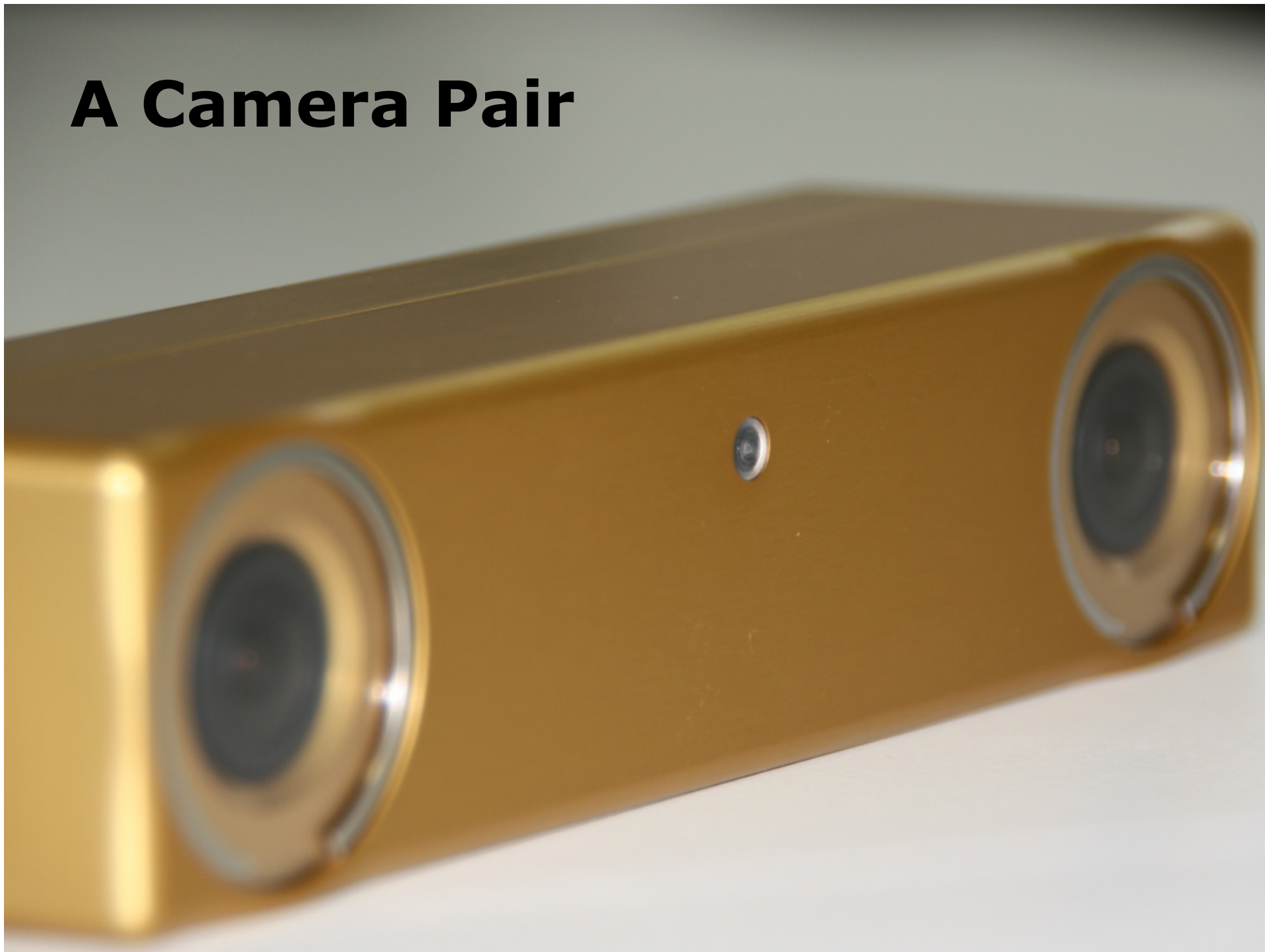# Photogrammetry & Robotics Lab
