MSC GEODETIC ENGINEERING

MSR-02: ADVANCED TECHNIQUES FOR MOBILE SENSING AND ROBOTICS (GEODESY TRACK)

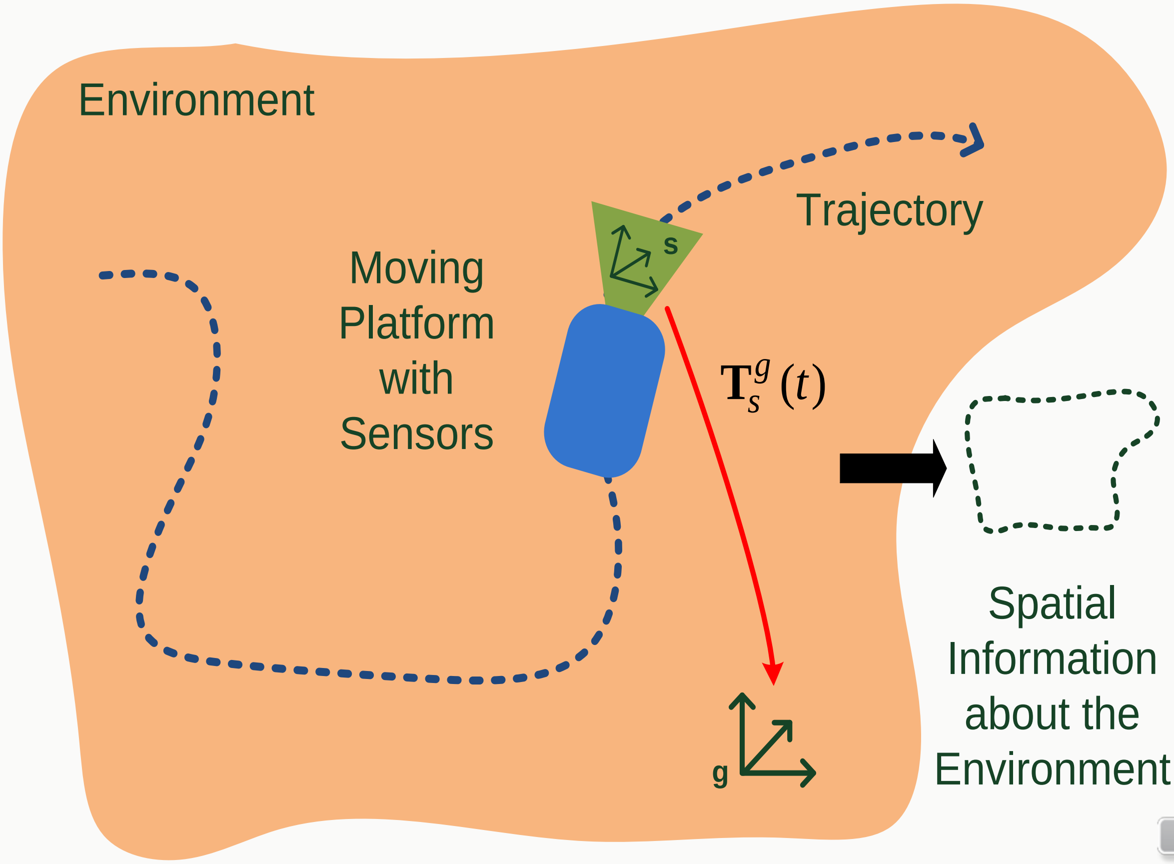
02: TRAJECTORY ESTIMATION

DR. LASSE KLINGBEIL

INSTITUTE FOR GEODESY AND GEOINFORMATION
UNIVERSITY OF BONN

ADVANCED TECHNIQUES FOR MOBILE SENSING AND ROBOTICS – LECTURE CONTENT

MOBILE MAPPING



CHAPTER 1: MOBILE LASER SCANNING

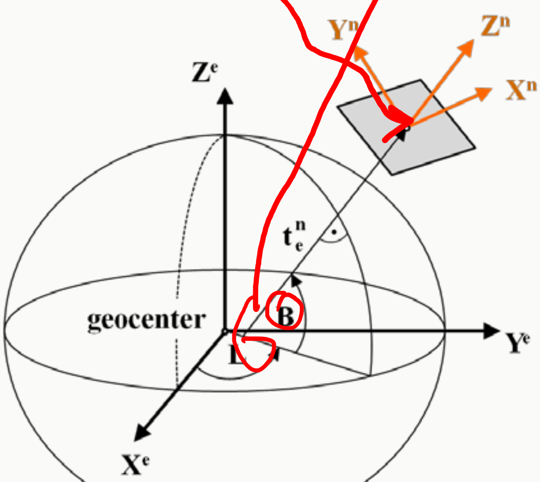
$$\mathbf{p}^{global}_{object}(t_s) \quad = \quad \mathbf{T}^{global}_{body}(t_s) \cdot \mathbf{T}^{body}_{sensor} \cdot \mathbf{p}^{sensor}_{object}(t_s)$$

- Review of involved **coordinate systems /frames**
- Derivation of detailed **georeferencing equation** for the example of **mobile laser scanning**

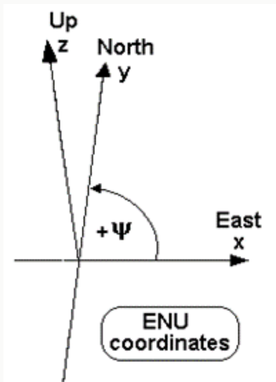
$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \mathbf{R}^e_n(L,B)\,\mathbf{R}^n_b(\phi,\theta,\psi) \cdot \left[\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \mathbf{R}^b_s(\alpha,\beta,\gamma) \cdot \begin{bmatrix} 0 \\ d \cdot \sin b \\ d \cdot \cos b \end{bmatrix} \right]$$

CHAPTER 2: TRAJECTORY PARAMETERS

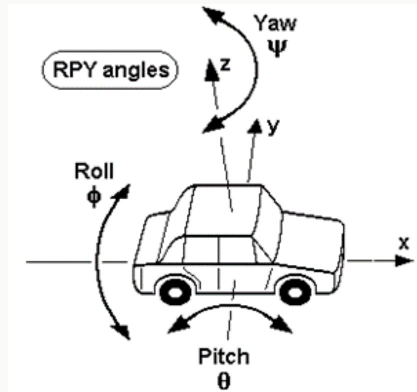
$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \mathbf{R}_n^e\,(L, B)\,\mathbf{R}_b^n\,(\phi, \theta, \psi) \cdot \left[\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \mathbf{R}_s^b\,(\alpha, \beta, \gamma) \cdot \begin{bmatrix} 0 \\ d \cdot \sin b \\ d \cdot \cos b \end{bmatrix} \right]$$



