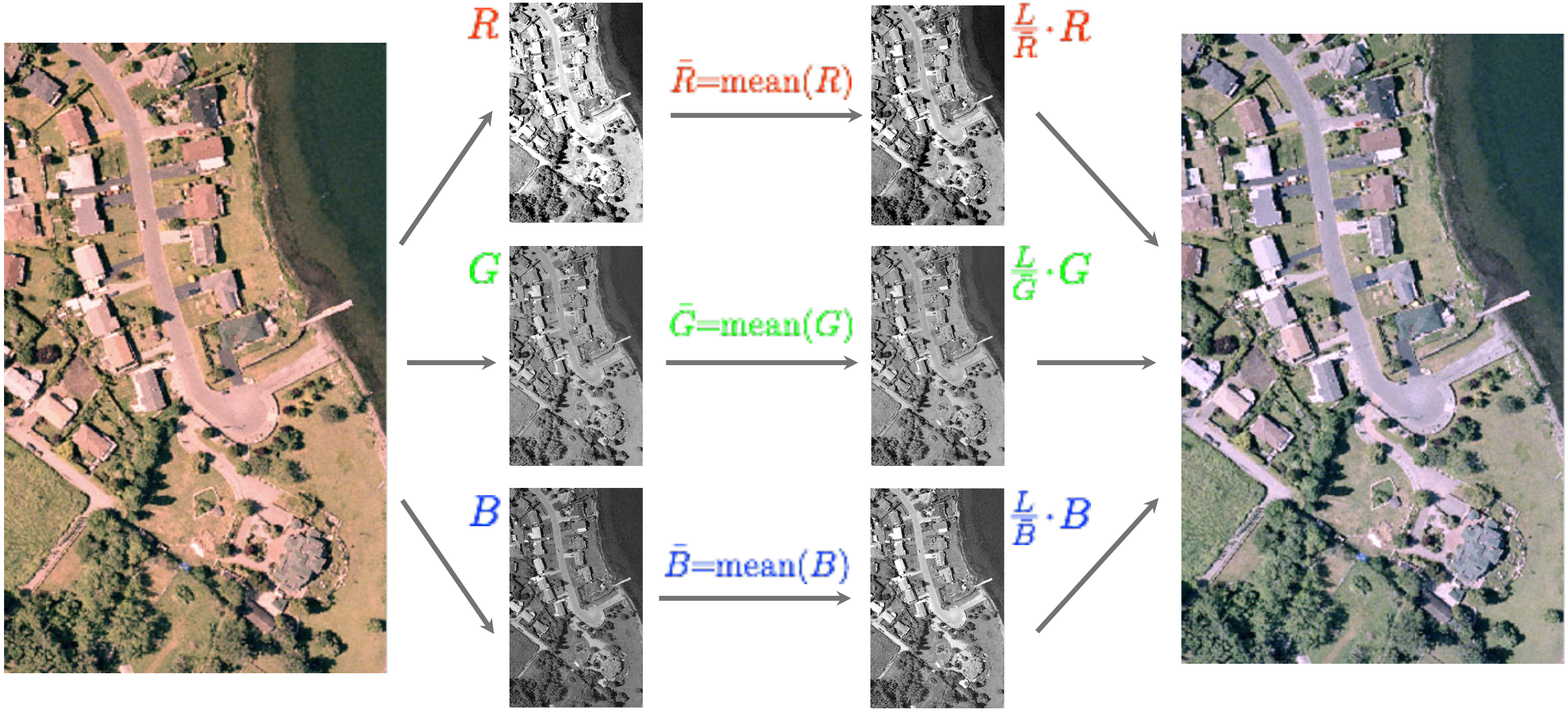


Image courtesy: Schindler

Lighting Normalization

- “On average, the world is gray”
- Change each image such that the average color is gray



$$L = \frac{1}{3}(\bar{R} + \bar{G} + \bar{B})$$

Image courtesy: Schindler

Blending

- Orthophotos inherit intensities from the taken images
- Intensities vary between images (lighting changes, exposure, ...)

Image courtesy: Schindler

Linear Blending

- Weighted averaging of intensity values
- Weights decrease linearly towards the boundary