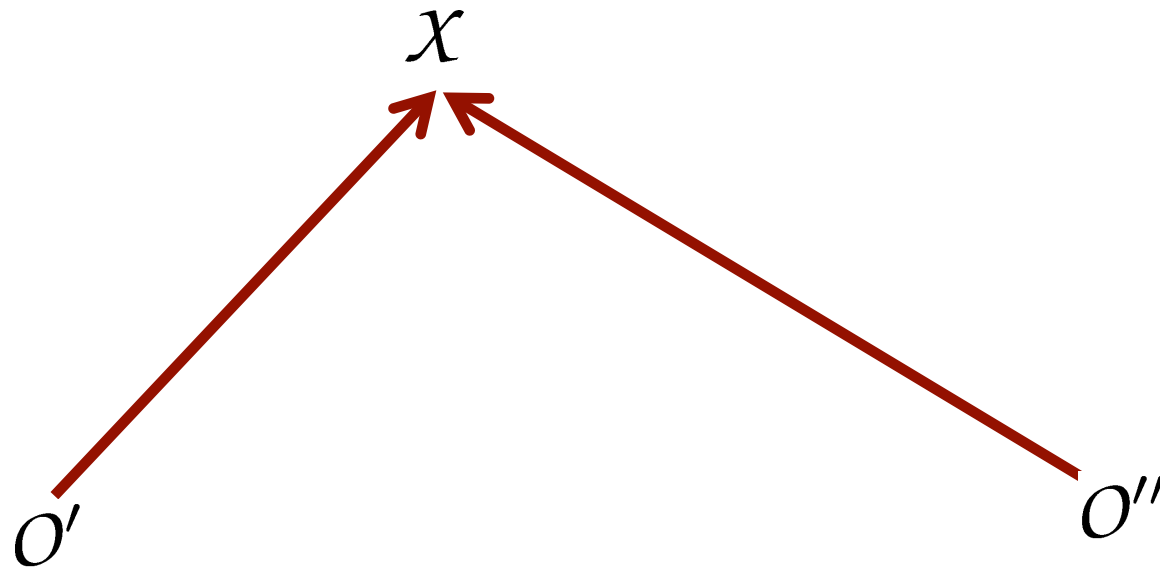
Epipolar Geometry