Earth frame

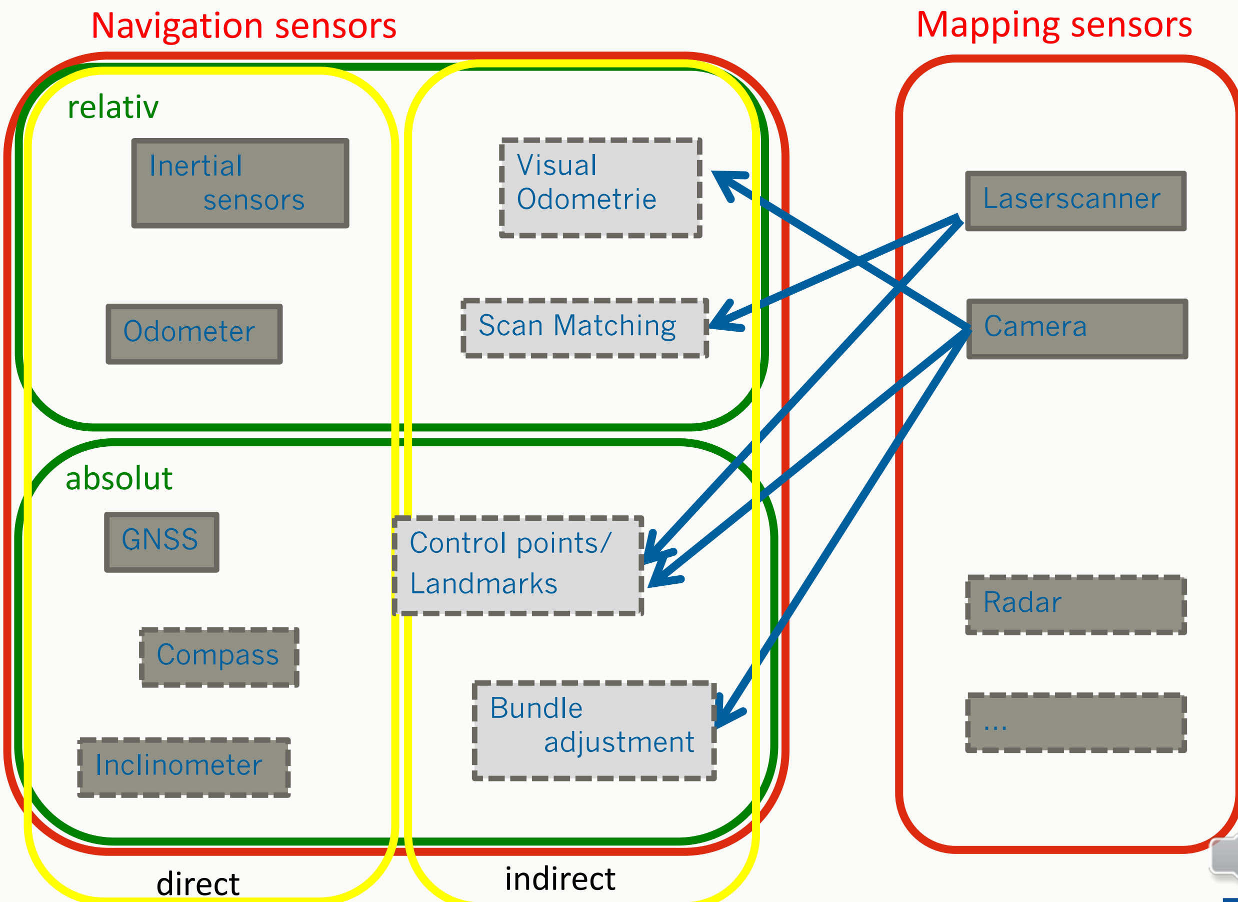


Navigation frame



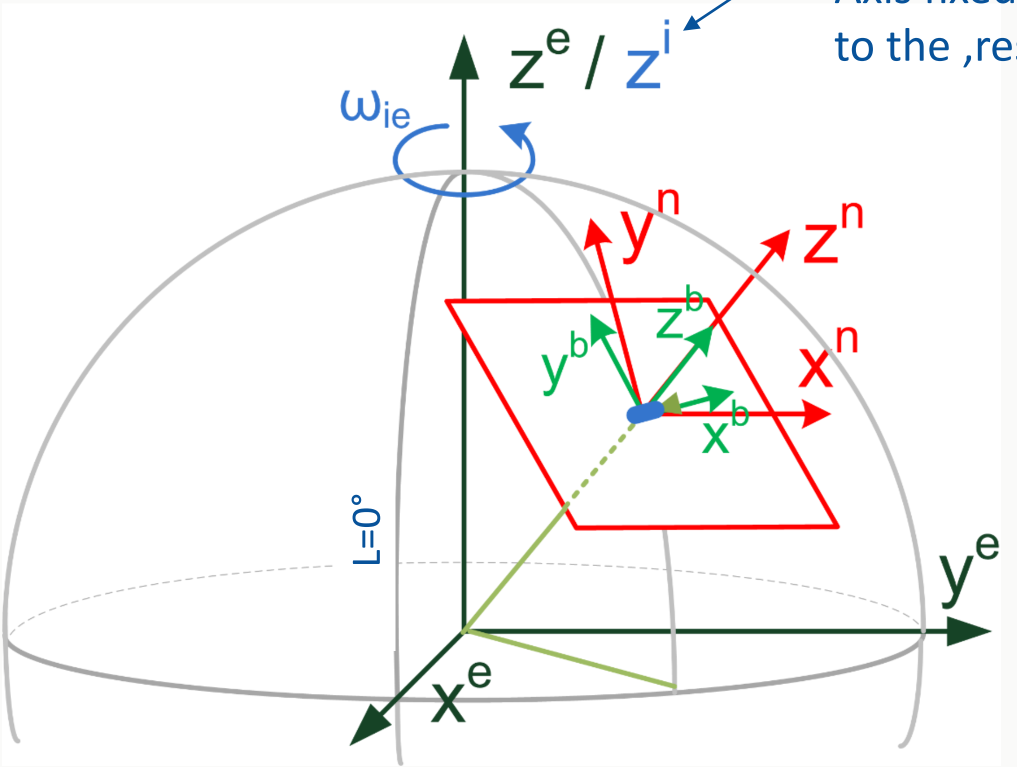
"body frame"

SENSORS OVERVIEW



INERTIAL SENSORS

COORDINATE SYSTEMS



Inertial frame

Axis fixed with respect to the ‚rest of the universe'

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INERTIAL SENSORS

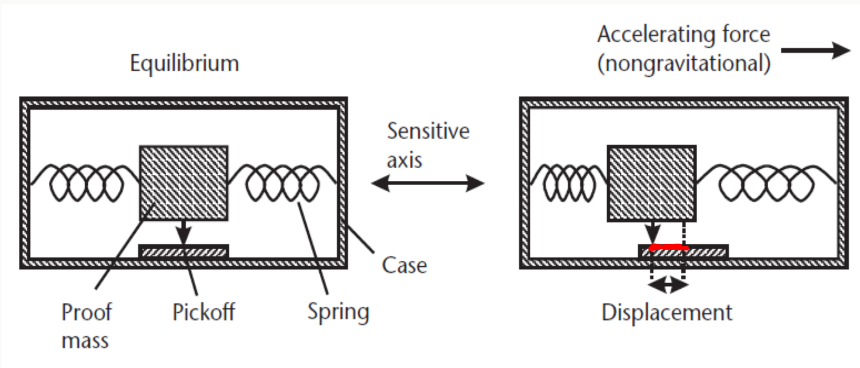
ACCELEROMETER

- Measurement of specific force
  (= non gravitational acceleration)

