Photogrammetry & Robotics Lab

Iterative Solution for the Relative Orientation

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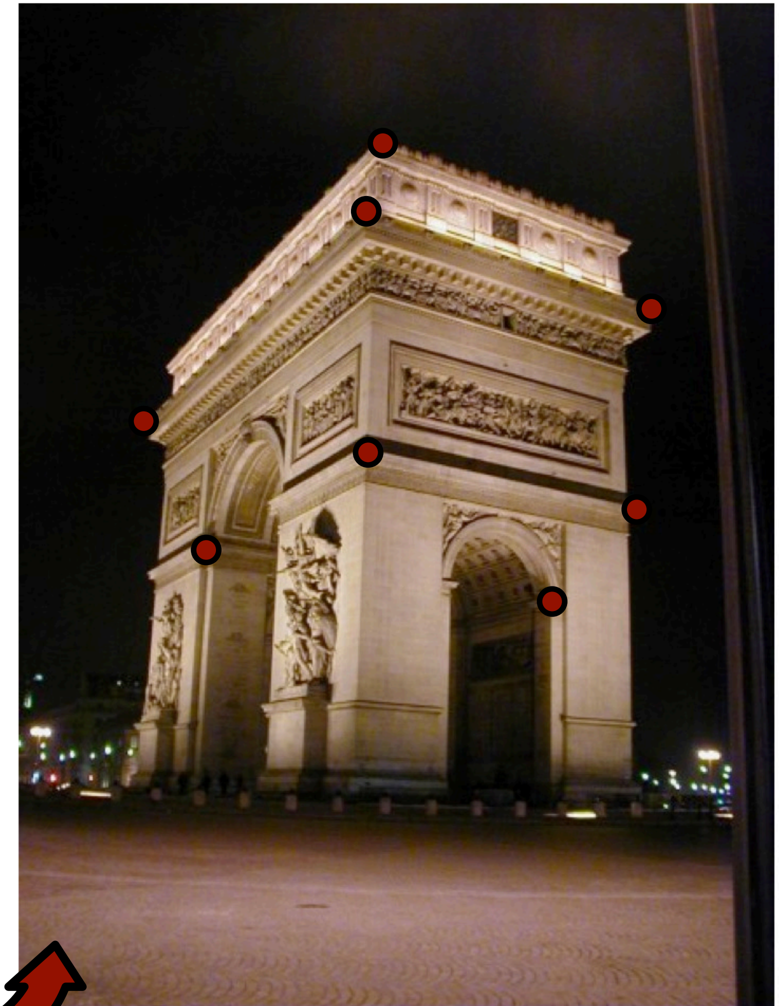
Motivation



E, B, R

Image courtesy: Collins

Table of Contents

Relative Orientation

Last lecture

Compute the essential matrix matrix given corresponding points using a **direct method**

Today's Lecture

- Compute the **essential matrix** given corresponding points with an **iterative least squares approach**
- Analyze the **quality of our solution**

Iterative Solution for the Relative Orientation from Corresponding Points

Reminder: Essential Matrix

- **Essential matrix encodes the R.O. for a calibrated camera pair**

$$\boxed{\mathsf{E} = R' \mathsf{S}_b R''^{\mathsf{T}}}$$

- Often parameterized through
  (parameterizations of dependent images)

$$\boxed{\mathsf{E} = \mathsf{S}_b R^{\mathsf{T}}}$$

- Coplanarity constraint

$$\boxed{{}^k\mathbf{x}'^{\mathsf{T}} \mathsf{E}\, {}^k\mathbf{x}'' = 0}$$

Coplanarity Constraint for N Corresponding Points

- For each point pair, we can formulate the coplanarity constraint:

$$ {}^{k}\mathbf{x}'^{\mathsf{T}}_{n}\ \mathsf{E}\ {}^{k}\mathbf{x}''_{n} = 0 \qquad n = 1, ..., N $$

- Expressed for the parameterizations of dependent images:

$$ {}^{k}\mathbf{x}'^{\mathsf{T}}_{n}\ \mathsf{S}_{b} R^{\mathsf{T}}\ {}^{k}\mathbf{x}''_{n} = 0 \qquad n = 1, ..., N $$

Estimate the Essential Matrix (Here: Stereo Normal Case)

- Estimate E through **least squares**
- **Coplanarity constraint** directly yields an **error function** in the parameters of the R.O.
- Coplanarity constraint is **non-linear** in the parameters
- Thus, we need to **iterate**

Non-Linear Error Function

- Coplanarity constraint yields a non-linear error function

Assumptions

- **Approximately stereo normal case**
- Classic photogrammetric parameteriz. of dependent images ($B_X = const.$, 5 parameters for the R.O.)

Problem Statement

Wanted: R.O. parameters $\boldsymbol{B}$, R (approximately stereo normal case)

Given:

- Observed image coordinates

$$(x'_n, y'_n) := ({}^k x'_n,\ {}^k y'_n) \qquad (x''_n, y''_n) = ({}^k x''_n,\ {}^k y''_n) \qquad n = 1, \ldots, N$$

- Uncertainty of the observations

$$\Sigma_{x'x'} \qquad \Sigma_{x''x''} \qquad n = 1, \ldots N$$

simplified: $\Sigma_{xx} = \sigma^2 \boldsymbol{I}$

- Initial guess for the R.O. parameters

$\boldsymbol{B}^a$, R^a parameters: $\boldsymbol{B}^a = [B_X, 0, 0]^\mathsf{T}$, $\mathsf{R}^a = \boldsymbol{I}_3$

Towards the Linearized Observation Equations

- Starting point: ${}^k\mathbf{x}'^{\mathsf{T}}\, \mathsf{S}_b R^{\mathsf{T}}\; {}^k\mathbf{x}'' = 0$
- Initial guess: $\boldsymbol{B}^a = [B_X, 0, 0]^{\mathsf{T}},\ R^a = I_3$
- **Next goal:** find the observation equation for the Gauss-Markov model:

observation + correction = coefficients times corrections in unknowns

Towards the Linearized Observation Equations

- Starting point: ${}^k\mathbf{x}'^{\mathsf{T}}\, \mathsf{S}_b R^{\mathsf{T}}\, {}^k\mathbf{x}'' = 0$
- Initial guess: $\boldsymbol{B}^a = [B_X, 0, 0]^{\mathsf{T}},\ R^a = I_3$
- "How do variations in the parameters effect the function itself?"

$${}^k\mathbf{x}' = {}^k\mathbf{x}'^a + \mathrm{d}\,{}^k\mathbf{x}'^a = \begin{bmatrix} x' \\ y' \\ c \end{bmatrix} + \begin{bmatrix} \mathrm{d}x' \\ \mathrm{d}y' \\ 0 \end{bmatrix}$$ correction in x'

$${}^k\mathbf{x}'' = {}^k\mathbf{x}''^a + \mathrm{d}\,{}^k\mathbf{x}''^a = \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} + \begin{bmatrix} \mathrm{d}x'' \\ \mathrm{d}y'' \\ 0 \end{bmatrix}$$ correction in x''

Basis

- Linearized equation for the basis

$$\mathbf{b} = \mathbf{b}^a + \mathrm{d}\mathbf{b} = \begin{bmatrix} B_X \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \mathrm{d}B_Y \\ \mathrm{d}B_Z \end{bmatrix}$$