Cyrill Stachniss

The slides have been created by Cyrill Stachniss.

A Camera Pair

Coplanarity Constraint

- Intersection of two corresponding rays
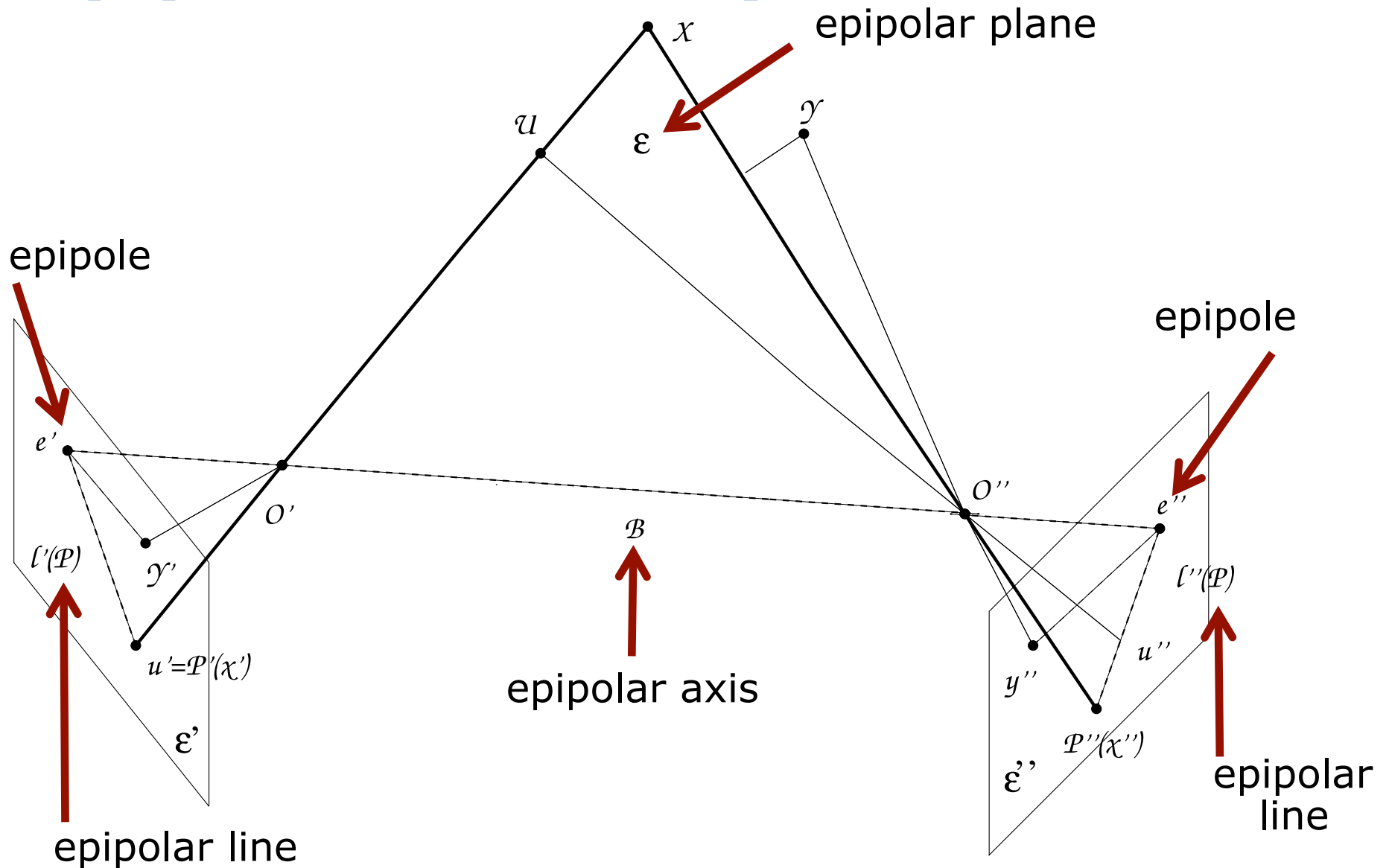- The rays lie in one plane in 3D

Epipolar Geometry – Motivation

- **Given** a point x' in the plane of the **first** image

- **Find** the corresponding point x'' in the **second** image plane

Epipolar Geometry

- Epipolar geometry is used to describe geometric relations in image pairs

- Enables efficient search for and prediction of corresponding points

- Given a straight-line preserving mapping, the search space reduces from 2D (whole image) to 1D (line)

Epipolar Geometry

epipolar plane

x

$\mathcal{U}$

ε

$\mathcal{Y}$

epipole

e'

$l'(\mathcal{P})$

O'

$\mathcal{Y}'$

$\mathcal{B}$

O''

e''

$l''(\mathcal{P})$

$u'=\mathcal{P}'(x')$

u''

y''

$\mathcal{P}''(x'')$

ε'

ε''