$F = 0$

$a = 9{,}81\ m/s^2$

$g = -9{,}81\ m/s^2$



Specific force of the body frame relativ to the inertial frame, given in body frame coordinates

gravitational acceleration

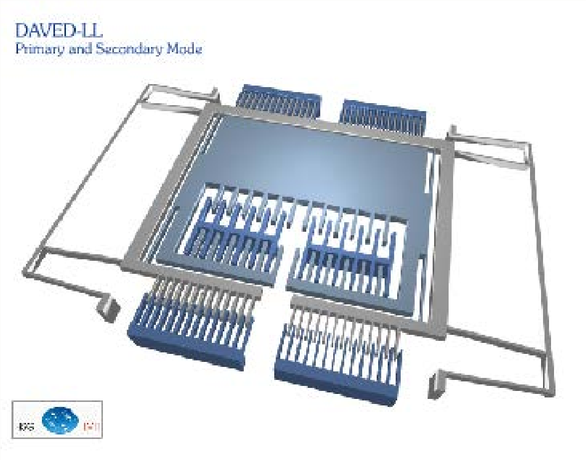
$$\mathbf{f}_{ib}^{b} = \underline{\mathbf{a}_{ib}^{b}} - \gamma_{ib}^{b}$$

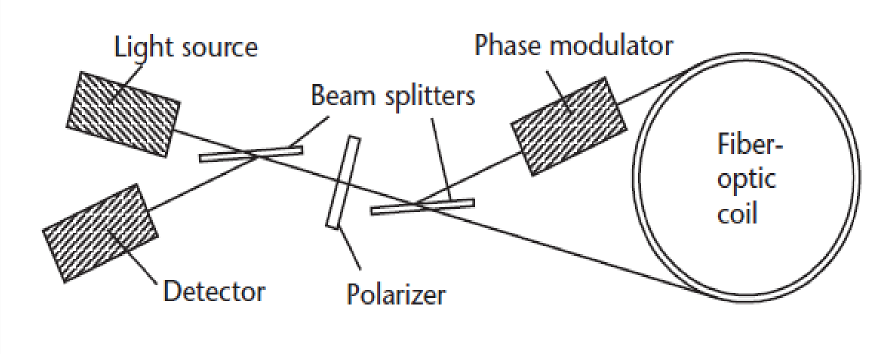
INERTIAL SENSORS

GYROSCOPE

- Measure angular rate of the body relative to the inertial frame given in body frame coordinates

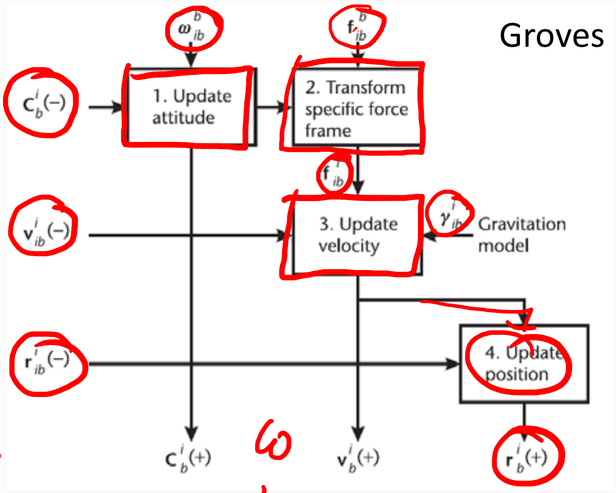
$$\omega_{ib}^{b}$$





STRAP DOWN INTEGRATION

INERTIAL FRAME EQUATIONS



- Use inertial frame as the system of resolving frame for position and orientation

- Update orientation

Integration errors

$$\mathbf{C}_b^i(+) \approx \mathbf{C}_b^i(-)(\mathbb{1} + \boldsymbol{\Omega}_{ib}^b \tau_i)$$

- Transform accelerometer measurement to inertial frame

$$\mathbf{f}_{ib}^i = \mathbf{C}_b^i \mathbf{f}_{ib}^b$$

- Correct for gravity

$$\mathbf{a}_{ib}^i = \mathbf{f}_{ib}^i + \gamma_{ib}^i$$

- Update velocity

$$\mathbf{v}_{ib}^i(+) \approx \mathbf{v}_{ib}^i(-) + \mathbf{a}_{ib}^i \tau_i$$

- Update position

$$\mathbf{r}_{ib}^i(+) \approx \mathbf{r}_{ib}^i(-) + \mathbf{v}_{ib}^i(-) \tau_i$$

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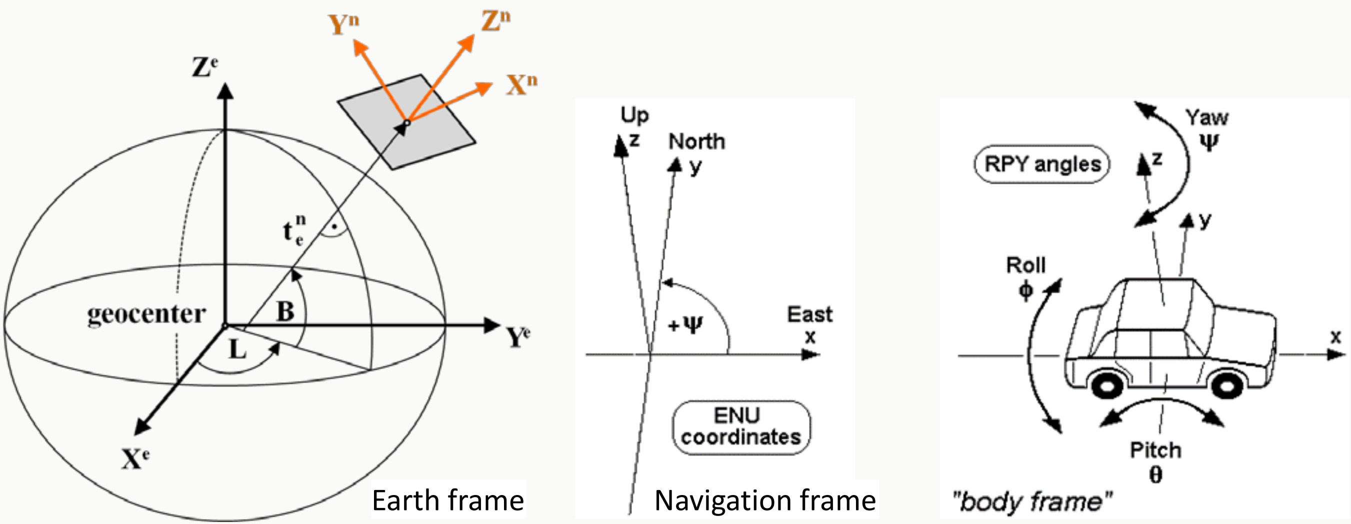
TRAJECTORY PARAMETERS