2 unknowns

- This leads to the skew-symmetric S_b

$$\mathsf{S}_b = \mathsf{S}_b^a + \mathrm{d}\mathsf{S}_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} + \begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & 0 \\ -\mathrm{d}B_Y & 0 & 0 \end{bmatrix}$$

correction in S_b

Rotation

- **Linearized** equation for the rotation

$$R^{\mathsf{T}} = R^{a\mathsf{T}} + \mathrm{d}R^{\mathsf{T}} = I_3 + S_{\mathrm{d}r}^{\mathsf{T}} = I_3 + \begin{bmatrix} 0 & \mathrm{d}\kappa & -\mathrm{d}\phi \\ -\mathrm{d}\kappa & 0 & \mathrm{d}\omega \\ \mathrm{d}\phi & -\mathrm{d}\omega & 0 \end{bmatrix}$$

correction in R **3 unknowns**

- Coplanarity constraint (~normal case)

$$\boxed{{}^{k}\mathbf{x}'^{\mathsf{T}} \begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & -B_X \\ -\mathrm{d}B_Y & B_X & 0 \end{bmatrix} \begin{bmatrix} 1 & -d\kappa & d\phi \\ d\kappa & 1 & -d\omega \\ -d\phi & d\omega & 1 \end{bmatrix}^{\mathsf{T}} {}^{k}\mathbf{x}'' = 0}$$

Linearized Observation Equation

- The coplanarity constraint

$$ {}^k\mathbf{x}'^{\mathsf{T}}\, \mathsf{S}_b R^{\mathsf{T}}\; {}^k\mathbf{x}'' = 0 $$

- The linearized error function through the initial guess and total differential

$$
\begin{aligned}
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\; {}^k\mathbf{x}''^{a} \quad &+ \\
\mathrm{d}\; {}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\; {}^k\mathbf{x}''^{a} \quad &+ \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}} \mathrm{d}\; {}^k\mathbf{x}''^{a} \quad &+ \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathrm{d}\mathsf{S}_b R^{a\mathsf{T}}\; {}^k\mathbf{x}''^{a} \quad &+ \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a \mathrm{d}R^{\mathsf{T}}\; {}^k\mathbf{x}''^{a} \quad &= \quad 0
\end{aligned}
$$

Linearized Observation Equation

$$
\begin{aligned}
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad + \\
\mathrm{d}\ {}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}} \mathrm{d}\ {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathrm{d}\mathsf{S}_b R^{a\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a \mathrm{d}R^{\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad = \quad 0
\end{aligned}
$$

normal case

$$[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +$$

change in x'

$$[\mathrm{d}x', \mathrm{d}y', 0] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +$$

change in x''

$$[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} \mathrm{d}x'' \\ \mathrm{d}y'' \\ 0 \end{bmatrix} \quad +$$

change in S_b

$$[x', y', c] \begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & 0 \\ -\mathrm{d}B_Y & 0 & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +$$

change in R

$$[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} 0 & \mathrm{d}\kappa & -\mathrm{d}\phi \\ -\mathrm{d}\kappa & 0 & \mathrm{d}\omega \\ \mathrm{d}\phi & -\mathrm{d}\omega & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad = \quad 0$$

Linearized Observation Equation

$$
\begin{aligned}
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\, {}^k\mathbf{x}''^a &\quad + \\
\mathrm{d}\, {}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\, {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}} \mathrm{d}\, {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathrm{d}\mathsf{S}_b R^{a\mathsf{T}}\, {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a \mathrm{d}R^{\mathsf{T}}\, {}^k\mathbf{x}''^a &\quad = \quad 0
\end{aligned}
$$

normal case

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

change in x'

$$
[\mathrm{d}x', \mathrm{d}y', 0] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

change in x''

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} \mathrm{d}x'' \\ \mathrm{d}y'' \\ 0 \end{bmatrix} \quad +
$$

change in S_b

$$
[x', y', c] \begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & 0 \\ -\mathrm{d}B_Y & 0 & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

change in R

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} 0 & \mathrm{d}\kappa & -\mathrm{d}\phi \\ -\mathrm{d}\kappa & 0 & \mathrm{d}\omega \\ \mathrm{d}\phi & -\mathrm{d}\omega & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad = \quad 0
$$

Linearized Observation Equation

$$
\begin{aligned}
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\; {}^k\mathbf{x}''^{a} &\quad + \\
\mathrm{d}\; {}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}}\; {}^k\mathbf{x}''^{a} &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a R^{a\mathsf{T}} \mathrm{d}\; {}^k\mathbf{x}''^{a} &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathrm{d}\mathsf{S}_b R^{a\mathsf{T}}\; {}^k\mathbf{x}''^{a} &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathsf{S}_b^a \mathrm{d}R^{\mathsf{T}}\; {}^k\mathbf{x}''^{a} &\quad = \quad 0
\end{aligned}
$$

normal case

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} +
$$

change in x′

$$
[\mathrm{d}x', \mathrm{d}y', 0] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} +
$$

$$
= [x', y', c] \begin{bmatrix} 0 \\ -B_X\, c \\ B_X\, y'' \end{bmatrix}
$$

change in

$$
\begin{bmatrix} \mathrm{d}x'' \\ \mathrm{d}y'' \\ 0 \end{bmatrix} +
$$

change in S_b

$$
[x', y', c] \begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & 0 \\ -\mathrm{d}B_Y & 0 & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} +
$$

change in R

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} 0 & \mathrm{d}\kappa & -\mathrm{d}\phi \\ -\mathrm{d}\kappa & 0 & \mathrm{d}\omega \\ \mathrm{d}\phi & -\mathrm{d}\omega & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} = 0
$$

Linearized Observation Equation

$$
\begin{aligned}
{}^k\mathbf{x}'^{a\mathsf{T}}\,\mathsf{S}^a_b R^{a\mathsf{T}}\;{}^k\mathbf{x}''^a &\quad + \\
\mathrm{d}\,{}^k\mathbf{x}'^{a\mathsf{T}}\,\mathsf{S}^a_b R^{a\mathsf{T}}\;{}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}}\,\mathsf{S}^a_b R^{a\mathsf{T}}\mathrm{d}\;{}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}}\,\mathrm{d}\mathsf{S}_b R^{a\mathsf{T}}\;{}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}}\,\mathsf{S}^a_b \mathrm{d}R^{\mathsf{T}}\;{}^k\mathbf{x}''^a &\quad = \quad 0
\end{aligned}
$$

normal case

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

$$
= [x', y', c] \begin{bmatrix} 0 \\ -B_X\, c \\ B_X\, y'' \end{bmatrix}
$$

$$
= -y'\, B_X\, c + c\, B_X\, y''
$$

Linearized Observation Equation

$$
\begin{aligned}
{}^k\mathbf{x}'^{a\mathsf{T}} S_b^a R^{a\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad + \\
\mathrm{d}\ {}^k\mathbf{x}'^{a\mathsf{T}} S_b^a R^{a\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} S_b^a R^{a\mathsf{T}} \mathrm{d}\ {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} \mathrm{d}S_b R^{a\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad + \\
{}^k\mathbf{x}'^{a\mathsf{T}} S_b^a \mathrm{d}R^{\mathsf{T}}\ {}^k\mathbf{x}''^a &\quad = \quad 0
\end{aligned}
$$

normal case

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

change in x'

$$
[\mathrm{d}x', \mathrm{d}y', 0] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

$$
= [x', y', c] \begin{bmatrix} 0 \\ -B_X\, c \\ B_X\, y'' \end{bmatrix}
$$

change in x''

$$
\begin{bmatrix} \mathrm{d}x'' \\ \mathrm{d}y'' \\ 0 \end{bmatrix} \quad +
$$

change in S

$$
[x', y', c] \begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & 0 \\ & 0 & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad +
$$

$$
= -y'\, B_X\, c + c\, B_X\, y''
$$

change in R

$$
[x', y', c] \begin{bmatrix} 0 & 0 & 0 \\ 0 & & -B_X \\ & & 0 \end{bmatrix} \begin{bmatrix} 0 & \mathrm{d}\kappa & -\mathrm{d}\phi \\ -\mathrm{d}\kappa & 0 & \mathrm{d}\omega \\ \mathrm{d}\phi & -\mathrm{d}\omega & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} \quad = \quad 0
$$