epipolar axis

epipolar line

epipole

epipolar line
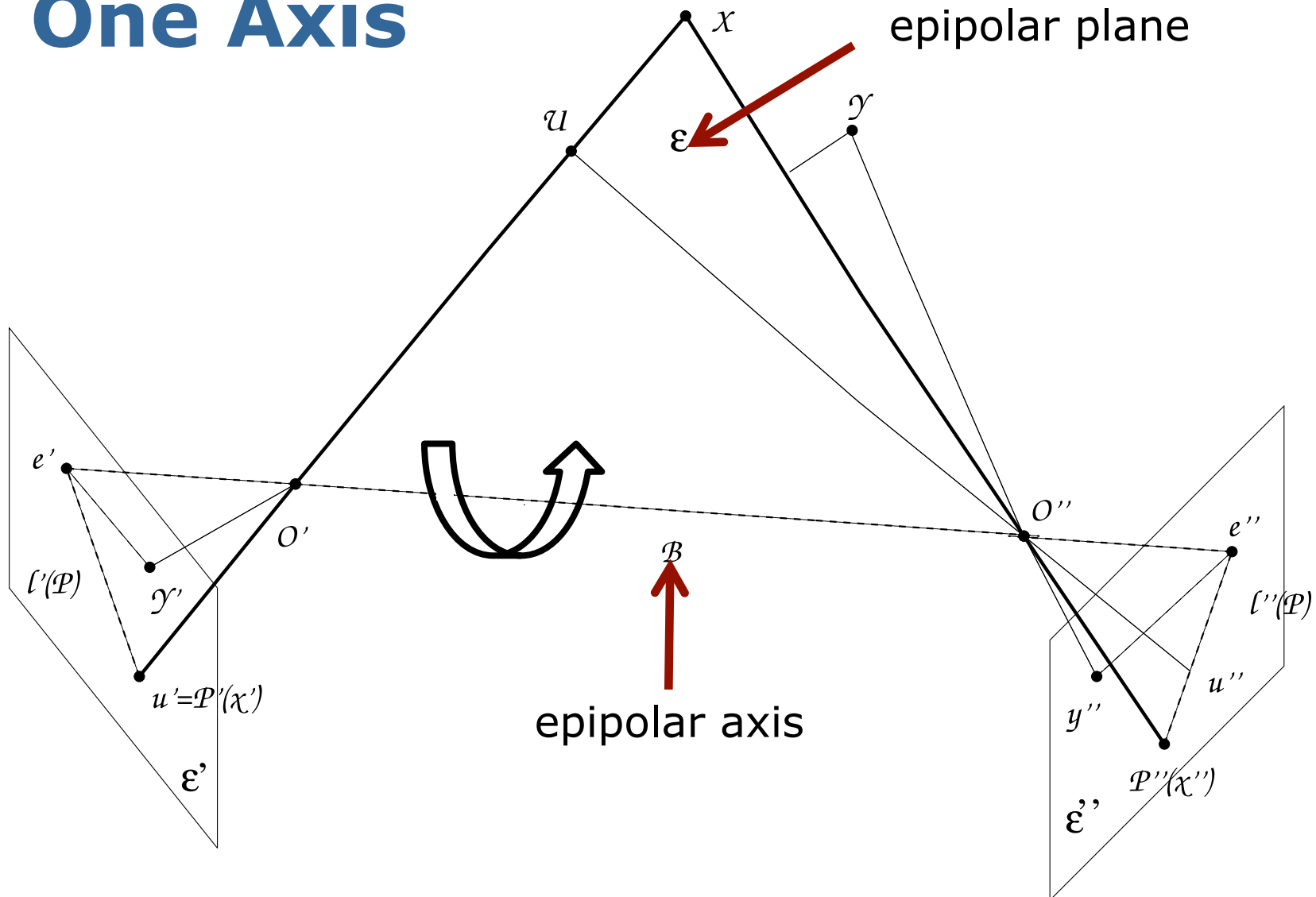
Image courtesy: Förstner   6

Important Elements (1)

- **Epipolar axis** $\mathcal{B} = (\mathcal{O}'\,\mathcal{O}'')$ is the line through the two projection centers
- **Epipolar plane** $\varepsilon = (\mathcal{O}'\,\mathcal{O}''\,\mathcal{X})$ depends on the projection centers and the point

Multiple Epipolar Planes, One Axis

epipolar plane

$\mathcal{E}$

epipolar axis

Image courtesy: Förstner   8

Important Elements (2)

- **Epipolar axis** $\mathcal{B} = (\mathcal{O}'\mathcal{O}'')$ is the line through the two projection centers
- **Epipolar plane** $\varepsilon = (\mathcal{O}'\mathcal{O}''X)$ depends on the projection centers and the point
- **Epipoles** $e' = (\mathcal{O}'')',\ e'' = (\mathcal{O}')''$ are the images of the projection centers
- **Epipolar lines** $l'(X) = (\mathcal{O}''X)',\ l''(X) = (\mathcal{O}'X)''$ are are the images of the rays $(\mathcal{O}''X)$ and $(\mathcal{O}'X)$ in the other image respectively

Epipoles and Epipolar Lines

- **Epipoles** $e' = (O'')', \; e'' = (O')''$ are the images of the projection centers

