#### **Photogrammetry & Robotics Lab**

#### Histogram Transformations: Histogram Equalization & Noise Variance Equalization

**Cyrill Stachniss** 

The slides have been created by Cyrill Stachniss.

#### **5 Minute Preparation for Today**



https://www.ipb.uni-bonn.de/5min/

#### **Image Histogram**







#### **Brightness and Contrast**

- A transformation that changes the mean, changes the image brightness
- Analogous for variance and contrast



#### Can We Compute the Output Histogram Knowing the Transformation Function?

## How Do Operators Affect the Distribution of Intensities?

- Monotonous function b = f(a)
- Histogram of the input image:  $h_a(a)$
- Compute the histogram h<sub>b</sub>(b) of the output image given h<sub>a</sub>(a) and f
- Use gained knowledge to design f

#### **Transformation of a PDF**



#### **Effect for Monotonous Point Operator**

- The gray "area" in the interval [a, a + da] is mapped to the interval [b, b + db]
- So we have

 $h_b(b) \, db = h_a(a) \, da$ 

("area stays the same")





#### This leads to



#### This leads to



$$h_b(b) db = h_a(a) da$$
$$h_b(b) = \frac{h_a(a)}{\left|\frac{db}{da}\right|} = \frac{h_a(a)}{\left|f'(a)\right|}$$



#### This leads to

 $h_b(b) db = h_a(a) da$  $h_b(b) = \frac{h_a(a)}{\left|\frac{db}{da}\right|} = \frac{h_a(a)}{|f'(a)|}$ 

function  $h_b(b)$  should depend on b, not on a

b

first derivative



$$= f(a) \to a = f^{-1}(b)$$

$$\implies h_b(b) = \frac{h_a(f^{-1}(b))}{|f'(f^{-1}(b))|}$$

#### **Example: Linear Function**

If we have the linear function

b = f(a) = k + ma

- with the first derivative
   f'(a) = m
- and the inverse

$$a = f^{-1}(b) = \frac{b-k}{m}$$
  
• this yields  $h_b(b) = \frac{h_a(f^{-1}(b))}{|f'(f^{-1}(b))|} = \frac{h_a\left(\frac{b-k}{m}\right)}{|m|}$ 

#### Example

- We can directly compute the output histogram given input and the (linear) transfer function
- Result

$$h_b(b) = \frac{h_a\left(\frac{b-k}{m}\right)}{|m|}$$

This corresponds to a shift by k and a scaling by 1/m

#### Can We Design Transformations Such That the Resulting Image Has Desired Properties?

#### **Sometimes One Can Barely See Anything...**



#### ... but a Change in the Intensity Values Can Help



### How to transform the image so that all intensities are used equally often?





Image courtesy: Phillip Capper 20

• Goal: same number of values in every histogram bin  $h_b(b) = \text{const.}$ 



- Goal: same number of values in every histogram bin  $h_b(b) = \text{const.}$
- With a monotonically increasing f

$$h_{b}(b) = k \qquad h_{b}(b) = \frac{h_{a}(a)}{\frac{db}{da}}$$
$$\implies k = \frac{h_{a}(a)}{\frac{db}{da}} \implies db = \frac{1}{k}h_{a}(a)da$$

# • How to solve $db = \frac{1}{k}h_a(a)da$ ?

- How to solve  $db = \frac{1}{k}h_a(a)da$  ?
- Solve via integratióň



Integral over the histogram of intensities H(a)

The equation

$$b + C_1 = \frac{1}{k} \int h_a(a) da$$

simplifies to

$$b + C_1 = \frac{1}{k}H(a) + C_2$$

• with the cumulative H(a) .

• Thus: 
$$b = f(a) = \frac{1}{k}H(a) + C$$

• The parameters k, C in

$$b = f(a) = \frac{1}{k}H(a) + C$$

- are typically chosen so that f(0) = 0 f(255) = 255
- This maps the spectrum of intensity values to the full spectrum (0 to 255)

#### **Mapping Illustration**



$$f(0) = 0$$
  
 $f(255) = 255$ 

Courtesy: Phillip Capper 27

- Chose the parameters k, C so that f(0) = 0 f(255) = 255
- From  $\frac{1}{k}H(0) + C = 0$   $\frac{1}{k}\underbrace{H(255)}_{=N} + C = 255$
- follows

$$k = \frac{N - H(0)}{255} \qquad C = -H(0)\frac{255}{N - H(0)}$$

 This results in the point operator for histogram equalization

$$f(a) = \text{round}\left(\frac{255(H(a) - H(0))}{N - H(0)}\right)$$

#### **This Results in...**









Courtesy: Phillip Capper 30

#### **Uniform Distribution?**



Courtesy: Phillip Capper 31

#### **Uniform Distribution?**





Equalized image





#### **Continuous vs. Discrete Case**



#### **Effect of the Hist. Equalization**

- Typically increases the contrast
- Areas of lower local contrast gain higher contrast
- Distributes the intensities over the histogram