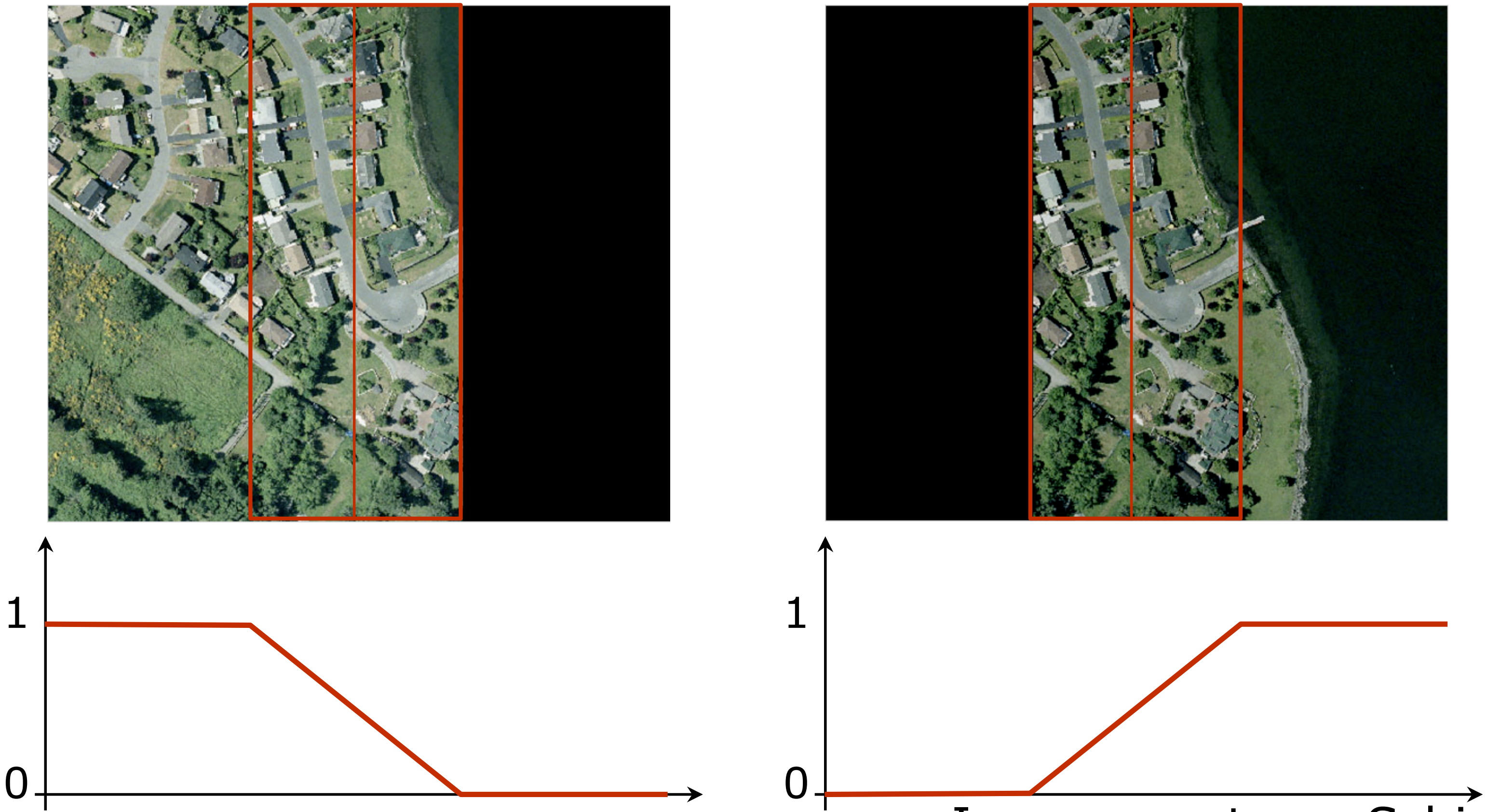


Image courtesy: Schindler

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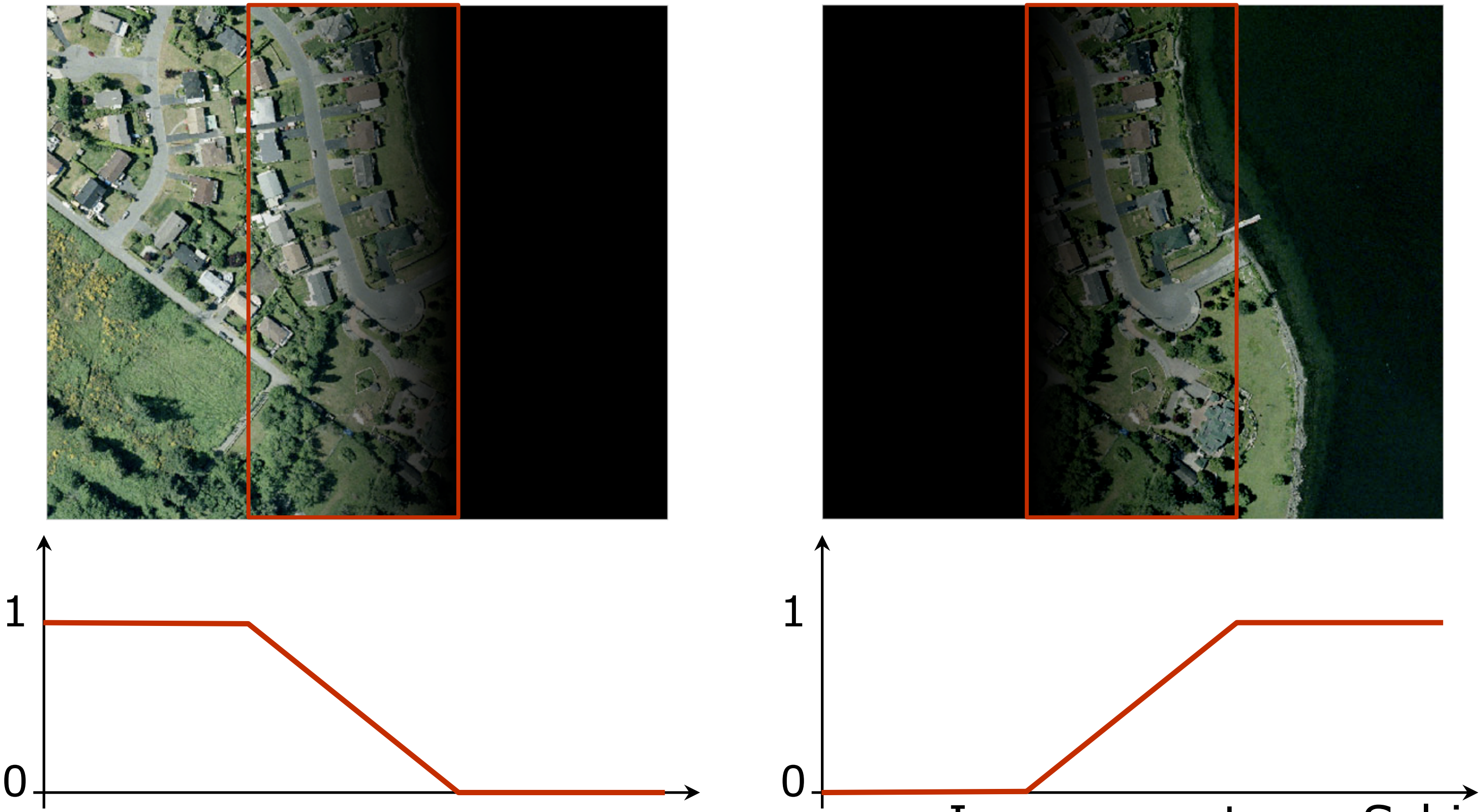