$$
= c\, B_X (y'' - y')
$$

Linearized Observation Equation

$$
\begin{aligned}
(x', y', c)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix}\begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} &\quad + \\
(\mathrm{d}x', \mathrm{d}y', 0)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix}\begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} &\quad + \\
(x', y', c)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix}\begin{bmatrix} \mathrm{d}x'' \\ \mathrm{d}y'' \\ 0 \end{bmatrix} &\quad + \\
(x', y', c)\begin{bmatrix} 0 & -\mathrm{d}B_Z & \mathrm{d}B_Y \\ \mathrm{d}B_Z & 0 & 0 \\ -\mathrm{d}B_Y & 0 & 0 \end{bmatrix}\begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} &\quad + \\
(x', y', c)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -B_X \\ 0 & B_X & 0 \end{bmatrix}\begin{bmatrix} 0 & \mathrm{d}\kappa & -\mathrm{d}\phi \\ -\mathrm{d}\kappa & 0 & \mathrm{d}\omega \\ \mathrm{d}\phi & -\mathrm{d}\omega & 0 \end{bmatrix}\begin{bmatrix} x'' \\ y'' \\ c \end{bmatrix} &\quad = \quad 0
\end{aligned}
$$

y-parallax p_y

y-parallax corr

x-parallax p_x

$$
\begin{aligned}
cB_X(y'' - y') \\
+cB_X(\mathrm{d}y'' - \mathrm{d}y') \\
-c\,\mathrm{d}B_Y(x'' - x') \\
-\mathrm{d}B_Z(x'y'' - x''y') \\
+B_X\mathrm{d}\omega\,(y'y'' + c^2) - B_X\mathrm{d}\phi\, y'x'' - cB_X\mathrm{d}\kappa\, x'' &\quad = \quad 0
\end{aligned}
$$

for the stereo normal case $y' \approx y''$

Linearized Observation Equation

$$
\begin{aligned}
& cB_X(y'' - y') \\
& +cB_X(\mathrm{d}y'' - \mathrm{d}y') \\
& -c\,\mathrm{d}B_Y(x'' - x') \\
& -\mathrm{d}B_Z(x'y'' - x''y') \\
& +B_X\mathrm{d}\omega\,(y'y'' + c^2) - B_X\mathrm{d}\phi\,y'x'' - cB_X\mathrm{d}\kappa\,x'' \quad = \quad 0
\end{aligned}
$$

Target p_y because this is the term to be zero (coplanarity constraint for stereo normal)

$$
\begin{aligned}
& cB_X\boxed{p_y} \\
& +cB_X\boxed{\mathrm{d}p_y} \\
& -c\,\mathrm{d}B_Y p_x \\
& +\mathrm{d}B_Z y' p_x \\
& +B_X\mathrm{d}\omega\,(y'y'' + c^2) \\
& -B_X\mathrm{d}\phi\,y'x'' \\
& -cB_X\mathrm{d}\kappa\,x'' \quad = \quad 0
\end{aligned}
$$

This Leads Us to

$$\begin{aligned} cB_X p_y & \\ +cB_X \mathrm{d}p_y & \\ -c\,\mathrm{d}B_Y p_x & \\ +\mathrm{d}B_Z y' p_x & \\ +B_X \mathrm{d}\omega\,(y'y'' + c^2) & \\ -B_X \mathrm{d}\phi\, y'x'' & \\ -cB_X \mathrm{d}\kappa\, x'' &= 0 \end{aligned}$$

$$\boxed{\begin{aligned} p_y + \mathrm{d}p_y &= \frac{p_x}{B_X}\mathrm{d}B_Y - \frac{p_x}{B_X}\frac{y'}{c}\mathrm{d}B_Z - \left(\frac{y'y''}{c} + c\right)\mathrm{d}\omega \\ &\quad + \frac{y'x''}{c}\mathrm{d}\phi + x''\mathrm{d}\kappa \end{aligned}}$$

Gauss Markov Model

$$\begin{aligned} cB_X p_y & \\ +cB_X \mathrm{d}p_y & \\ -c\,\mathrm{d}B_Y p_x & \\ +\mathrm{d}B_Z y' p_x & \\ +B_X \mathrm{d}\omega\,(y'y'' + c^2) & \\ -B_X \mathrm{d}\phi\, y' x'' & \\ -cB_X \mathrm{d}\kappa\, x'' &= 0 \end{aligned}$$

$$\boxed{\begin{aligned} p_y + \mathrm{d}p_y &= \frac{p_x}{B_X}\mathrm{d}B_Y - \frac{p_x}{B_X}\frac{y'}{c}\mathrm{d}B_Z - \left(\frac{y'y''}{c} + c\right)\mathrm{d}\omega \\ &\quad + \frac{y'x''}{c}\mathrm{d}\phi + x''\mathrm{d}\kappa \end{aligned}}$$

GM: observation +

GM: corrections =

GM: coefficients times the corrections in the unknowns

Observation Equation Written Using Vectors

$$
\begin{aligned}
p_y + \mathrm{d}p_y &= \frac{p_x}{B_X}\mathrm{d}B_Y - \frac{p_x}{B_X}\frac{y'}{c}\mathrm{d}B_Z - \left(\frac{y'y''}{c} + c\right)\mathrm{d}\omega \\
&\quad + \frac{y'x''}{c}\mathrm{d}\phi + x''\mathrm{d}\kappa \\
&= \underbrace{\begin{bmatrix} \frac{p_x}{B_X} \\ -\frac{p_x}{B_X}\frac{y'}{c} \\ -\left(\frac{y'y''}{c} + c\right) \\ \frac{y'x''}{c} \\ x'' \end{bmatrix}^{\mathsf{T}}}_{\boldsymbol{a}_n^{\mathsf{T}}} \underbrace{\begin{bmatrix} \mathrm{d}B_Y \\ \mathrm{d}B_Z \\ \mathrm{d}\omega \\ \mathrm{d}\phi \\ \mathrm{d}\kappa \end{bmatrix}}_{\Delta\boldsymbol{x}} \\
&= \boldsymbol{a}_n^{\mathsf{T}}\Delta\boldsymbol{x}
\end{aligned}
$$

coefficients times the corrections in the unknowns

For All Observations, We Obtain

$$\Delta l_n + v_n \quad = \quad \boldsymbol{a}_n^\mathsf{T} \Delta \boldsymbol{x} \quad \Rightarrow \quad \boxed{\Delta \boldsymbol{l} + \boldsymbol{v} \quad = \quad A \Delta \boldsymbol{x}}$$

$$\underbrace{\begin{pmatrix} p_{y_1} \\ p_{y_2} \\ .. \\ p_{y_n} \\ .. \\ p_{y_N} \end{pmatrix}}_{\Delta \boldsymbol{l}} + \underbrace{\begin{pmatrix} v_{p_{y_1}} \\ v_{p_{y_2}} \\ .. \\ v_{p_{y_n}} \\ .. \\ v_{p_{y_N}} \end{pmatrix}}_{\boldsymbol{v}} = \underbrace{\begin{pmatrix} \frac{p_{x_1}}{B_X} & -\frac{p_{x_1}}{B_X}\frac{y_1'}{c} & -\left(\frac{y_1' y_1''}{c} + c\right) & \frac{y_1' x_1''}{c} & x_1'' \\ \frac{p_{x_2}}{B_X} & -\frac{p_{x_2}}{B_X}\frac{y_2'}{c} & -\left(\frac{y_2' y_2''}{c} + c\right) & \frac{y_2' x_2''}{c} & x_2'' \\ .. & .. & .. & .. & .. \\ \frac{p_{x_n}}{B_X} & -\frac{p_{x_n}}{B_X}\frac{y_n'}{c} & -\left(\frac{y_n' y_n''}{c} + c\right) & \frac{y_n' x_n''}{c} & x_n'' \\ .. & .. & .. & .. & .. \\ \frac{p_{x_N}}{B_X} & -\frac{p_{x_N}}{B_X}\frac{y_N'}{c} & -\left(\frac{y_N' y_N''}{c} + c\right) & \frac{y_N' x_N''}{c} & x_N'' \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} dB_Y \\ dB_Z \\ d\omega \\ d\phi \\ d\kappa \end{pmatrix}}_{\Delta \boldsymbol{x}}$$