#### **Another Example**

**Original image** 





#### **Another Example**



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#### **Another Example**

Original image



Equalized image





#### Variants of Histogram Equal.

- There exist variants of HE
- Adaptive HE (AHE) performs HE in local patches, not the whole image
- AHE has issues in low contrast regions as it over-amplifies the corrections
- Contrast limited AHE (CLAHE) limits this over-amplifications in homogeneous regions

- Intensity measurements are noisy
- Sensor: variance of a measured intensity depends on the intensity itself (Poisson distribution)
- Goal: adjust the variance of the intensities to a fixed value
- Useful for statistical analysis of images (all pixels have the same variance)

#### **Poisson Distribution**

"The Poisson distribution is a **discrete** probability distribution that expresses the probability of a given number k of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event."

$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

#### **Poisson Distribution**



Image courtesy: Wikipedia 42

#### **Distribution About the Number** of Incoming Photons

Poisson distribution

$$P(k) = \frac{(\beta t)^k}{k!} e^{-\beta t}$$

- with
  - β: avg. number of incoming photons per second
  - t : exposure time
- Distribution about the number of photons reaching the sensor

#### **Properties of the Poisson Dist.**

Mean and variance are then

$$\mu_k = \beta t \qquad \sigma_k^2 = \beta t$$

- and thus a standard deviation of  $\sigma_k = \sqrt{\beta t}$
- and a relative precision of

$$\frac{\sigma_k}{\mu_k} = \frac{\sqrt{\beta t}}{\beta t} = \frac{1}{\sqrt{\beta t}} = \frac{1}{\sqrt{\mu_k}}$$



- Consider a proportional relationship between variance and intensity  $\sigma_a^2 = ma$
- Goal:  $\sigma_b = \sigma_0$  for all intensities

- Consider a proportional relationship between variance and intensity  $\sigma_a^2 = ma$
- Goal:  $\sigma_b = \sigma_0$  for all intensities
- Variance propagation yields

$$\sigma_b = \frac{db}{da} \sigma_a \stackrel{!}{=} \sigma_0 = \text{const.}$$

(given a monotonously increasing function)

We have

$$\sigma_b = \frac{db}{da}\sigma_a = \sigma_0 \qquad \sigma_a^2 = ma$$

We can rewrite that as

$$db = \frac{\sigma_0}{\sigma_a} da \qquad \qquad \sigma_a = \sqrt{ma}$$

Directly yields

$$db = \frac{\sigma_0}{\sqrt{m}} a^{-\frac{1}{2}} da$$

The equation

$$db = \frac{\sigma_0}{\sqrt{m}} a^{-\frac{1}{2}} da$$

can be solved by integration to

$$b = f(a) = \frac{2\sigma_0}{\sqrt{m}}\sqrt{a} + C$$

- Choose C = 0 to obtain  $a = 0 \rightarrow b = 0$
- Chose  $\sigma_0$  so that  $a = 255 \rightarrow b = 255$

- Thus, we set C = 0
- For other variables:

$$255 = \frac{2\,\sigma_0}{\sqrt{m}}\sqrt{255}$$

• This means 
$$b = f(a) = \sqrt{255}\sqrt{a}$$

 Variance equalization yields a square root function with black=black and white=while

$$a = 0 \rightarrow b = 0 \qquad a = 255 \rightarrow b = 255$$

• Result: 
$$b = f(a) = \sqrt{255}\sqrt{a}$$

 The square root stretches the dark areas and compresses bright areas



#### **Summary**

- Image histograms represent the image intensity distributions
- Point operators to manipulate images
- Designing transformations such that images have certain properties
- Histogram equalization to obtain a uniform distribution of intensities
- Noise variance equalization: All pixels intensities have the same noise

#### **Slide Information**

- The slides have been created by Cyrill Stachniss as part of the photogrammetry and robotics courses.
- I tried to acknowledge all people from whom I used images or videos. In case I made a mistake or missed someone, please let me know.
- The photogrammetry material heavily relies on the very well written lecture notes by Wolfgang Förstner and the Photogrammetric Computer Vision book by Förstner & Wrobel.
- Parts of the robotics material stems from the great
   Probabilistic Robotics book by Thrun, Burgard and Fox.
- If you are a university lecturer, feel free to use the course material. If you adapt the course material, please make sure that you keep the acknowledgements to others and please acknowledge me as well. To satisfy my own curiosity, please send me email notice if you use my slides.

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