Photogrammetry & Robotics Lab

Iterative Closest Point: Point Cloud Alignment

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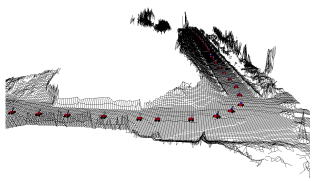
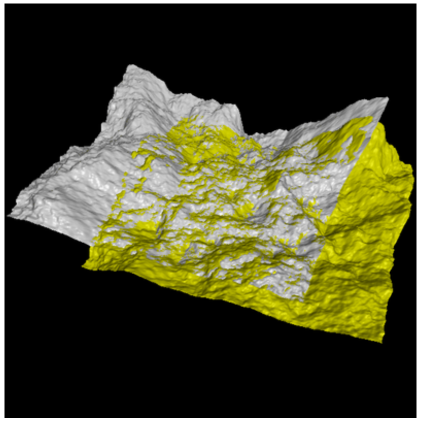
Alignment of 3D Data Points

- Find the parameters of the transformation that best align corresponding data points
- Optimization / search for parameters
  - Least squares and robust least squares
  - **Iterative closest point (ICP)**

Scan Alignment in Mapping



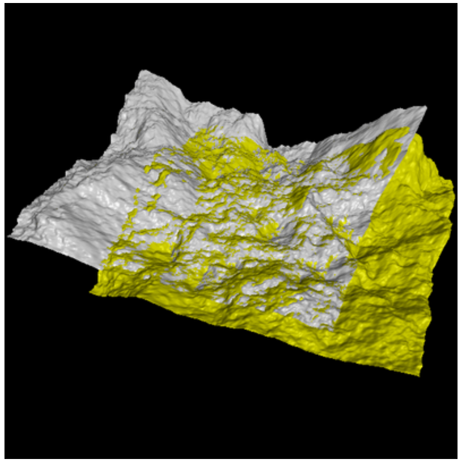


Goal: Find local transformation to align points

Iterative Closest Point (ICP)

[Besl & McKay 92]

Find Local Transformation to Align Points or Surfaces



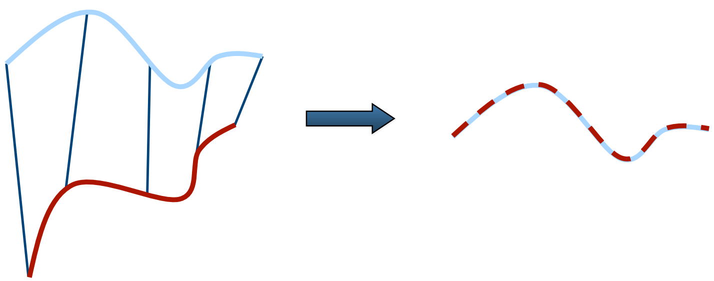
The Problem

- Given two point sets:
  $Q = \{\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N\} \qquad P = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_M\}$
  with correspondences $C = \{(i,j)\}$
- Wanted: Translation $\boldsymbol{t}$ and rotation R that minimize the sum of the squared errors:

$$E(R, \boldsymbol{t}) = \sum_{(i,j) \in C} \|\boldsymbol{q}_i - R\boldsymbol{p}_j - \boldsymbol{t}\|^2$$

Key Idea

If the correct correspondences are known, the correct relative rotation/translation can be computed directly



Key Idea

If the correct correspondences are known, the correct relative rotation/translation can be computed directly

- Shift via the center of mass
- Rotational alignment

Center of Mass

- The centers of mass of the correspond. points in both sets

$$\boldsymbol{\mu}_Q = \frac{1}{|C|} \sum_{(i,j)\in\mathcal{C}} \boldsymbol{q}_i \quad \boldsymbol{\mu}_P = \frac{1}{|C|} \sum_{(i,j)\in\mathcal{C}} \boldsymbol{p}_j$$

- Subtract the corresponding center of mass from every point

$$\begin{aligned} Q' &= \{\boldsymbol{q}_i - \boldsymbol{\mu}_Q\} = \{\boldsymbol{q}'_i\} \\ P' &= \{\boldsymbol{p}_j - \boldsymbol{\mu}_P\} = \{\boldsymbol{p}'_j\} \end{aligned}$$