Image courtesy: Schindler

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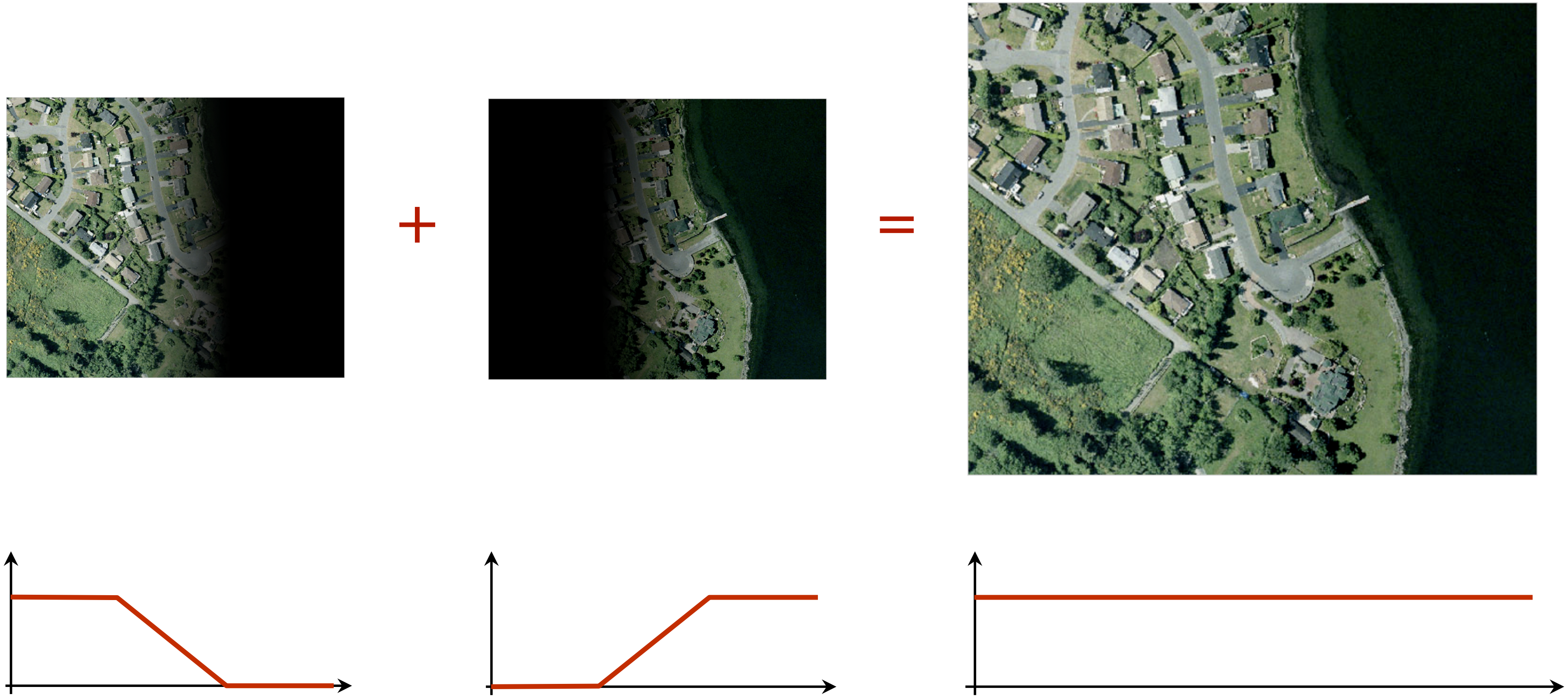
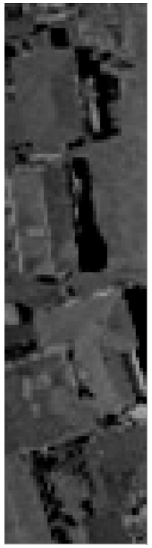