Uncertainties

- Uncertainty in the y-parallax

$$\sigma^2_{p_{y_n}} = \sigma^2_{y'_n} + \sigma^2_{y''_n}$$

- In case both coordinates are measured equally accurate

$$\sigma_{p_{y_n}} = \sqrt{2}\,\sigma_{y'}$$

- Assuming no correlation between corresponding points

$$\boxed{\Sigma_{ll} = \mathrm{Diag}(\sigma^2_{p_{y_n}})}$$

n by n
diagonal
matrix

System of Normal Equations

- We computed the linearized error eqn
- We have the observation cov matrix
- This leads to the normal equations

$$A^{\mathsf{T}} \Sigma_{ll}^{-1} A \, \Delta \boldsymbol{x} = A^{\mathsf{T}} \Sigma_{ll}^{-1} \Delta \boldsymbol{l}$$

- And thus the parameter corrections

$$\widehat{\Delta \boldsymbol{x}} = (A^{\mathsf{T}} \Sigma_{ll}^{-1} A)^{-1} A^{\mathsf{T}} \Sigma_{ll}^{-1} \Delta \boldsymbol{l}$$

- For the observations (y-parallaxes)

$$\widehat{\boldsymbol{v}} = A \widehat{\Delta \boldsymbol{x}} - \Delta \boldsymbol{l} \quad \text{or} \quad \widehat{\boldsymbol{v}}_n = \boldsymbol{a}_n \widehat{\Delta \boldsymbol{x}} - \Delta \boldsymbol{l}_n$$

Summary so far

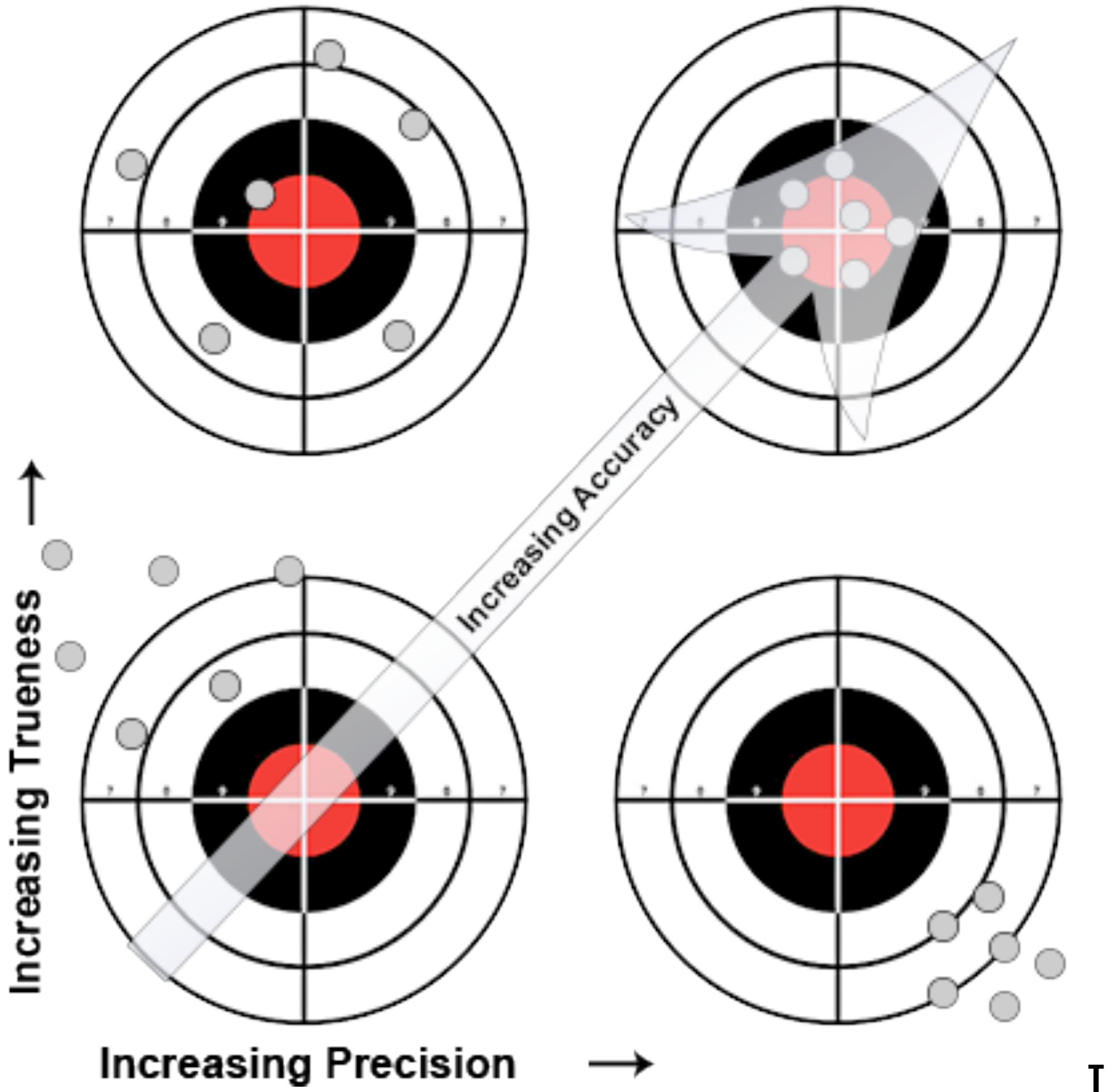
- Iterative least squares approach to estimate the relative orientation for calibrated cameras
- We used the coplanarity constraint as our error function
- Linearization
- Yields GM model
- Setup of a linear system
- Solving it yields the corrections

Quality of the Result
“How Good is a Solution?”

Precision, Trueness, Accuracy

- **Precision (DE: Präzision)**
  The closeness of agreement between independent test results obtained under the same conditions.
- **Trueness (DE: Richtigkeit)**
  The closeness of agreement between the average value obtained from a large series of measurements and the true value.
- **Accuracy (DE: äußere Genauigk.)**
  The closeness of agreement between a test result and the true value.

Precision, Trueness, Accuracy



ISO5725-1

English vs. German

- Precision
  DE: Präzision (or innere Genauigkeit, Wiederholgenauigkeit)
- Trueness
  DE: Richtigkeit
- Accuracy
  DE: äußere Genauigkeit
- Reliability
  DE: Zuverlässigkeit
- "Genauigkeit"... innere oder äußere?

Precision for the Relative Orientation

- **Precision:** How large is the influence of random noise on the result?

Precision & Reliability for the Relative Orientation

- **Precision:** How large is the influence of random noise on the result?

- **Reliability:** Can we detect measurement errors/outliers?