Orthogonal Procrustes Problem

- Minimizing $E(R,\boldsymbol{t}) = \sum_{(i,j)\in\mathcal{C}} \|\boldsymbol{q}_i - R\boldsymbol{p}_j - \boldsymbol{t}\|^2$
- Equivalent to minimizing

$$E'(R) = \|[\boldsymbol{q}'_1 \ldots \boldsymbol{q}'_n] - R[\boldsymbol{p}'_1 \ldots \boldsymbol{p}'_n]\|_F^2$$

- Called Orthogonal Procrustes problem
- Can be solved through SVD

See: Söderkvist, Using SVD for some fitting problems

Singular Value Decomposition

- Compute the cross-covariance matrix

$$W = \sum_{(i,j)\in\mathcal{C}} \boldsymbol{q}'_i \boldsymbol{p}'^{\mathsf{T}}_j$$

- Use the SVD to decompose

$$W = UDV^{\mathsf{T}}$$

- The matrices U, V are 3 by 3 matrices
- U, V are rotation matrices
- Diagonal matrix $D = \mathrm{Diag}(\sigma_1, \sigma_2, \sigma_3)$

Singular Value Decomposition

- If $\mathrm{rank}(W) = 3$, the parameters minimizing $E(R, \boldsymbol{t})$ are unique and given by:

$$\begin{aligned} R &= UV^{\mathsf{T}} \\ \boldsymbol{t} &= \boldsymbol{\mu}_Q - R\boldsymbol{\mu}_P \end{aligned}$$

SVD-Based Alignment Summary

- Form the cross-covariance matrix

$$W = \sum \boldsymbol{q}'_i \boldsymbol{p}'^{\mathsf{T}}_j$$

- Compute SVD

$$W = UDV^{\mathsf{T}}$$

- The rotation matrix is

$$R = UV^{\mathsf{T}}$$

- Translate and rotate points:

$$\boldsymbol{p}_j \leftarrow R(\boldsymbol{p}_j - \boldsymbol{\mu}_P) + \mu_Q$$

SVD-Based Alignment Summary

Alignment through translation and rotation $\boldsymbol{p}_j \leftarrow R(\boldsymbol{p}_j - \boldsymbol{\mu}_P) + \mu_Q$

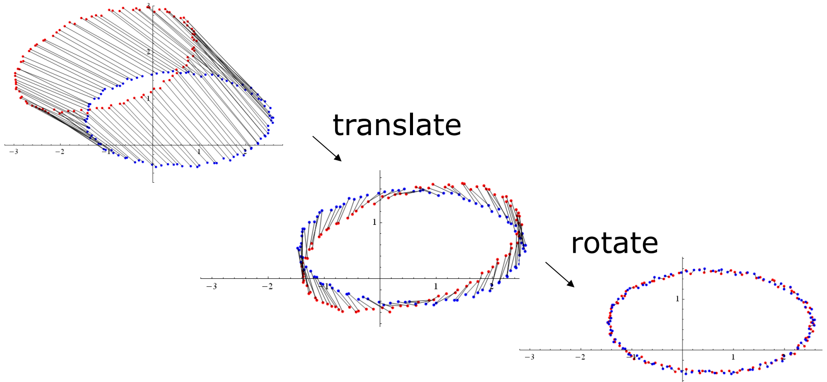
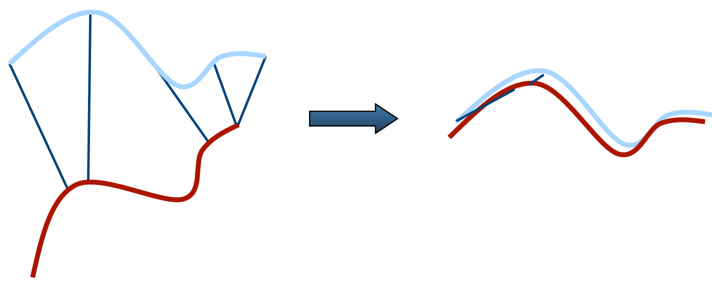


Image courtesy: Ju

Unknown Data Association

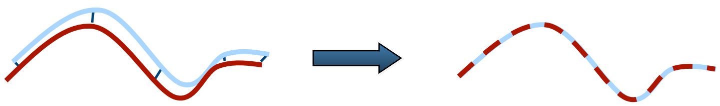
ICP with Unknown Data Association

If the correct correspondences are not known, it is generally impossible to determine the optimal relative rotation and translation in one step



Iterative Closest Point (ICP) Algorithm

- Idea: Iterate to find alignment
- Iterative Closest Points
  [Besl & McKay 92]
- Converges if starting positions are "close enough"