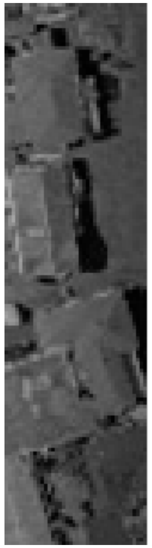
Image courtesy: Schindler

Finding Seams

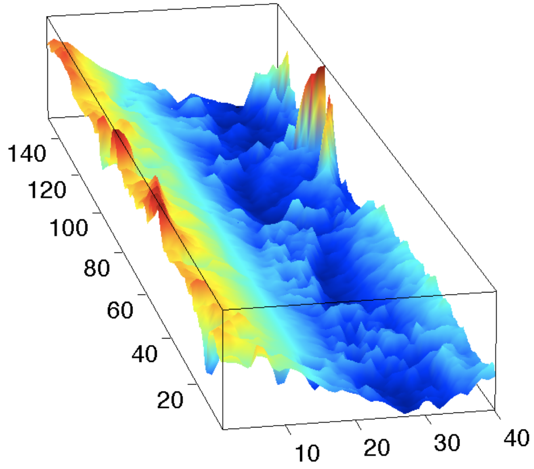
- Optimal seam by computing a shortest path
- Method is globally optimal: among all possible paths connecting the “upper” and “lower” borders of the region, the seam has the lowest cumulative error



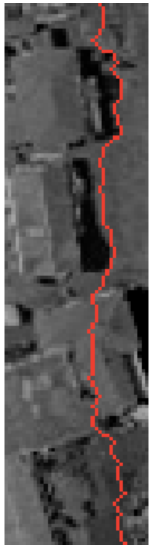
overlap
region in I



overlap
region in J



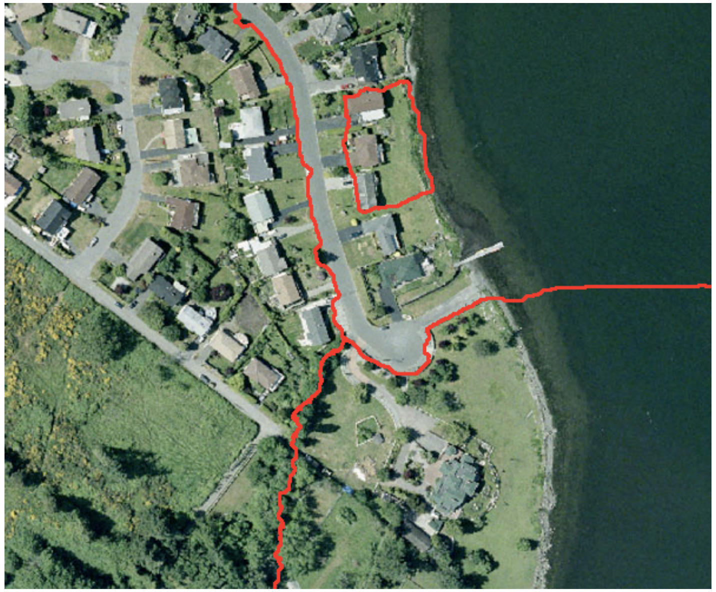
error
$e = \sqrt{(I-J)^2}$



optimal
seam

Distance Transform Blending

- More general form of blending
- Weight depends on distance from the seam
- Weights are normalized to sum to 1
- Also called "feathering"