Precision

- To analyze the precision, we need the covariance matrix of the unknowns
- **Theoretical precision**

$$\Sigma_{\widehat{x}\widehat{x}} = (A^\mathsf{T} \Sigma_{ll}^{-1} A)^{-1}$$

- **Empirical precision**

$$\widehat{\Sigma}_{\widehat{x}\widehat{x}} = \widehat{\sigma}_0^2 \Sigma_{\widehat{x}\widehat{x}} = \widehat{\sigma}_0^2 (A^\mathsf{T} \Sigma_{ll}^{-1} A)^{-1}$$

- Empirical and theoretical precision related through the variance factor

Variance Factor

- Computation of the variance factor

$$\widehat{\sigma}_0^2 = \frac{\Omega}{R}$$

- Weighted sum of the squared corrections in the parallaxes after convergence

$$\Omega = \widehat{\boldsymbol{v}}^{\mathsf{T}} \Sigma_{ll}^{-1} \widehat{\boldsymbol{v}} = \sum_n \widehat{\boldsymbol{v}}_n^{\mathsf{T}} \Sigma_{l_n l_n}^{-1} \widehat{\boldsymbol{v}}_n$$

- Redundancy

$$R = N - \#\text{unknowns} = N - 5$$

Empirical Precision

- With a redundancy of R>30, we obtain realistic estimates of the precision of our unknown relative orientation

$$\widehat{\Sigma}_{\widehat{x}\widehat{x}} = \underbrace{\frac{\widehat{\boldsymbol{v}}^{\mathsf{T}} \Sigma_{ll}^{-1} \widehat{\boldsymbol{v}}}{N-5}}_{\widehat{\sigma}_0^2} (A^{\mathsf{T}} \Sigma_{ll}^{-1} A)^{-1}$$

$$\widehat{\sigma}_{\widehat{B}_Y} = \sqrt{\widehat{\Sigma}_{\widehat{x}_1\widehat{x}_1}} \quad \widehat{\sigma}_{\widehat{B}_Z} = \sqrt{\widehat{\Sigma}_{\widehat{x}_2\widehat{x}_2}} \quad \widehat{\sigma}_{\widehat{\omega}} = \sqrt{\widehat{\Sigma}_{\widehat{x}_3\widehat{x}_3}}$$

$$\widehat{\sigma}_{\widehat{\phi}} = \sqrt{\widehat{\Sigma}_{\widehat{x}_4\widehat{x}_4}} \quad \widehat{\sigma}_{\widehat{\kappa}} = \sqrt{\widehat{\Sigma}_{\widehat{x}_5\widehat{x}_5}}$$

Correlation

- We can also compute the correlation of the parameters

$$\rho_{x_i x_j} = \frac{\Sigma_{\widehat{x}_i \widehat{x}_j}}{\sigma_{x_i} \sigma_{x_j}}$$

- Large correlation values (=> +1/-1) between parameters can be a reason for instabilities of the solution

Reliability

- Covariance matrix of the corrections

$$\Sigma_{vv} = \Sigma_{ll} - A\Sigma_{\widehat{x}\widehat{x}}A^{\mathsf{T}}$$

- Σ_{vv} is smaller than Σ_{ll}
- Redundancy components r_n of observations are defined as

$$r_n = \frac{\sigma^2_{v_n}}{\sigma^2_{l_n}} \in [0, 1]$$

- Sum over all r_n gives the redundancy

$$R = \sum r_n$$

Reliability

- Redundancy components $r_n = \sigma_{v_n}^2 \sigma_{l_n}^{-2}$ tells which fraction of original errors we see in the residual parallaxes v_n after the adjustment

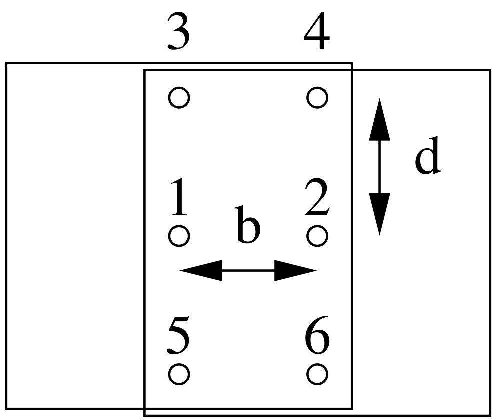
$$\Delta v_n = -r_n \, \Delta l_n$$

- Small values for r_n indicate that outliers are hard to detect

Quality of the Relative Orientation for the Stereo Normal Case

Quality of the R.O. for the Stereo Normal Case

- Depends on the exact configuration
- Difficult in the general case
- Here: stereo normal case with Gruber points



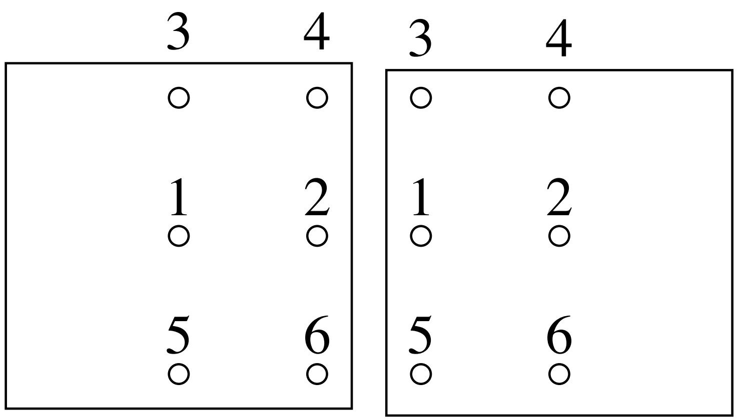


Image courtesy: Förstner

Assumptions

- Six corresponding points (Gruber points) in the overlapping area
- Image overlap: 60%
- Identical uncertainty in y-parallaxes (weight=1, $\sigma_0 = \sigma_{p_y}$)
- Basis $B_X = M\, b_X$ (image scale number times image basis)

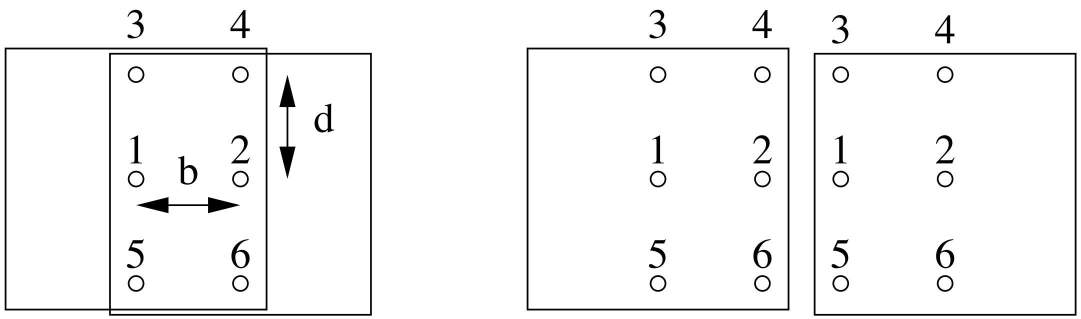
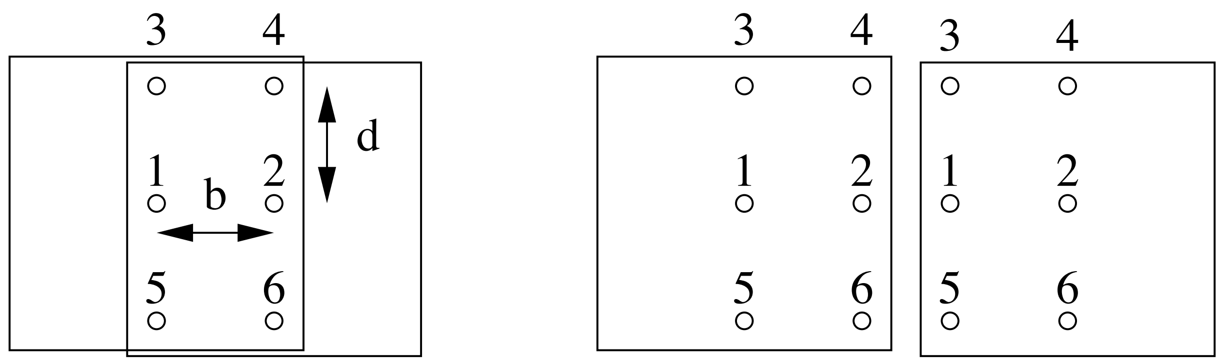