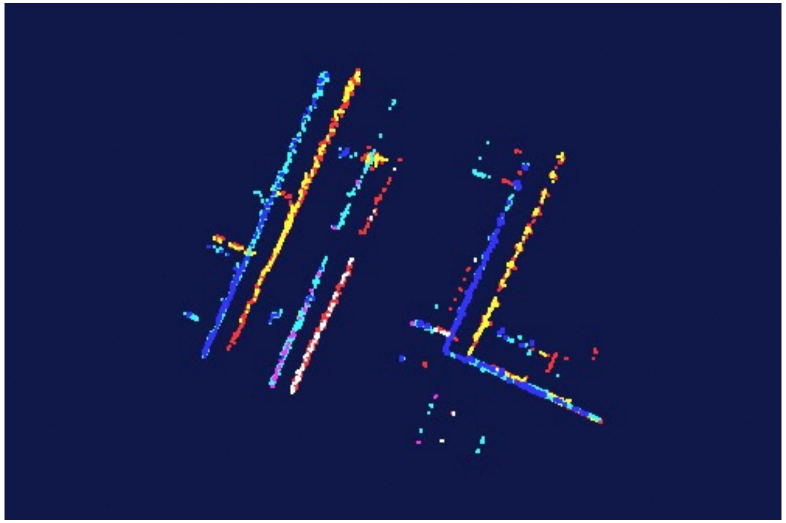
Basic ICP Algorithm

error = inf

while (error decreased and error > threshold)

- Determine corresponding points
- Compute rotation R, translation t via SVD
- Apply R and t to the points of the set to be registered
- error = $E(R, \boldsymbol{t})$

ICP Example



ICP Variants

Variants on the following stages of ICP have been proposed:

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

Performance of Variants

Various aspects of performance:

- Speed
- Stability (local minima)
- Tolerance w.r.t. noise and outliers
- Basin of convergence (maximum initial misalignment)

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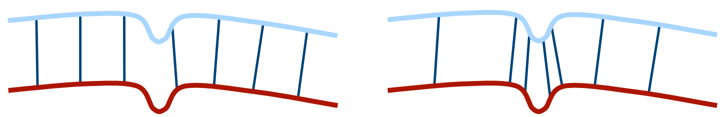
Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based sampling
- Normal-space sampling (Ensure that samples have normals distributed as uniformly as possible)

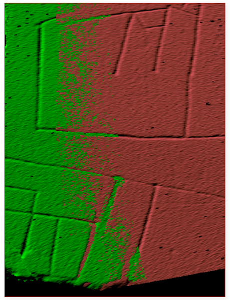
Normal-Space Sampling

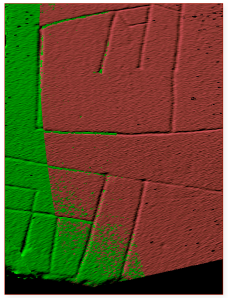


uniform sampling   normal-space sampling

Comparison

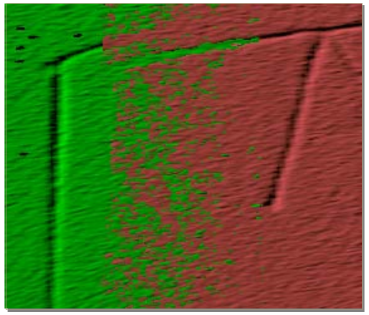
- Normal-space sampling better for mostly smooth areas with sparse features [Rusinkiewicz et al., 01]

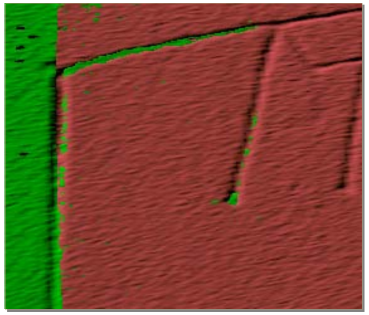

random sampling


normal-space sampling

Comparison

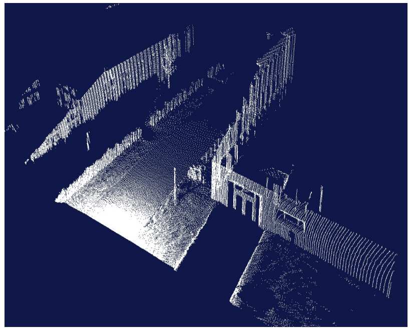
- Normal-space sampling better for mostly smooth areas with sparse features [Rusinkiewicz et al., 01]