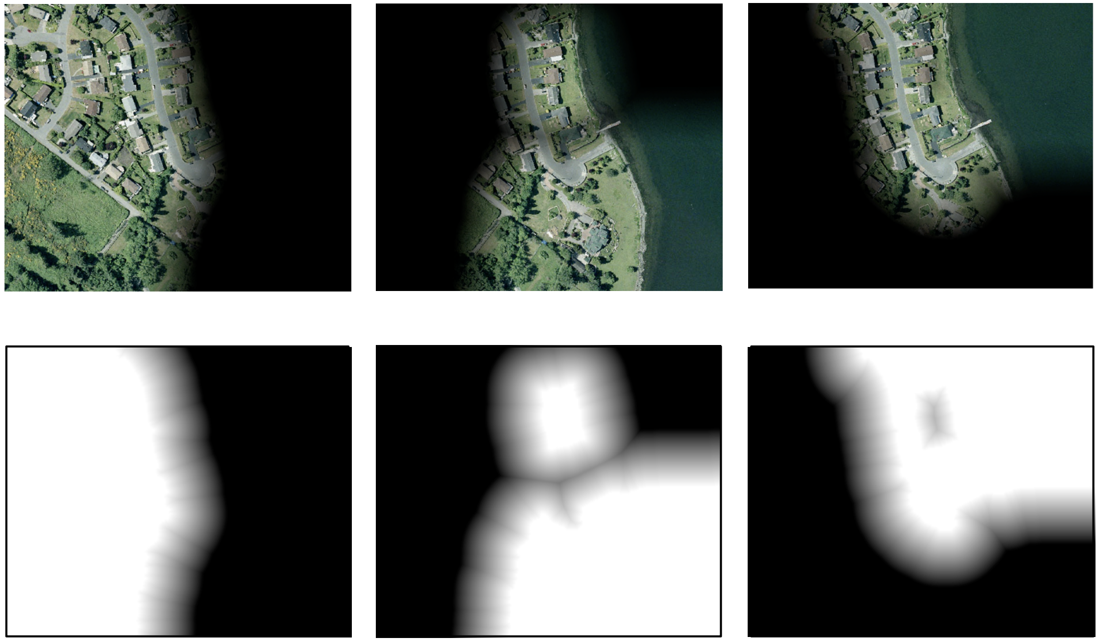


Image courtesy: Schindler

Orthophotos from Aerial Images

- Given aerial images and 3D data, we can compute an orthophoto
- Precise height information is key
- Combine multiple photos to avoid occlusions (true orthophoto)
- Combining multiple orthophotos requires stitching

Special Case:
Orthophoto of a Planar Surface

Rectification For the Surface of a Planar Object

Orthophoto plane to object plane

- **Affine**, 6 parameters: position, orientation, anisotropic scaling and shear of coordinate axis

Object plane and image plane

- **Projectivity** (line-preserving): 8 parameters for homography
- Combination is also a **projectivity**

$$\mathrm{x}' = \mathrm{H}\mathrm{x}$$

3x3 matrix

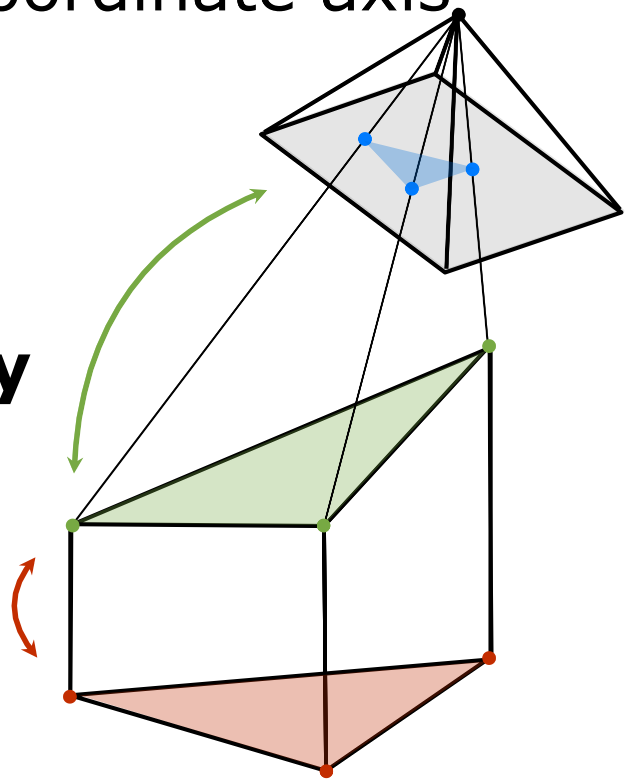


Image courtesy: Schindler

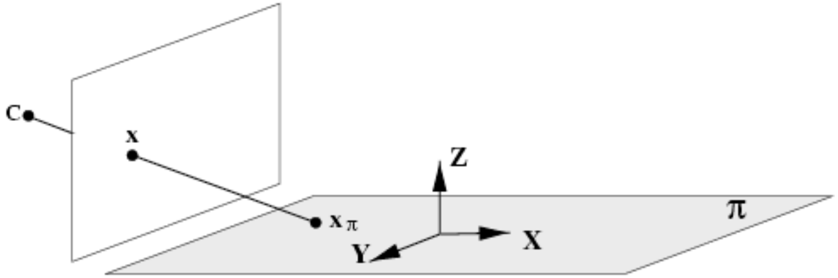
Rectification For the Surface of a Planar Object