Image courtesy: Förstner

Image Coordinates



Gruber point	x'	y'	x''	y''
1	0	0	$-b$	0
2	b	0	0	0
3	0	d	$-b$	d
4	b	d	0	d
5	0	$-d$	$-b$	$-d$
6	b	$-d$	0	$-d$

Image courtesy: Förstner

Coefficient Matrix

$$
\begin{aligned}
p_y + \mathrm{d}p_y \quad &= \quad \frac{p_x}{B_X}\mathrm{d}B_y - \frac{p_x}{B_X}\frac{y'}{c}\mathrm{d}B_Z - \left(\frac{y'y''}{c} + c\right)\mathrm{d}\omega \\
&\quad + \frac{y'x''}{c}\mathrm{d}\phi + x''\mathrm{d}\kappa
\end{aligned}
$$

$$
A = -\left[\begin{array}{ccccc}
\frac{b}{B_X} & 0 & c & 0 & b \\
\frac{b}{B_X} & 0 & c & 0 & 0 \\
\frac{b}{B_X} & -\frac{bd}{B_X\, c} & \frac{d^2}{c} + c & \frac{bd}{c} & b \\
\frac{b}{B_X} & -\frac{bd}{B_X\, c} & \frac{d^2}{c} + c & 0 & 0 \\
\frac{b}{B_X} & \frac{bd}{B_X\, c} & \frac{d^2}{c} + c & -\frac{bd}{c} & b \\
\frac{b}{B_X} & \frac{bd}{B_X\, c} & \frac{d^2}{c} + c & 0 & 0
\end{array}\right]
\begin{array}{l}
\text{point 1} \\
\\
\dots \\
\\
\\
\text{point 6}
\end{array}
$$

Matrix of the Normal Equations

$$A = -\begin{bmatrix} \frac{b}{B_X} & 0 & c & 0 & b \\ \frac{b}{B_X} & 0 & c & 0 & 0 \\ \frac{b}{B_X} & -\frac{bd}{B_X\,c} & \frac{d^2}{c}+c & \frac{bd}{c} & b \\ \frac{b}{B_X} & -\frac{bd}{B_X\,c} & \frac{d^2}{c}+c & 0 & 0 \\ \frac{b}{B_X} & \frac{bd}{B_X\,c} & \frac{d^2}{c}+c & -\frac{bd}{c} & b \\ \frac{b}{B_X} & \frac{bd}{B_X\,c} & \frac{d^2}{c}+c & 0 & 0 \end{bmatrix}$$

$$A^{\mathsf{T}}A = \begin{bmatrix} 6\,\frac{b^2}{{B_X}^2} & 0 & 2\,\frac{b\left(3\,c^2+2\,d^2\right)}{B_X\,c} & 0 & 3\,\frac{b^2}{B_X} \\ 0 & 4\,\frac{b^2 d^2}{{B_X}^2 c^2} & 0 & -2\,\frac{b^2 d^2}{B_X\,c^2} & 0 \\ 2\,\frac{b\left(3\,c^2+2\,d^2\right)}{B_X\,c} & 0 & 2\,\frac{3\,c^4+2\,d^4+4\,d^2 c^2}{c^2} & 0 & \frac{b\left(3\,c^2+2\,d^2\right)}{c} \\ 0 & -2\,\frac{b^2 d^2}{B_X\,c^2} & 0 & 2\,\frac{b^2 d^2}{c^2} & 0 \\ 3\,\frac{b^2}{B_X} & 0 & \frac{b\left(3\,c^2+2\,d^2\right)}{c} & 0 & 3\,b^2 \end{bmatrix}$$

Covariance Matrix

- This directly yields the covariance matrix of the parameter through

$$\widehat{\Sigma}_{\widehat{x}\widehat{x}} = \sigma_0^2 (A^\mathsf{T} A)^{-1}$$

$$= \sigma_0^2 \begin{bmatrix} \frac{1}{12}\frac{{B_X}^2\left(9\,c^4+8\,d^4+12\,d^2c^2\right)}{b^2d^4} & 0 & -\frac{1}{4}\frac{\left(3\,c^2+2\,d^2\right)B_X\,c}{bd^4} & 0 & -\frac{1}{3}\frac{B_X}{b^2} \\ 0 & \frac{1}{2}\frac{{B_X}^2c^2}{b^2d^2} & 0 & \frac{1}{2}\frac{B_X\,c^2}{b^2d^2} & 0 \\ -\frac{1}{4}\frac{\left(3\,c^2+2\,d^2\right)B_X\,c}{bd^4} & 0 & \frac{3}{4}\frac{c^2}{d^4} & 0 & 0 \\ 0 & \frac{1}{2}\frac{B_X\,c^2}{b^2d^2} & 0 & \frac{c^2}{b^2d^2} & 0 \\ -\frac{1}{3}\frac{B_X}{b^2} & 0 & 0 & 0 & \frac{2}{3}\frac{1}{b^2} \end{bmatrix}$$

Uncertainty in the Parameters

$$
\begin{aligned}
\widehat{\Sigma}_{\widehat{x}\widehat{x}} &= \sigma_0^2 (A^\mathsf{T} A)^{-1} \\
&= \sigma_0^2 \begin{bmatrix}
\frac{1}{12}\frac{{B_X}^2\left(9\,c^4+8\,d^4+12\,d^2c^2\right)}{b^2d^4} & 0 & -\frac{1}{4}\frac{\left(3\,c^2+2\,d^2\right)B_X\,c}{bd^4} & 0 & -\frac{1}{3}\frac{B_X}{b^2} \\
0 & \frac{1}{2}\frac{{B_X}^2c^2}{b^2d^2} & 0 & \frac{1}{2}\frac{B_X\,c^2}{b^2d^2} & 0 \\
-\frac{1}{4}\frac{\left(3\,c^2+2\,d^2\right)B_X\,c}{bd^4} & 0 & \frac{3}{4}\frac{c^2}{d^4} & 0 & 0 \\
0 & \frac{1}{2}\frac{B_X\,c^2}{b^2d^2} & 0 & \frac{c^2}{b^2d^2} & 0 \\
-\frac{1}{3}\frac{B_X}{b^2} & 0 & 0 & 0 & \frac{2}{3}\frac{1}{b^2}
\end{bmatrix}
\end{aligned}
$$

standard deviation
of the y-parallaxes

$$\sigma_0 = \sigma_{p_y} = \sqrt{2}\sigma_{y'}$$

scale number: $M \approx B_X / b$

$$
\begin{aligned}
\sigma_{B_Y} &= M\frac{\sqrt{9c^4+8d^4+12d^2c^2}}{d^2\sqrt{6}}\,\sigma_{y'} \\
\sigma_{B_Z} &= M\frac{c}{d}\,\sigma_{y'} \\
\sigma_{\omega} &= \sqrt{\frac{3}{2}\frac{c}{d^2}}\,\sigma_{y'} \\
\sigma_{\phi} &= \sqrt{2}\frac{c}{bd}\,\sigma_{y'} \\
\sigma_{\kappa} &= \frac{2}{\sqrt{3}}\frac{1}{b}\,\sigma_{y'}
\end{aligned}
$$