- They can also be written as

$$e' = (O'\,O'') \cap \varepsilon' \qquad e'' = (O'\,O'') \cap \varepsilon''$$

- **Epipolar lines** $l'(X) = (O''X)', \; l''(X) = (O'X)''$ are are the images of the rays $(O''X)$ and $(O'X)$ in the other image respectively and can be written as

$$l'(X) = \varepsilon \cap \varepsilon' \qquad l''(X) = \varepsilon \cap \varepsilon''$$

Epipolar Geometry

$$\varepsilon = (O'O''X)$$

$$x$$

$$u$$

$$\varepsilon$$

$$y$$

$$e' = (O'')'$$

$$o'$$

$$l'(\mathcal{P})$$

$$\mathcal{Y}'$$

$$\mathcal{B} = (O'O'')$$

$$o''$$

$$e'' = (O')''$$

$$l''(\mathcal{P})$$

$$u' = \mathcal{P}'(x')$$

$$\varepsilon'$$

$$y''$$

$$u''$$

$$\mathcal{P}''(x'')$$

$$\varepsilon''$$

$$l'(X) = (O''X)'$$
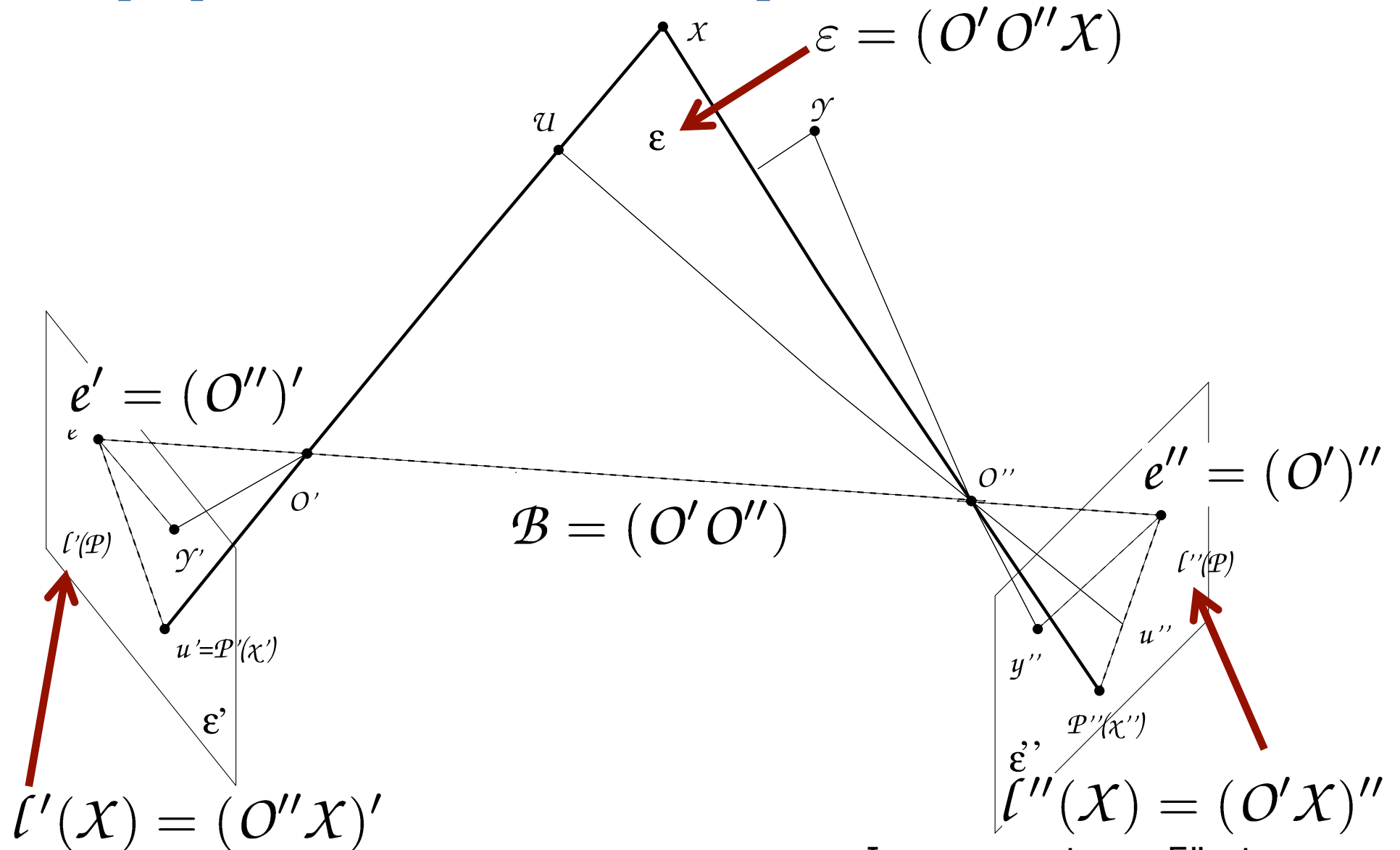
$$l''(X) = (O'X)''$$