- Assumption: object in (X,Y)-plane
  (can always be achieved by a rigid body transformation)

$$\mathbf{x} = \begin{bmatrix} \tilde{p}_{11} & \tilde{p}_{12} & \tilde{p}_{14} \\ \tilde{p}_{21} & \tilde{p}_{22} & \tilde{p}_{24} \\ \tilde{p}_{31} & \tilde{p}_{32} & \tilde{p}_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathsf{H} = \begin{bmatrix} \tilde{p}_{11} & \tilde{p}_{12} & \tilde{p}_{14} \\ \tilde{p}_{21} & \tilde{p}_{22} & \tilde{p}_{24} \\ \tilde{p}_{31} & \tilde{p}_{32} & \tilde{p}_{34} \end{bmatrix}^{-1}$$

$$\mathbf{x}' = \mathsf{H}\mathbf{x}$$



- Note: other than perspective projection, the projectivity is bijective and thus invertible

Image courtesy: Schindler

Planar Rectification Direct Estimation of the Homography

- All we need are the 8 unknowns of H
- A pair of corresponding points yield 2 linearly independent equations
- ≥4 correspondences (no 3 of them on a line) yield ≥8 equations
- Solve via singular value decomposition
- Neither the interior nor the exterior orientation need to be known!

Setting Up the Equations for Determining the Parameter of H

- For multiple (min 4) points, we obtain

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \underset{3\times3}{\mathsf{H}} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix} \qquad i = 1, \ldots, I$$

- Compute the values for H from eqns

Setting Up the Equations for Determining the Parameter of H

- For multiple (min 4) points, we obtain

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \underset{3\times 3}{\mathsf{H}} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix} \qquad i = 1, \ldots, I$$

- Compute the values for H from eqns
- **Identical solution** to computing the **DLT / Zhang's method** for calibration (Photogrammetry 1)

(reminder)

Rearrange the Equation

$$\mathbf{x}_i = \underset{3\times 3}{\mathsf{H}} \mathbf{X}_i = \begin{bmatrix} \boxed{p_{11} \quad p_{12} \quad p_{13}} \\ \boxed{p_{21} \quad p_{22} \quad p_{23}} \\ \boxed{p_{31} \quad p_{32} \quad p_{33}} \end{bmatrix} \mathbf{X}_i$$

$$= \begin{bmatrix} \boxed{\mathbf{A}^\mathsf{T}} \\ \boxed{\mathbf{B}^\mathsf{T}} \\ \boxed{\mathbf{C}^\mathsf{T}} \end{bmatrix} \mathbf{X}_i$$

(reminder)

Rearrange the Equation

$$\mathbf{x}_i = \underset{3\times3}{\mathsf{H}}\,\mathbf{X}_i = \begin{bmatrix} \boxed{p_{11} \quad p_{12} \quad p_{13}} \\ \boxed{p_{21} \quad p_{22} \quad p_{23}} \\ \boxed{p_{31} \quad p_{32} \quad p_{33}} \end{bmatrix} \mathbf{X}_i$$

$$= \begin{bmatrix} \boxed{\mathbf{A}^\mathsf{T}} \\ \boxed{\mathbf{B}^\mathsf{T}} \\ \boxed{\mathbf{C}^\mathsf{T}} \end{bmatrix} \mathbf{X}_i$$

$$\Updownarrow$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}^\mathsf{T}\mathbf{X}_i \\ \mathbf{B}^\mathsf{T}\mathbf{X}_i \\ \mathbf{C}^\mathsf{T}\mathbf{X}_i \end{bmatrix}$$

(reminder)

Rearrange the Equation

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}^\top \mathbf{X}_i \\ \mathbf{B}^\top \mathbf{X}_i \\ \mathbf{C}^\top \mathbf{X}_i \end{bmatrix}$$

➡

$$x_i = \frac{u_i}{w_i} = \frac{\mathbf{A}^\top \mathbf{X}_i}{\mathbf{C}^\top \mathbf{X}_i} \qquad y_i = \frac{v_i}{w_i} = \frac{\mathbf{B}^\top \mathbf{X}_i}{\mathbf{C}^\top \mathbf{X}_i}$$

(reminder)

Rearrange the Equation

$$x_i = \frac{\mathbf{A}^\top \mathbf{X}_i}{\mathbf{C}^\top \mathbf{X}_i} \quad \Rightarrow \quad x_i\, \mathbf{C}^\top \mathbf{X}_i - \mathbf{A}^\top \mathbf{X}_i = 0$$

$$y_i = \frac{\mathbf{B}^\top \mathbf{X}_i}{\mathbf{C}^\top \mathbf{X}_i} \quad \Rightarrow \quad y_i\, \mathbf{C}^\top \mathbf{X}_i - \mathbf{B}^\top \mathbf{X}_i = 0$$

Leads to a system of equations, which is **linear in the parameters $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$**

$$\begin{aligned} -\mathbf{X}_i^\top \mathbf{A} \qquad\qquad\qquad & +x_i\, \mathbf{X}_i^\top \mathbf{C} = 0 \\ -\mathbf{X}_i^\top \mathbf{B} \quad & +y_i\, \mathbf{X}_i^\top \mathbf{C} = 0 \end{aligned}$$