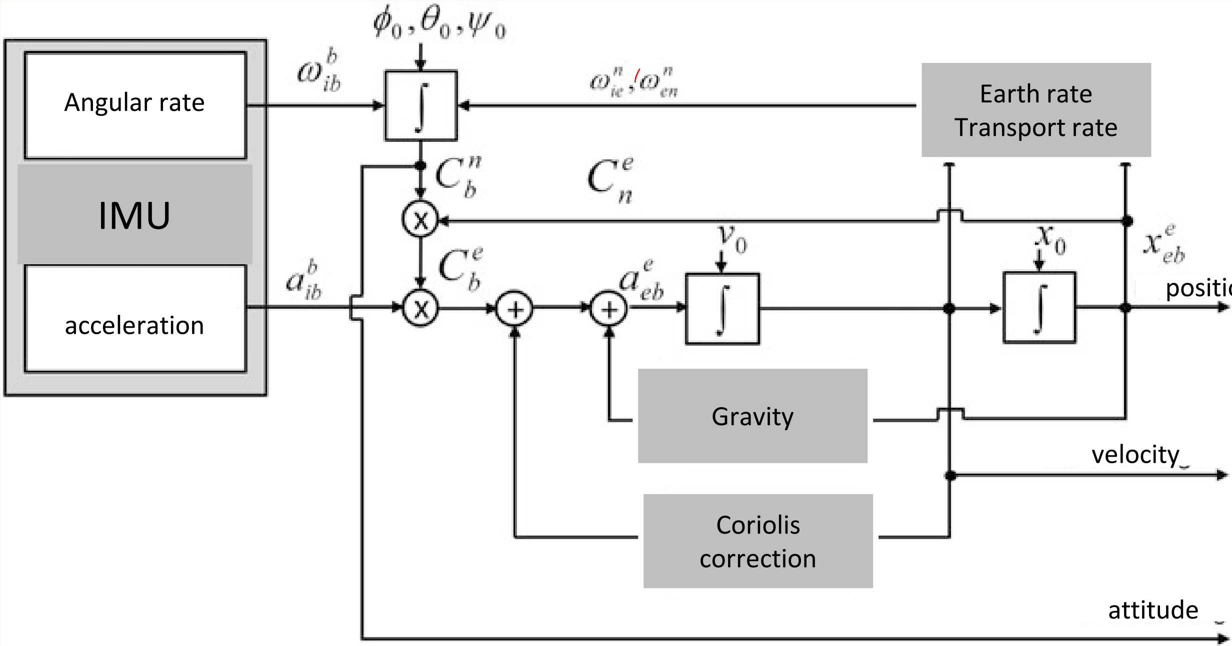
$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \mathbf{R}_n^e(L,B)\,\mathbf{R}_b^n(\phi,\theta,\psi)\cdot\left[\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \mathbf{R}_s^b(\alpha,\beta,\gamma)\cdot\begin{bmatrix} 0 \\ d\cdot\sin b \\ d\cdot\cos b \end{bmatrix}\right]$$



STRAPDOWN INTEGRATION

WANTED:

$$\begin{pmatrix} \mathbf{q}_b^n \\ \mathbf{p}_{eb}^e \\ \mathbf{v}_{eb}^n \end{pmatrix}_k = f\left(\begin{pmatrix} \mathbf{q}_b^n \\ \mathbf{p}_{eb}^e \\ \mathbf{v}_{eb}^n \end{pmatrix}_{k-1}, \begin{pmatrix} \mathbf{a}_{ib}^b \\ \boldsymbol{\omega}_{ib}^b \end{pmatrix}_{k-1} \right)$$



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INITIALIZATION AND ALIGNEMENT

HOW TO GET THE STARTING VALUES?

- Position
  - Use external systems, such as GNSS
  - Start from a known position (e.g. car has parked at known position)
- Orientation
  - Accelerometer levelling
  - Gyro Compassing
  - Magnetometer/Compass
  - Multiple GNSS Antennas
- Velocity
  - Use external system (e.g. GNSS)
  - Start in a static situation (v=0)

INS ERROR PROPAGATION

Orientation update $$\mathbf{C}_b^i(+) \approx \mathbf{C}_b^i(-)(\mathbb{1} + \mathbf{\Omega}_{ib}^b \tau_i)$$

- Constant gyroscope error leads to orientation error

$$\delta\phi_b^n(t) = \mathbf{b}_\omega t$$

  - e.g. 0.1 ° gyro bias → 30° orientation error after 5 minutes

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INS ERROR PROPAGATION

Position/velocity update

$$\mathbf{v}_{ib}^{i}(+) \approx \mathbf{v}_{ib}^{i}(-) + \mathbf{a}_{ib}^{i}\tau_i$$
$$\mathbf{r}_{ib}^{i}(+) \approx \mathbf{r}_{ib}^{i}(-) + \mathbf{v}_{ib}^{i}(-)\tau_i$$

- Constant velocity error integrates to position error

$$\delta\mathbf{r}_{eb}^{n}(t) \approx \delta\mathbf{v}_{eb}^{n}t$$

  - e.g. 0.1 m/s velocity error ➔ 30m position error after 5 minutes

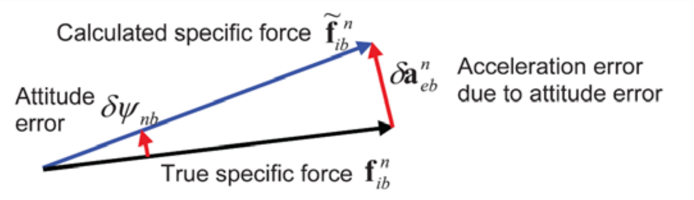
- Constant accelerometer bias leads to velocity and position errors

$$\delta\mathbf{v}_{eb}^{n}(t) \approx \mathbf{C}_{b}^{n}\mathbf{b}_a t, \qquad \delta\mathbf{r}_{eb}^{n}(t) \approx \frac{1}{2}\mathbf{C}_{b}^{n}\mathbf{b}_a t^2$$

  - e.g. 0.01 m/s^2 accelerometer bias ➔ 450m position error after 5 minutes

INS ERROR PROPAGATION

- Constant attitude errors lead to wrong transformations of the measured specific-force into the resoving frame
  - „Gravity compensation“ is wrong, parts of the measured reaction to gravity are considered as acceleration leading to acceleration offsets