Image courtesy: Förstner 11

In the Epipolar Plane...

Using a distortion-free lens,

- the projection centers O', O''
- the point X
- the epipolar lines $l'(X), l''(X)$
- the epipoles e', e''
- the image points X', X''

all lie in the epipolar plane ε

This simplifies the task of predicting the location of corresponding a point in the other image

Predicting the Location of Corresponding Points

- Task: Predict the location x'' given x'
- The epipolar plane through $\varepsilon = (O' O'' X')$
- The intersection of the epipolar plane and the second image plane ε'' yields the epipolar line $l''(x)$
- The corresponding point x'' lies on the epipolar line $l''(x)$

Reduces the search space from 2D to 1D

Computing the Key Elements of the Epipolar Geometry

Computing the Key Elements of the Epipolar Geometry

- We described the important quantities geometrically, but do not know yet how to obtain them

- We will show how to compute them based on the **projection matrices** and the **fundamental matrix**
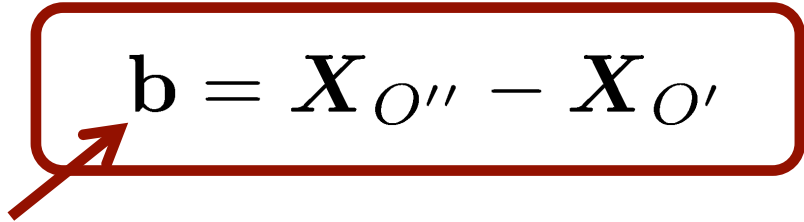
Epipolar Axis

- The direction of the epipolar axis can directly be computed from the projection centers

$$\mathbf{b} =$$

Epipolar Axis

- The direction of the epipolar axis can directly be computed from the projection centers

$$\mathbf{b} = \boldsymbol{X}_{O''} - \boldsymbol{X}_{O'}$$
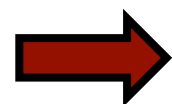
homogenous vector

- This equation expresses that we only know the **direction, not the length**

Computing Epipolar Lines

- The image points lie on the epipolar lines, i.e., $x' \in l'$, $x'' \in l''$

- For x':
$$\mathbf{x'}^{\top}\mathbf{l'} \;=\; 0$$

- We can exploit the coplanarity constraint for both points x', x''

$$\mathbf{x'}^{\top} \underbrace{\mathsf{F}\mathbf{x''}}_{\mathbf{l'}} \;=\; 0$$

$$\Rightarrow \boxed{\mathbf{l'} \;=\; \mathsf{F}\mathbf{x''}}$$

18
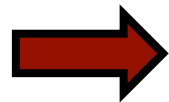
Computing Epipolar Lines

- The same for the point x''

$$\mathbf{l''}^\top \mathbf{x''} = 0$$

- We can again exploit the coplanarity constraint $\mathbf{x'}^\top F \mathbf{x''} = 0$ and obtain

$$\mathbf{l''}^\top = \mathbf{x'}^\top F$$

$$\Longrightarrow \boxed{\mathbf{l''} = F^\top \mathbf{x'}}$$ **this is our prediction (1D space)**