(reminder)

Estimating the Elements of H

- Collect the elements of H within a parameter vector $\boldsymbol{p}$

$$\boldsymbol{p} = (p_k) = \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \end{bmatrix} = \mathrm{vec}(\mathsf{H}^{\mathsf{T}})$$

rows of H as column-vectors, one below the other (9x1)

(reminder)

Estimating the Elements of H

- Rewrite

$$-\mathbf{X}_i^\mathsf{T}\mathbf{A} + x_i\,\mathbf{X}_i^\mathsf{T}\mathbf{C} = 0$$

$$-\mathbf{X}_i^\mathsf{T}\mathbf{B} + y_i\,\mathbf{X}_i^\mathsf{T}\mathbf{C} = 0$$

- as

$$\boldsymbol{a}_{x_i}^\mathsf{T}\boldsymbol{p} = 0$$

$$\boldsymbol{a}_{y_i}^\mathsf{T}\boldsymbol{p} = 0$$

(reminder)

Estimating the Elements of H

- Rewrite

$$\begin{aligned} -\mathbf{X}_i^\top \mathbf{A} \qquad\qquad & + x_i\, \mathbf{X}_i^\top \mathbf{C} = 0 \\ -\mathbf{X}_i^\top \mathbf{B} & + y_i\, \mathbf{X}_i^\top \mathbf{C} = 0 \end{aligned}$$

- as

$$\begin{aligned} \boldsymbol{a}_{x_i}^\top \boldsymbol{p} &= 0 \\ \boldsymbol{a}_{y_i}^\top \boldsymbol{p} &= 0 \end{aligned}$$

- with

$$\begin{aligned} \boldsymbol{p} &= (p_k) = \text{vec}(\mathsf{H}^\top) \\ \boldsymbol{a}_{x_i}^\top &= (-X_i,\, -Y_i,\, -1, 0,\, 0,\, 0, x_i X_i,\, x_i Y_i,\, x_i) \\ \boldsymbol{a}_{y_i}^\top &= (0,\, 0,\, 0, -X_i,\, -Y_i,\, -1, y_i X_i,\, y_i Y_i,\, y_i) \end{aligned}$$

- Homogeneous equation => use SVD!

(reminder)

Solution Summary

1. Vectorize H : $\boldsymbol{p} = (p_k) = \mathrm{vec}(\mathsf{H}^{\mathsf{T}})$
2. Build the M for the linear system

$$\mathsf{M} = \begin{bmatrix} \boldsymbol{a}_{x_1}^{\mathsf{T}} \\ \boldsymbol{a}_{y_1}^{\mathsf{T}} \\ \cdots \\ \boldsymbol{a}_{x_I}^{\mathsf{T}} \\ \boldsymbol{a}_{y_I}^{\mathsf{T}} \end{bmatrix}$$

$$\underset{\substack{\uparrow \\ \text{2Ix9}}}{\mathsf{M}}\;\underset{\substack{\nwarrow \\ \text{9x1}}}{\boldsymbol{p}} \overset{!}{=} 0$$

with

$$\begin{aligned} \boldsymbol{a}_{x_i}^{\mathsf{T}} &= (-X_i,\, -Y_i,\, -1, 0,\, 0,\, 0, x_i X_i,\, x_i Y_i,\, x_i) \\ \boldsymbol{a}_{y_i}^{\mathsf{T}} &= (0,\, 0,\, 0, -X_i,\, -Y_i,\, -1, y_i X_i,\, y_i Y_i,\, y_i) \end{aligned}$$

(reminder)

Solution Summary

3. Solve by SVD $\mathrm{M} = U\,\mathrm{S}\,V^{\mathrm{T}}$
   Solution is last column of V

$$\widehat{\boldsymbol{p}} = \boldsymbol{v}_9 \;\Rightarrow\; \widehat{\mathrm{H}} = \begin{bmatrix} \widehat{p_1} & \widehat{p_2} & \widehat{p_3} \\ \widehat{p_4} & \widehat{p_5} & \widehat{p_6} \\ \widehat{p_7} & \widehat{p_8} & \widehat{p_9} \end{bmatrix}$$