  - e.g. 0.057° attitude error ➔ 440m position error after 5 minutes
- Constant gyroscope bias leads to attitude error leads to position error

$$\delta\mathbf{r}^n_{eb}(t) \propto t^3$$

  - e.g. 2.1°/hr bias ➔ 440m position error after 5 minutes

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INS ERROR PROPAGATION

CONSTANT ERRORS

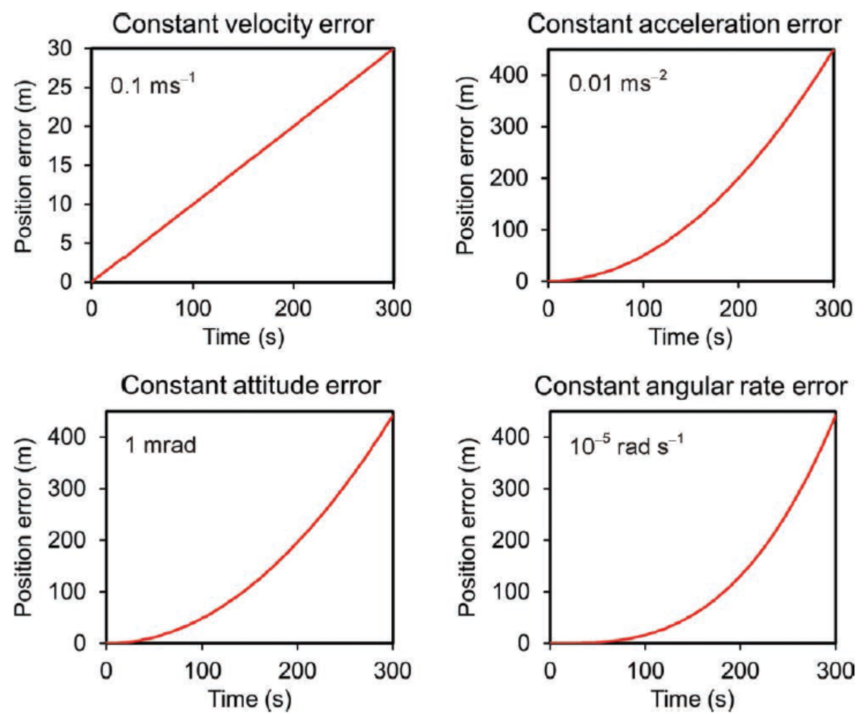

Fig. 10.
Short-term straight-line position error growth per axis for different error sources. (From [3] © Paul Groves 2013. Reproduced with permission.)

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INS ERROR PROPAGATION

SENSOR NOISE

- Noisy measurements of accelerometers and gyroscopes also lead to noisy position errors (uncertainty)

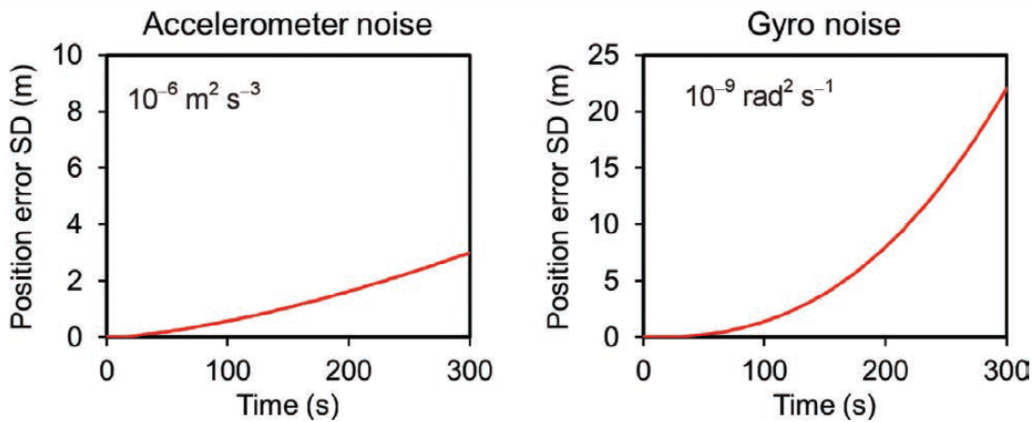

Fig. 12.
Short-term straight-line position error standard deviation growth per axis because of inertial sensor noise. (From [3] © Paul Groves 2013. Reproduced with permission.)

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NON-LINEAR FILTERING

- Linear Kalman Filter:
  - linear system and measurement models

$$\mathbf{x}_k = \mathbf{\Phi}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_k + \mathbf{G}\mathbf{w}_{s,k}$$
$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{w}_{m,k}$$

- General case:
  - predicted state is a function of last state, control input and noise

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{w}_{s,k})$$

  - Measurement is a function of the state

$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{w}_{m,k})$$

KALMAN FILTER

Predicted estimate

Predicted covariance

System noise

$$\mathbf{x}_k^- = \boldsymbol{\Phi}\mathbf{x}_{k-1}$$

$$P_k^- = \boldsymbol{\Phi} P_{k-1}\boldsymbol{\Phi}^T + \mathbf{Q}$$

Prediction

measurement noise

Kalman Gain

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

$$\mathbf{x}_k = \mathbf{x}_k^- + K_k(z_k - H\mathbf{x}_k^-)$$

$$P_k = (I - K_k H)P_k^-$$

Correction

measurement

Predicted measurement

Updated estimate

Updated covariance

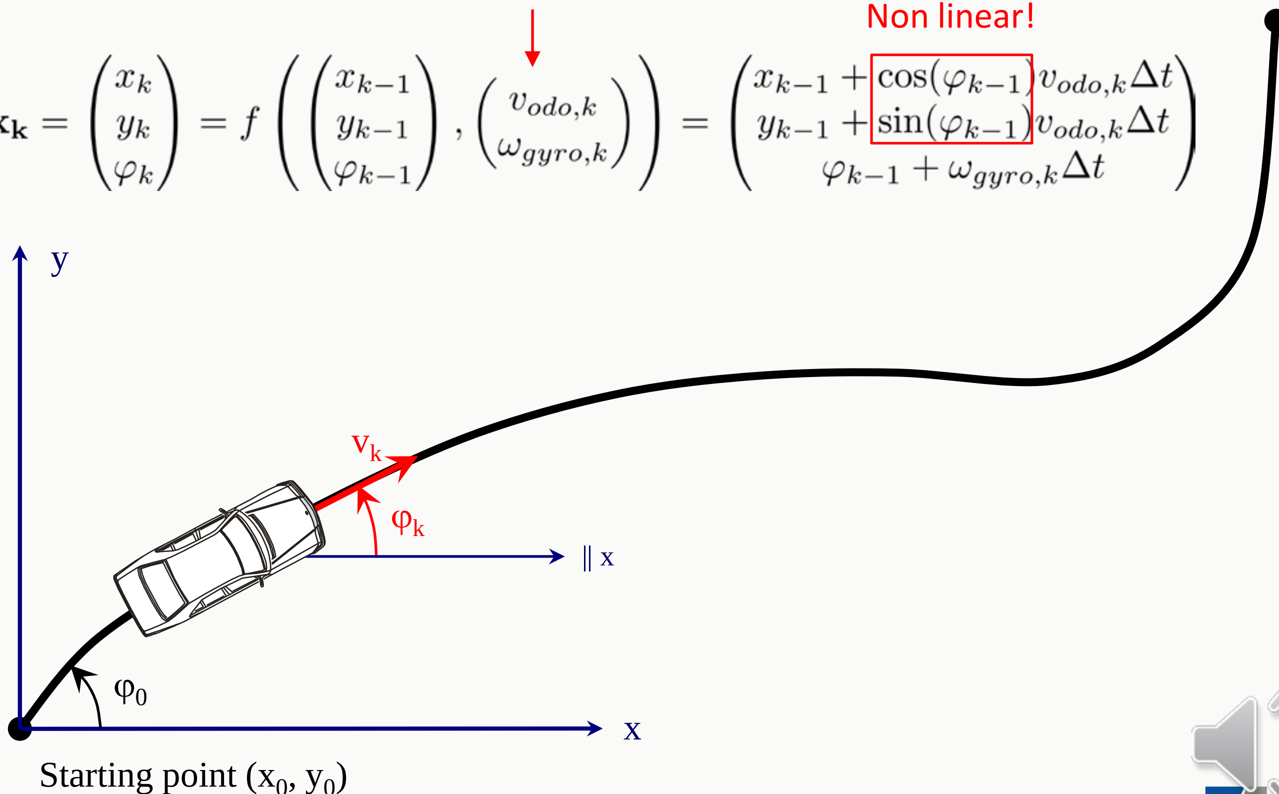
NON LINEAR EXAMPLE (DEAD RECKONING)