19

Epipolar Lines Summary

- The image points lie on the epipolar lines, i.e., $x' \in l'$, $x'' \in l''$
- For the points x', x'', we can exploit the coplanarity constraint
$$\mathbf{x'}^\top \mathsf{F} \mathbf{x''} = 0$$
- This is valid if
$$\boxed{\mathbf{l'} = \mathsf{F} \mathbf{x''} \qquad \mathbf{l''} = \mathsf{F}^\top \mathbf{x'}}$$
- as then
$$\mathbf{x'}^\top \mathbf{l'} = 0 \qquad \mathbf{x''}^\top \mathbf{l''} = 0$$

Computing the Epipoles

- The epipoles are the projections of the projection centers in the other image
- Both can be computed easily using the projection matrices

$$\mathbf{e}' = \qquad\qquad \mathbf{e}'' = $$

Computing the Epipoles

- The epipoles are the projections of the projection centers in the other image
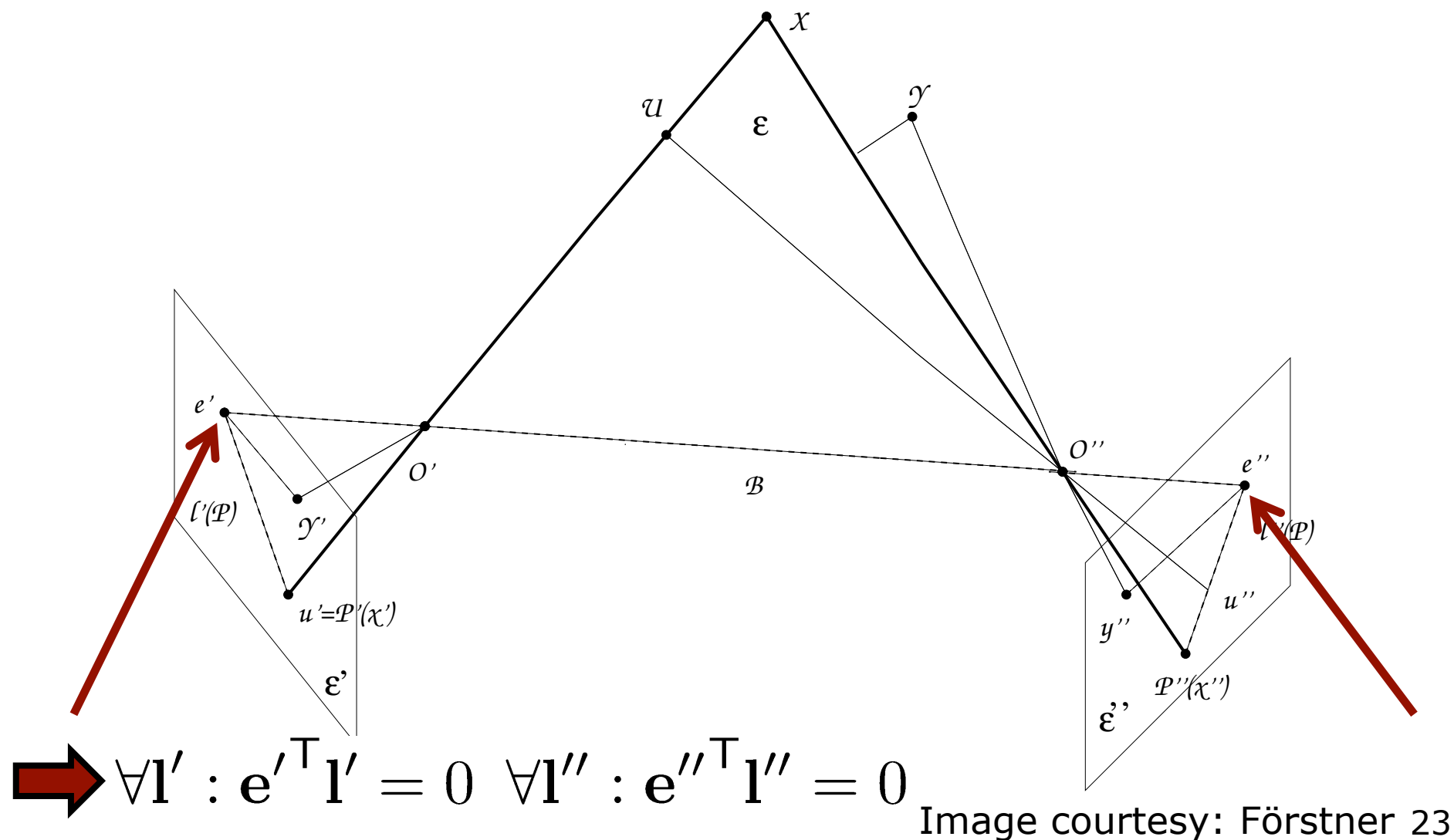- Both can be computed easily using the projection matrices