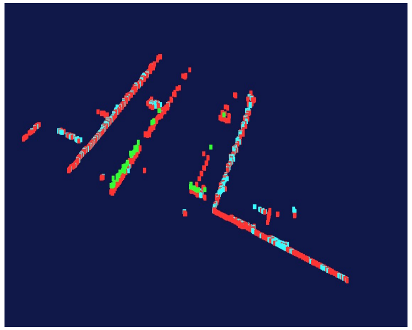

random sampling


normal-space sampling

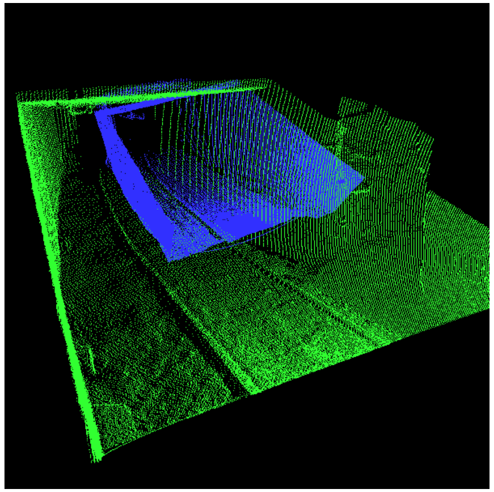
Feature-Based Sampling

- Try to find "important" points
- Simplifies the search for correspondences
- Higher efficiency and higher accuracy
- Requires preprocessing


3D Scan (~200.000 Points)


Extracted Features (~5.000 Points)

ICP with Uniform Sampling



Video courtesy: Nuechter

ICP Variants

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Re-Weighting

- **Weight the corresponding pairs**
- Noise: Weighting based on sensor uncertainty
- Outlier: Assign **lower weights** for points with **higher point-point distances**
- Determine transformation that minimizes the weighted error function

ICP Variants

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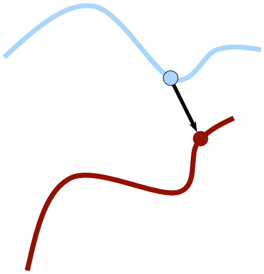
Data Association

- Has greatest effect on convergence and speed
- Matching methods:
  - Closest point
  - Normal shooting
  - Closest compatible point
  - Point-to-plane
  - Projection-based approaches
  - …

Closest-Point Matching

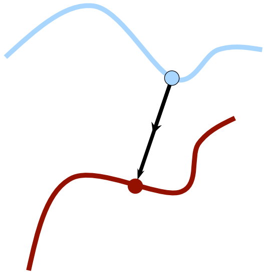
- Find closest point in other the point set (using kd-trees)



Generally stable, but slow convergence and requires preprocessing

Normal Shooting

- Project along normal, intersect other point set



Slightly better convergence results than closest point for smooth structures, worse for noisy or complex structures

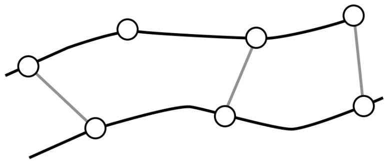
Closest Compatible Point

- Robustification by considering the **compatibility** of the points
- Only matches compatible points
- Compatibility can be based on
  - Normals
  - Colors
  - Curvature
  - Higher-order derivatives
  - Other local features

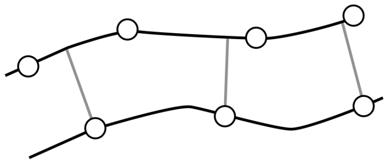
Point-to-Plane Error Metric

Minimize the sum of the squared distances between a point and the tangent plane at its correspondence point [Chen & Medioni 91]

Point-to-Point vs Point-to-Plane



point-to-point



point-to-plane

Point-to-Plane Error Metric

- Each iteration generally slower than the point-to-point version, however, often significantly better convergence rates [Rusinkiewicz01]
- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]

ICP Variants

Variants on the following stages of ICP have been proposed:

1. Point subsets (from one or both point sets)
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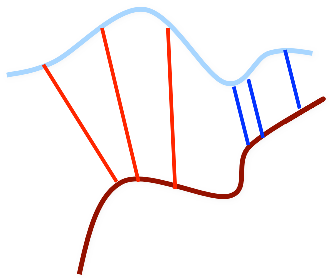
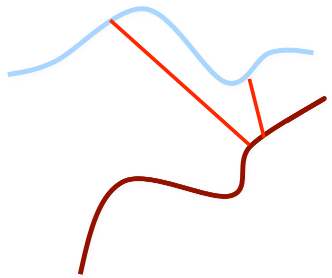
Rejecting (Outlier) Point Pairs