SYSTEM MODEL

control input

Non linear!

$$\mathbf{x_k} = \begin{pmatrix} x_k \\ y_k \\ \varphi_k \end{pmatrix} = f\left(\begin{pmatrix} x_{k-1} \\ y_{k-1} \\ \varphi_{k-1} \end{pmatrix}, \begin{pmatrix} v_{odo,k} \\ \omega_{gyro,k} \end{pmatrix} \right) = \begin{pmatrix} x_{k-1} + \boxed{\cos(\varphi_{k-1})} v_{odo,k} \Delta t \\ y_{k-1} + \boxed{\sin(\varphi_{k-1})} v_{odo,k} \Delta t \\ \varphi_{k-1} + \omega_{gyro,k} \Delta t \end{pmatrix}$$

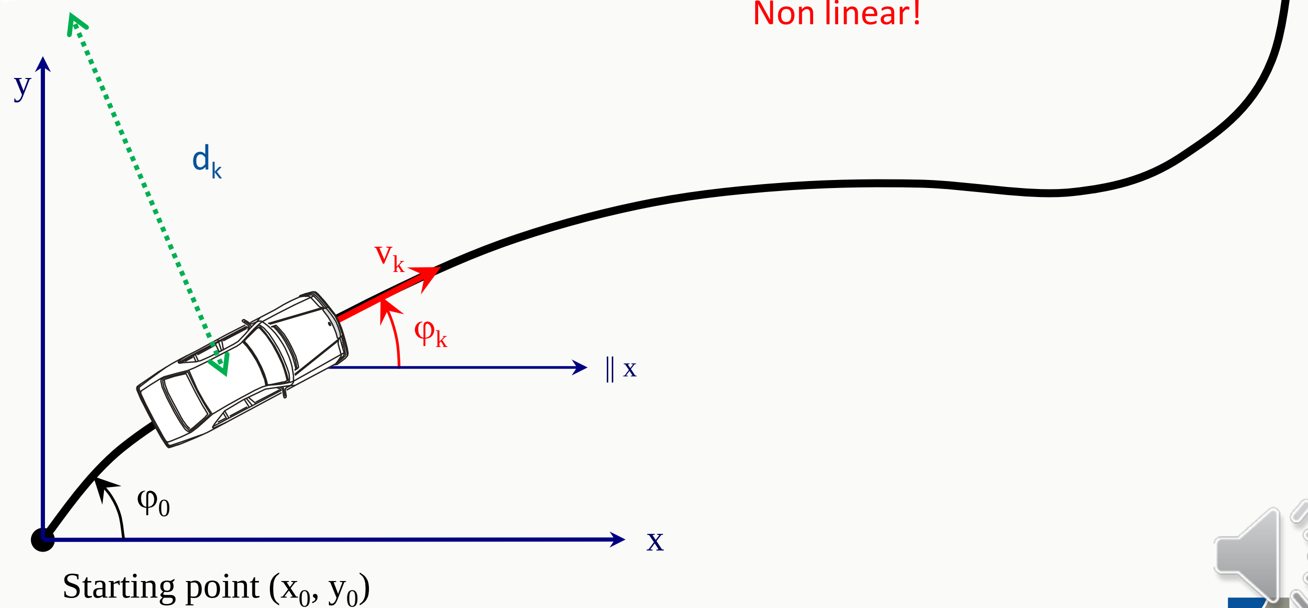


NON LINEAR EXAMPLE (DEAD RECKONING)

MEASUREMENT MODEL (E.G. GPS, DISTANCE TO KNOWN POSITION)

$$z_k = d_k = h(\mathbf{x}_k) = \boxed{\sqrt{(x_k - x_{sat,k})^2 + (y_k - y_{sat,k})^2}}$$

Non linear!



SUMMARY EXTENDED KALMAN FILTER

Prediction

$$
\begin{aligned}
\mathbf{x}_k^- &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) \\
\mathbf{P}_k^- &= \mathbf{\Phi} \mathbf{P}_{k-1} \mathbf{\Phi}^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T
\end{aligned}
$$

Correction

$$
\begin{aligned}
\mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H}\mathbf{P}_k^- \mathbf{H}^T + \mathbf{R})^{-1} \\
\mathbf{x}_k &= \mathbf{x}_k^- + \mathbf{K}_k(\mathbf{z}_k - h(\mathbf{x}_k^-)) \\
\mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_k^-
\end{aligned}
$$

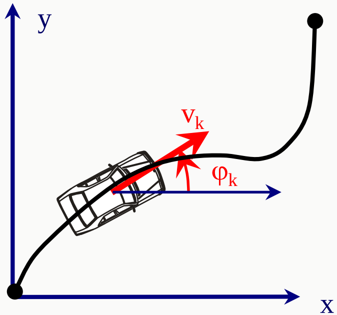
$$
\mathbf{\Phi} = \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \bigg|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}, \mathbf{u}=\mathbf{u}_k}, \quad \mathbf{G} = \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{w}} \bigg|_{\dots}
$$

$$
\mathbf{H}_k = \frac{\partial h(\mathbf{x})}{\partial \mathbf{x}} \bigg|_{\mathbf{x}=\hat{\mathbf{x}}_k^-}
$$

2D KF EXAMPLE

SYSTEM MODEL

- No assumed (and wrong) model on the temporal evolutionof motion states
- Relative measurements (path increment and angle change) are modelled as control input
- System noise is represented as control noise



control input

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \boxed{\mathbf{u}_k}, \boxed{\mathbf{w}_k})$$