$$\mathbf{e}' = \mathsf{P}'\mathbf{X}_{O''} \qquad \mathbf{e}'' = \mathsf{P}''\mathbf{X}_{O'}$$

Epipole is the Intersection of All Epipolar Lines in an Image

$$\forall \mathbf{l}' : \mathbf{e}'^{\top} \mathbf{l}' = 0 \quad \forall \mathbf{l}'' : \mathbf{e}''^{\top} \mathbf{l}'' = 0$$

Image courtesy: Förstner 23

Epipoles and Fundamental Mat.

- The epipoles are the intersection of all epipolar lines, i.e.,

$$\forall \mathbf{l}' : {\mathbf{e}'}^{\top} \mathbf{l}' = 0 \qquad \forall \mathbf{l}'' : {\mathbf{e}''}^{\top} \mathbf{l}'' = 0$$

- Combining this with the epipolar lines defined through F, i.e., $\mathbf{l}' = \mathsf{F}\mathbf{x}''$ yields for all points $\mathbf{x}''$

$$\boxed{{\mathbf{e}'}^{\top} \mathsf{F}\mathbf{x}'' = 0}$$

- The epipole is the null space of $\mathsf{F}^{\top}$

Epipoles and Fundamental Mat.

- Analogous, we obtain for the second epipole $\mathbf{x'}^\top \mathsf{F} \mathbf{e''} = 0$

- Thus,

$$\mathrm{null}(\mathsf{F}^\top) = \mathbf{e'} \qquad \mathrm{null}(\mathsf{F}) = \mathbf{e''}$$

Eigenvectors of F

The epipoles are the left and right eigenvectors of the fundamental matrix

$$\mathbf{e'}^{\mathsf{T}}\mathsf{F} = \mathbf{0} \qquad \mathsf{F}\mathbf{e''} = \mathbf{0}$$

$$\mathrm{null}(\mathsf{F}^{\mathsf{T}}) = \mathbf{e'} \qquad \mathrm{null}(\mathsf{F}) = \mathbf{e''}$$

(they correspond to an eigenvalue of zero)

Epipolar Geometry Summary

- We only assumed a straight-line preserving (uncalibrated) camera
- We discussed the epipolar geometry and key elements such as epipolar lines, axis, plane, and epipoles
- Insight: epipolar geometry reduces the search for correspondences in image pairs from a 2D space to 1D space

Literature

- Förstner, Wrobel: Photogrammetric Computer Vision, Ch. 12.2.1 – 12.2.7

Slide Information

- The slides have been created by Cyrill Stachniss as part of the photogrammetry and robotics courses.

- I tried to acknowledge all people from whom I used images or videos. In case I made a mistake or missed someone, please let me know.

- The photogrammetry material heavily relies on the very well written lecture notes by Wolfgang Förstner and the Photogrammetric Computer Vision book by Förstner & Wrobel.

- Parts of the robotics material stems from the great Probabilistic Robotics book by Thrun, Burgard and Fox.

- If you are a university lecturer, feel free to use the course material. If you adapt the course material, please make sure that you keep the acknowledgements to others and please acknowledge me as well. To satisfy my own curiosity, please send me email notice if you use my slides.

Cyrill Stachniss, cyrill.stachniss@igg.uni-bonn.de, 2014