- Point-to-point distance larger than a given threshold



Rejecting (Outlier) Point Pairs

- Point-to-point distance larger than a given threshold
- Rejection of pairs that are not consistent with their neighboring pairs
  [Dorai 98]



Rejecting (Outlier) Point Pairs

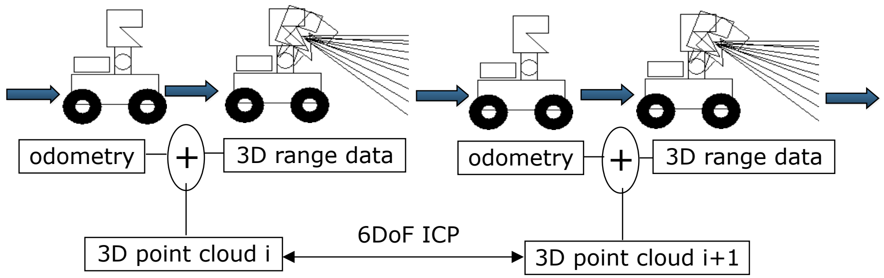
- Point-to-point distance larger than a given threshold
- Rejection of pairs that are not consistent with their neighboring pairs
  [Dorai 98]
- Trimmed ICP: Sort correspondences w.r.t. their error, ignore the worst t%
  [Chetverikov et al. 02]
  - t is related to overlap and outlier ratio
  - Knowledge about the overlap has to be estimated

ICP Algorithm

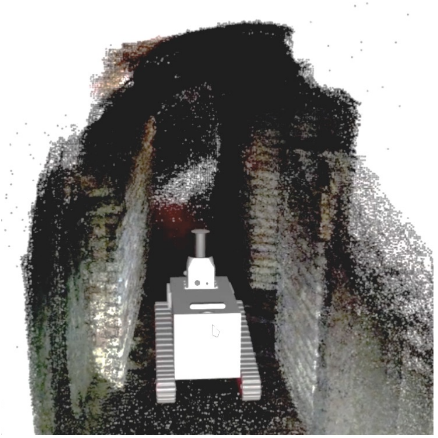
- Potentially subsample point clouds
- Determine corresponding points
- Potentially weight or reject pairs
- Compute rotation *R*, translation *t* (SVD)
- Apply *R* and *t* to all points of the set to be registered
- Compute the error $E(R, \boldsymbol{t})$
- While error decreased and error > threshold
  - Repeat to determine correspondences etc.
- Output final alignment

Common IPC Applications

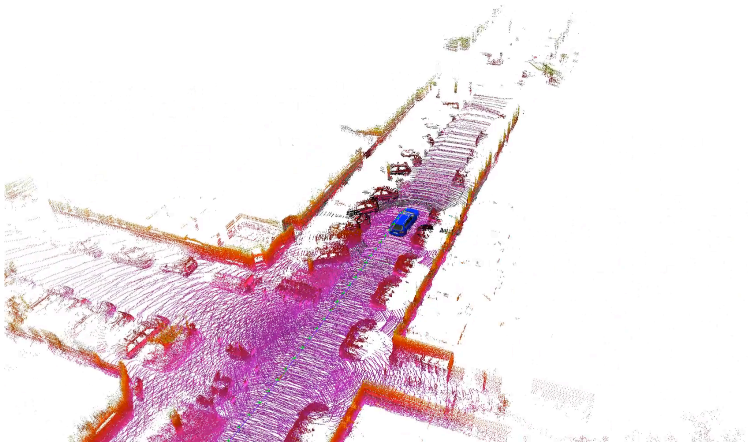
- Laser scan matching
- Range image matching



Kinect-Based Mapping



LiDAR ICP & SLAM



Video courtesy: Behley

Summary

- Alignment of 2D and 3D data points is an important task in perception
- Gold standard algorithm for calculating the transform between scans
- Estimates translation and rotation between the scans
- Given the correct data associations, the transformation can be computed efficiently using SVD

Summary

- The major problem is to determine the correct data associations
- Initial guess is needed for data association
- Iterative approach
- Several variants exist
- In practice, ICP does not always converge to the correct solution