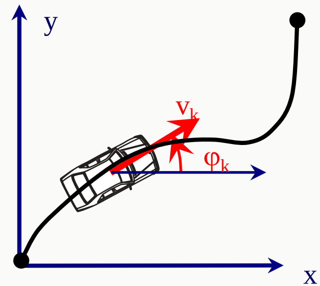
$$\begin{bmatrix} x_k \\ y_k \\ \varphi_k \end{bmatrix} = f\left(\begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \varphi_{k-1} \end{bmatrix}, \begin{bmatrix} \Delta s_k \\ \omega_k \end{bmatrix}, \begin{bmatrix} w_{odo} \\ w_{gyro} \end{bmatrix} \right) = \begin{bmatrix} x_{k-1} + \cos(\varphi_{k-1})(\boxed{\Delta s_k} \boxed{+ w_{odo}}) \\ y_{k-1} + \sin(\varphi_{k-1})(\boxed{\Delta s_k} \boxed{+ w_{odo}}) \\ \varphi_{k-1} + (\boxed{\omega_{gyro,k}} + \boxed{w_{gyro}})\Delta t \end{bmatrix}$$

System/control noise

2D KF EXAMPLE

TRANSITION MATRIX

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \varphi \end{bmatrix}$$



$$\boldsymbol{\Phi}_{k-1} = \frac{\partial}{\partial \mathbf{x}} \begin{bmatrix} x_{k-1} + \cos(\varphi_{k-1})(\Delta s_k + w_{odo}) \\ y_{k-1} + \sin(\varphi_{k-1})(\Delta s_k + w_{odo}) \\ \varphi_{k-1} + (\omega_{gyro,k} + w_{gyro})\Delta t \end{bmatrix}_{\mathbf{x}=\hat{\mathbf{x}}_{k-1},\mathbf{u}=\mathbf{u}_k}$$

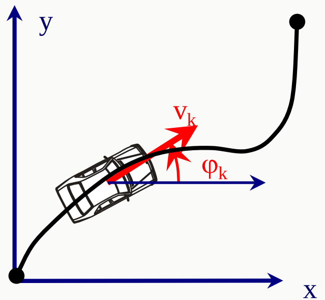
$$= \begin{bmatrix} 1 & 0 & -\sin(\varphi_{k-1})\Delta s_k \\ 0 & 1 & \cos(\varphi_{k-1})\Delta s_k \\ 0 & 0 & 1 \end{bmatrix}_{\mathbf{x}=\hat{\mathbf{x}}_{k-1},\mathbf{u}=\mathbf{u}_k}$$

2D KF EXAMPLE

$$\begin{aligned} \mathbf{x}_k^- &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) \\ \mathbf{P}_k^- &= \mathbf{\Phi} \mathbf{P}_{k-1} \mathbf{\Phi}^T + \boxed{\mathbf{G}\mathbf{Q}\mathbf{G}^T} \end{aligned}$$

SYSTEM NOISE

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \varphi \end{bmatrix}$$



$$\mathbf{Q} = \begin{bmatrix} \sigma_{odo}^2 & 0 \\ 0 & \sigma_{gyro}^2 \end{bmatrix}$$

$$\mathbf{G} = \frac{\partial f(\mathbf{x}, \mathbf{u}, \mathbf{w})}{\partial \mathbf{w}} = \begin{bmatrix} \cos(\varphi_{k-1}) & 0 \\ \sin(\varphi_{k-1}) & 0 \\ 0 & \Delta t \end{bmatrix}_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}, \mathbf{u}=\mathbf{u}_k}$$

$$\mathbf{G}\mathbf{Q}\mathbf{G}^T = \begin{bmatrix} \cos^2(\varphi_{k-1})\sigma_{odo}^2 & \cos(\varphi_{k-1})\sin(\varphi_{k-1})\sigma_{odo}^2 & 0 \\ \cos(\varphi_{k-1})\sin(\varphi_{k-1})\sigma_{odo}^2 & \sin^2(\varphi_{k-1})\sigma_{odo}^2 & 0 \\ 0 & 0 & \sigma_{gyro}^2 \Delta t^2 \end{bmatrix}$$

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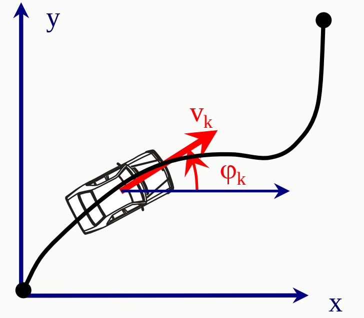
2D KF EXAMPLE

MEASUREMENT MODEL

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \varphi \end{bmatrix}$$



$$\begin{aligned} \mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H}\mathbf{P}_k^- \mathbf{H}^T + \boxed{\mathbf{R}})^{-1} \\ \mathbf{x}_k &= \mathbf{x}_k^- + \mathbf{K}_k(\mathbf{z}_k - \boxed{h(\mathbf{x}_k^-)}) \\ \mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k\mathbf{H})\mathbf{P}_k^- \end{aligned}$$

- Measurement is only GPS Position x_{gps}, y_{gps}

$$\mathbf{z}_k = \begin{bmatrix} x_{gps,k} \\ y_{gps,k} \end{bmatrix} = h(\mathbf{x}_k) = \mathbf{H}\mathbf{x}_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}_k$$

- Measurement Noise

$$\mathbf{R} = \begin{bmatrix} \sigma^2_{gps,x} & 0 \\ 0 & \sigma^2_{gps,y} \end{bmatrix}$$

GPS/IMU INTEGRATION

- Estimate the position and orientation of the system in all dimensions
- Use inertial sensors (acc + gyro) as control input: Strapdown inertial integration as system model
- Because acc + gyro have time dependant systematic errors (bias), estimate the biases as part of the state vector
- Use GPS as measurements
  - Use of positions: ➔ loosely coupled integration
  - Use of GPS raw data (double differences, carrier phases) ➔ tightly-coupled integration

GPS/IMU INTEGRATION

- State Vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{q}_b^n \\ \mathbf{x}_{eb}^e \\ \mathbf{v}_{eb}^n \\ \mathbf{b}_{gyro} \\ \mathbf{b}_{acc} \end{bmatrix}$$

Orientation quaternion

IMU bias values

- Control input

$$\mathbf{u} = \begin{bmatrix} \mathbf{f}_{ib}^b \\ \boldsymbol{\omega}_{ib}^b \end{bmatrix}$$