Discussion

- Impact of the pixel measurements

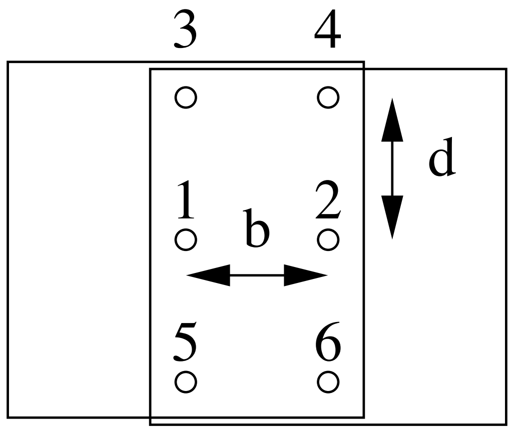
$$
\begin{aligned}
\sigma_{B_Y} &= M \frac{\sqrt{9c^4 + 8d^4 + 12d^2c^2}}{d^2\sqrt{6}} \boxed{\sigma_{y'}} \\
\sigma_{B_Z} &= M \frac{c}{d} \boxed{\sigma_{y'}} \\
\sigma_{\omega} &= \sqrt{\frac{3}{2}} \frac{c}{d^2} \boxed{\sigma_{y'}} \\
\sigma_{\phi} &= \sqrt{2} \frac{c}{bd} \boxed{\sigma_{y'}} \\
\sigma_{\kappa} &= \frac{2}{\sqrt{3}} \frac{1}{b} \boxed{\sigma_{y'}}
\end{aligned}
$$

"the more accurate one can measure the parallaxes, the better the result

Discussion

- Size of the scene and overlap

$$
\begin{aligned}
\sigma_{B_Y} &= M\frac{\sqrt{9c^4+8d^4+12d^2c^2}}{d^2\sqrt{6}}\,\sigma_{y'} \\
\sigma_{B_Z} &= M\frac{c}{\boxed{d}}\,\sigma_{y'} \\
\sigma_{\omega} &= \sqrt{\frac{3}{2}\frac{c}{\boxed{d^2}}}\,\sigma_{y'} \\
\sigma_{\phi} &= \sqrt{2}\frac{c}{\boxed{bd}}\,\sigma_{y'} \\
\sigma_{\kappa} &= \frac{2}{\sqrt{3}}\frac{1}{\boxed{b}}\,\sigma_{y'}
\end{aligned}
$$



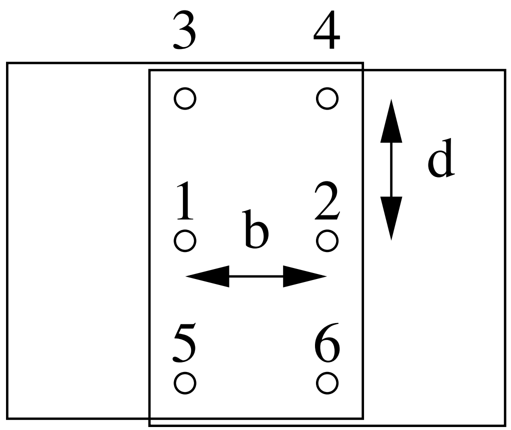
"the larger the scene and the overlap, the better the result

Discussion

- Size of the scene and overlap

$$\sigma_\omega = \sqrt{\frac{3}{2}}\frac{c}{\boxed{d^2}}\sigma_{y'}$$

$$\sigma_\phi = \sqrt{2}\frac{c}{\boxed{bd}}\sigma_{y'}$$



“the spread of the points in the plane (b, d) strongly impacts roll and pitch”

Discussion

- Camera constant

$$
\begin{aligned}
\sigma_{B_Y} &= M \frac{\sqrt{9\boxed{c^4} + 8d^4 + 12d^2\boxed{c^2}}}{d^2\sqrt{6}}\,\sigma_{y'} \\
\sigma_{B_Z} &= M \frac{\boxed{c}}{d}\,\sigma_{y'} \\
\sigma_{\omega} &= \sqrt{\frac{3}{2}}\,\frac{\boxed{c}}{d^2}\,\sigma_{y'} \\
\sigma_{\phi} &= \sqrt{2}\,\frac{\boxed{c}}{bd}\,\sigma_{y'} \\
\sigma_{\kappa} &= \frac{2}{\sqrt{3}}\,\frac{1}{b}\,\sigma_{y'}
\end{aligned}
$$

"the smaller the camera constant (at identical images), the better the result

Discussion

- Scale number and the baseline

$$\sigma_{B_Y} = \boxed{M}\frac{\sqrt{9c^4 + 8d^4 + 12d^2c^2}}{d^2\sqrt{6}}\,\sigma_{y'}$$

$$\sigma_{B_Z} = \boxed{M}\frac{c}{d}\,\sigma_{y'}$$

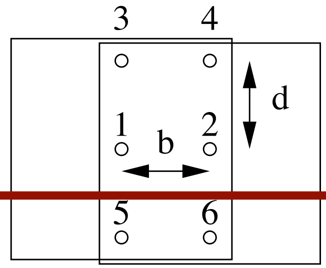
image scale number: $M \approx B_X/b$

"the smaller the image scale number (or the larger the image scale), the better the resulting baseline

Discussion

- All quantities are proportional to $\sigma_{p_{y'}}$
- σ_{B_Y}, σ_{B_Z} increase with the scale number
- d strongly influences roll (ω) and pitch (ϕ)
- If $b = d$, all quantities become more accurate with a larger basis b
- The more the overlap is exploited, the better

$$
\begin{aligned}
\sigma_{B_Y} &= M \frac{\sqrt{9c^4 + 8d^4 + 12d^2c^2}}{d^2\sqrt{6}} \sigma_{y'} \\
\sigma_{B_Z} &= M \frac{c}{d} \sigma_{y'} \\
\sigma_{\omega} &= \sqrt{\frac{3}{2}\frac{c}{d^2}} \sigma_{y'} \\
\sigma_{\phi} &= \sqrt{2}\frac{c}{bd} \sigma_{y'} \\
\sigma_{\kappa} &= \frac{2}{\sqrt{3}}\frac{1}{b} \sigma_{y'}
\end{aligned}
$$



Reliability

- Covariance matrix of the **corrections**

$$\Sigma_{vv} = \sigma_0^2 \left[\begin{array}{cc|cc|cc} \frac{1}{3} & -\frac{1}{3} & -\frac{1}{6} & \frac{1}{6} & -\frac{1}{6} & \frac{1}{6} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & -\frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \\ \hline -\frac{1}{6} & \frac{1}{6} & \frac{1}{12} & -\frac{1}{12} & \frac{1}{12} & -\frac{1}{12} \\ \frac{1}{6} & -\frac{1}{6} & -\frac{1}{12} & \frac{1}{12} & -1/12 & \frac{1}{12} \\ \hline -\frac{1}{6} & \frac{1}{6} & \frac{1}{12} & -\frac{1}{12} & \frac{1}{12} & -\frac{1}{12} \\ \frac{1}{6} & -\frac{1}{6} & -\frac{1}{12} & \frac{1}{12} & -\frac{1}{12} & \frac{1}{12} \end{array}\right]$$

$$\Sigma_{vv} = \underbrace{\Sigma_{ll}}_{\sigma_{p_{y'}}^2 I} - A\Sigma_{\widehat{x}\widehat{x}}A^{\mathsf{T}}$$

$$\sigma_{l_n} = 1$$

- and thus the redundancy components

$$r_1 = r_2 = \frac{1}{3} \qquad r_3 = r_4 = r_5 = r_6 = \frac{1}{12}$$

Reliability

- Covariance matrix of the corrections

Low redundancy components!