Example: Facade Orthophoto

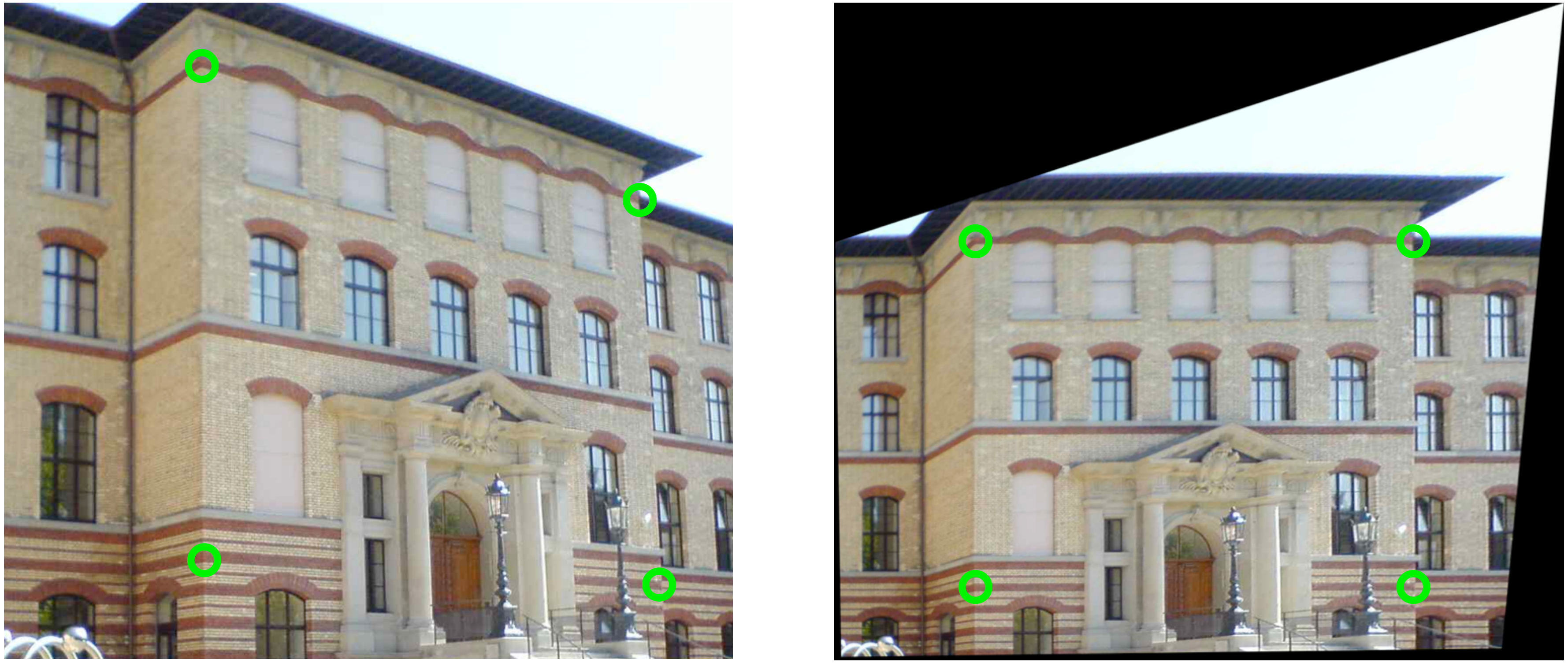


Image courtesy: Schindler

Summary

- Orthophotos are an important element for aligning aerial images with maps
- Allow for measuring in the orthophoto
- Prerequisites
  - Aerial image(s)
  - Orientation parameters
  - 3D information (DSM)
- Other solutions for special cases (planes)

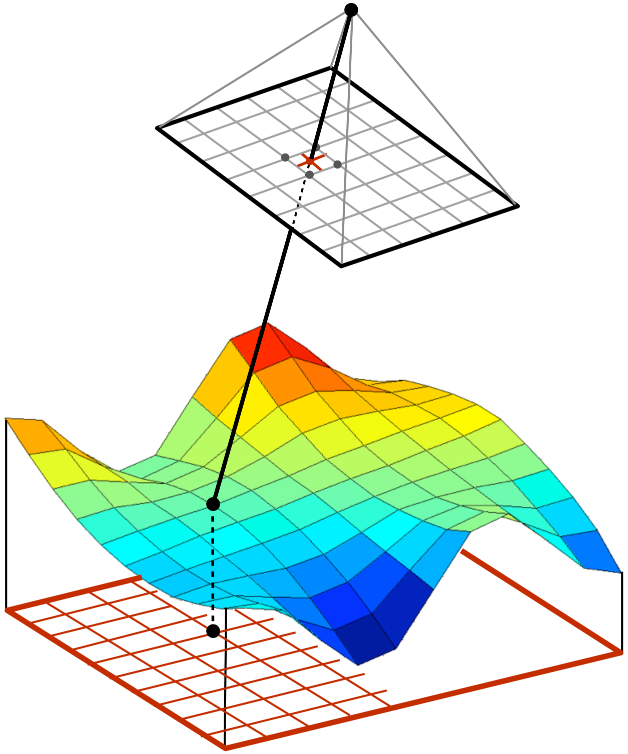


Image courtesy: Schindler

Slide Information

- These slides are an adapted version of the orthophoto slides by Konrad Schindler from ETH Zurich. Providing his slides has been highly appreciated.
- The slides have been adapted by Cyrill Stachniss as part of the Photogrammetry II course.
- I tried to acknowledge all people from whom I used images or videos. In case I made a mistake or missed someone, please let me know.
- If you are a university lecturer, feel free to use the course material. If you adapt the course material, please make sure that you keep the acknowledgements to others and please acknowledge me as well. To satisfy my own curiosity, please send me email notice if you use my slides.

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