$$\Delta v_n = -r_n \, \Delta l_n$$

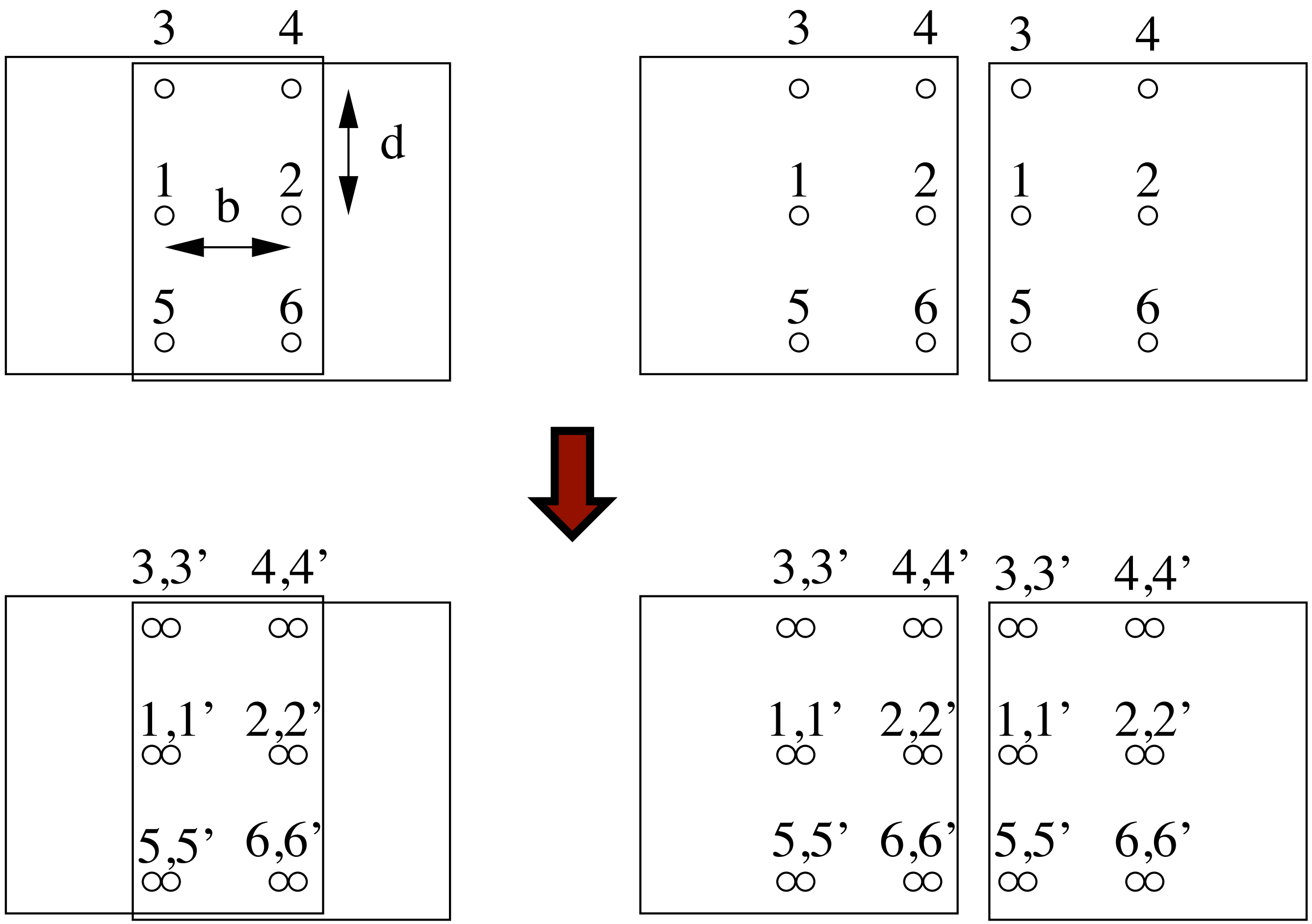
Gross errors in the y-parallaxes must be large compared to the standard deviation of the parallaxes in order to be detectable.

- and thus the redundancy components

$$r_1 = r_2 = \frac{1}{3} \qquad r_3 = r_4 = r_5 = r_6 = \frac{1}{12}$$

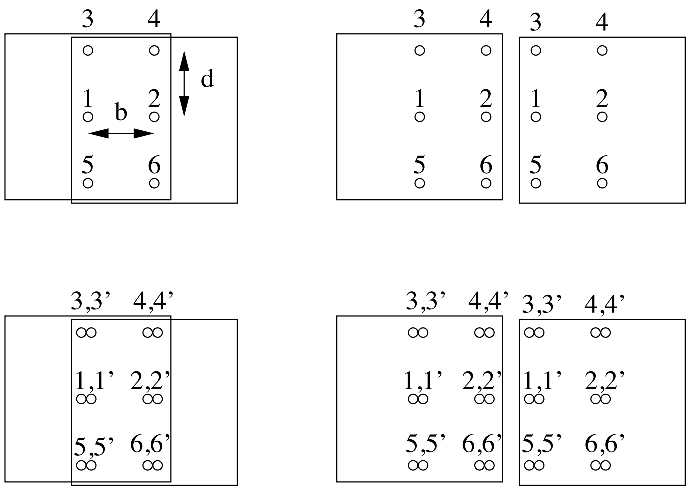
Double Points/12 Gruber Points

Improving the result with 12 points



Double Points

Improving the result with 12 points



$$\Sigma_{xx}(12) = \frac{1}{2}\Sigma_{xx}(6)$$

no. points

**Furthermore:
The more points we have, the easier we can detect outliers!**

Double Points

Covariance of the parallax corrections

$$\Sigma_{vv}(12) = \begin{bmatrix} \Sigma_{ll} & \mathbf{0} \\ \mathbf{0} & \Sigma_{ll} \end{bmatrix} - \begin{bmatrix} \mathsf{A} \\ \mathsf{A} \end{bmatrix} \Sigma_{xx}(12)(\mathsf{A}^{\mathsf{T}}\ \mathsf{A}^{\mathsf{T}})$$

which leads to the redundancy components

$$r_n = \frac{2}{3} \quad n = 1, 1', 2, 2' \qquad\qquad r_n = \frac{7}{12} \quad n = 3, 3', \ldots, 6, 6'$$

Double Points

Covariance of the parallax corrections

$$\Sigma_{vv}(12) = \begin{bmatrix} \Sigma_{ll} & \mathbf{0} \\ \mathbf{0} & \Sigma_{ll} \end{bmatrix} - \begin{bmatrix} A \\ A \end{bmatrix} \Sigma_{xx}(12)(A^{\mathsf{T}}\ A^{\mathsf{T}})$$

which leads to the redundancy components

$$r_n = \frac{2}{3} \quad n = 1, 1', 2, 2' \qquad\qquad r_n = \frac{7}{12} \quad n = 3, 3', \ldots, 6, 6'$$

Outliers are much easier to detect with Gruber "*double*" points!

Double Points

Covariance of the parallax corrections

$$\Sigma_{vv}(12) = \begin{bmatrix} \Sigma_{ll} & \mathbf{0} \\ \mathbf{0} & \Sigma_{ll} \end{bmatrix} - \begin{bmatrix} A \\ A \end{bmatrix} \Sigma_{xx}(12)(A^\mathsf{T} \; A^\mathsf{T})$$

The more points we have, the easier we can detect outliers!

which leads to the redundancy components

$$r_n = \frac{2}{3} \quad n = 1, 1', 2, 2' \qquad\qquad r_n = \frac{7}{12} \quad n = 3, 3', \ldots, 6, 6'$$

Outliers are much easier to detect with Gruber "*double*" points!

Summary

- Estimating the relative orientation using a least squares approach
- Solution for the normal stereo case (done without relinearizing)
- Statistically optimal solution
- Analysis of the solution based on Gruber points
- More points improve the results

Literature

- Förstner, Skript Photogrammetrie II, Chapter 1.3
- Förstner, Wrobel: Photogrammetric Computer Vision, Ch. 12.3.6 & 3.3.3

Slide Information

- These slides have been created by Cyrill Stachniss as part of the Photogrammetry II course taught in 2014/15.
- The material heavily relies on the very well written scripts by Wolfgang Förstner and the (upcoming) Photogrammetric Computer Vision book by Förstner and Wrobl.
- I tried to acknowledge all people that contributed image or video material. In case I missed something, please let me know. If you adapt this course material, please make sure you keep the acknowledgements.
- Feel free to use and change the slides. If you use them, I would appreciate an acknowledgement as well. To satisfy my own curiosity, please send me short email notice.

Cyrill Stachniss, 2014
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