IMU sensor readings

- Measurement $\mathbf{z} = \mathbf{x}_{gps}$

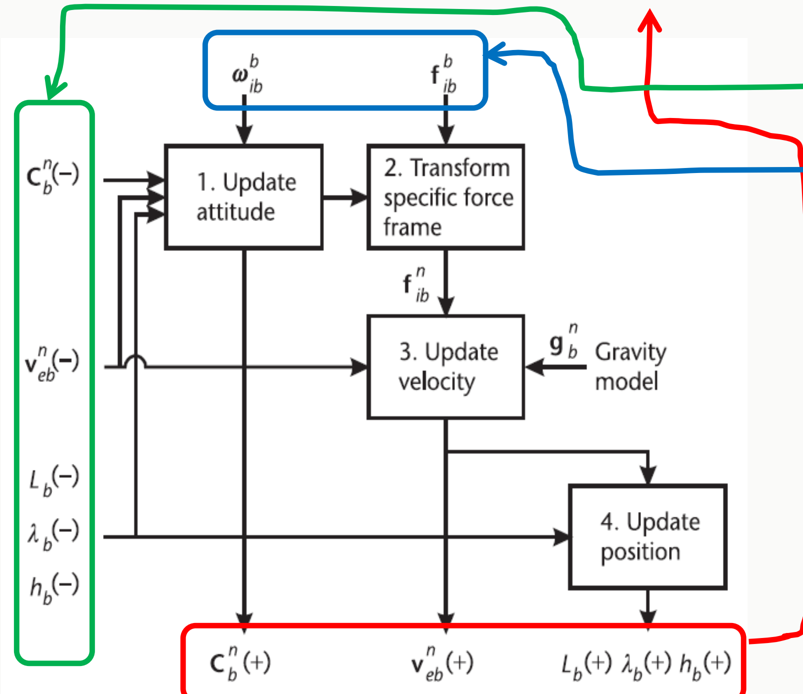
Processed GNSS position

GPS/IMU INTEGRATION

SYSTEM MODEL
STRAPDOWN ALGORITHM

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{w}_k)$$

$$\begin{bmatrix} \mathbf{q}_b^n \\ \mathbf{p}_{eb}^e \\ \mathbf{v}_{eb}^n \\ \mathbf{b}_{gyro} \\ \mathbf{b}_{acc} \end{bmatrix}_k = f\left(\begin{bmatrix} \mathbf{q}_b^n \\ \mathbf{p}_{eb}^e \\ \mathbf{v}_{eb}^n \\ \mathbf{b}_{gyro} \\ \mathbf{b}_{acc} \end{bmatrix}_k, \begin{bmatrix} \mathbf{a}_{ib}^b \\ \boldsymbol{\omega}_{ib}^b \end{bmatrix}_k, \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_a \end{bmatrix}_k \right)$$



System noise is IMU noise

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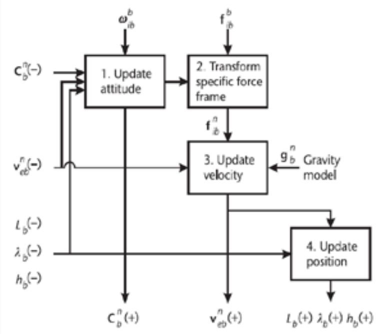
GPS/IMU INTEGRATION

MEASUREMENT MODEL

$$\mathbf{x} = \begin{bmatrix} \mathbf{q}_b^n \\ \mathbf{x}_{eb}^e \\ \mathbf{v}_{eb}^n \\ \mathbf{b}_{gyro} \\ \mathbf{b}_{acc} \end{bmatrix}$$



$$\begin{aligned} \mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H}\mathbf{P}_k^- \mathbf{H}^T + \boxed{\mathbf{R}})^{-1} \\ \mathbf{x}_k &= \mathbf{x}_k^- + \mathbf{K}_k(\mathbf{z}_k - \boxed{h(\mathbf{x}_k^-)}) \\ \mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H})\mathbf{P}_k^- \end{aligned}$$

- Measurement is only GPS Position $x_{gps}, y_{gps}, z_{gps}$

$$\mathbf{z}_k = \begin{bmatrix} x_{gps,k} \\ y_{gps,k} \\ z_{gps,k} \end{bmatrix} = h(\mathbf{x}_k) = \mathbf{H}\mathbf{x}_k = \begin{bmatrix} \dots & 1 & 0 & 0 & \dots \\ \dots & 0 & 1 & 0 & \dots \\ \dots & 0 & 0 & 1 & \dots \end{bmatrix} \mathbf{x}_k$$

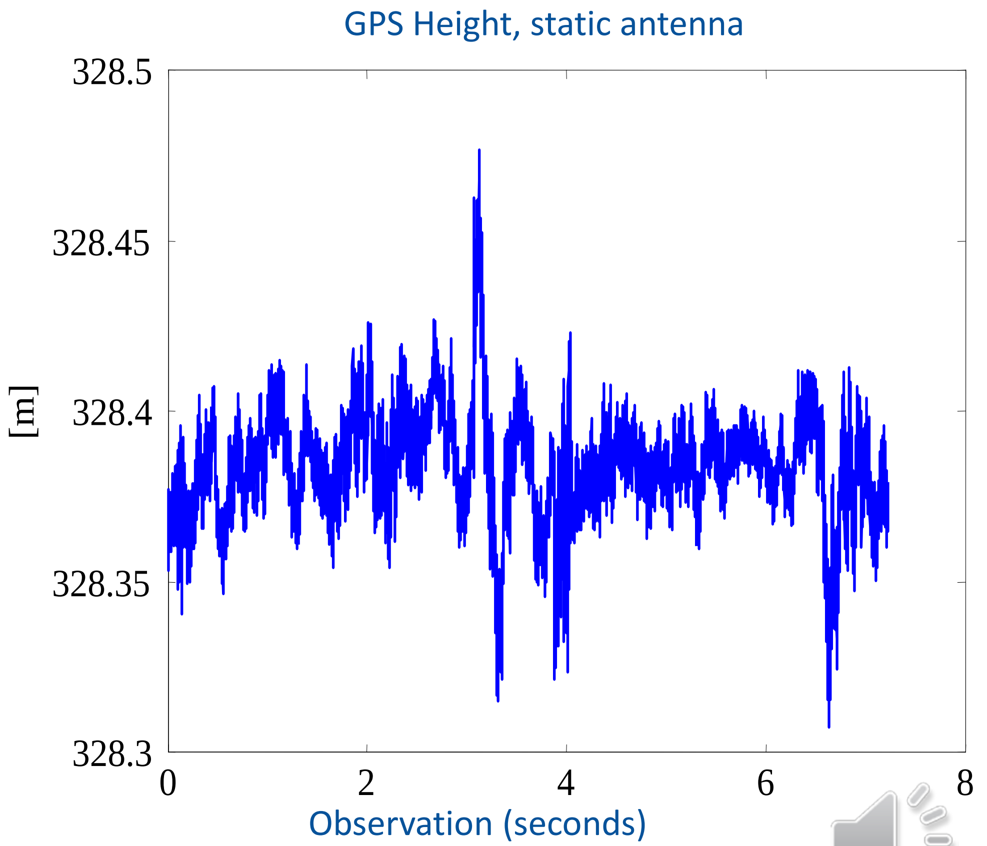
- Measurement Noise

$$\mathbf{R} = \begin{bmatrix} \sigma_{gps,x}^2 & 0 & 0 \\ 0 & \sigma_{gps,y}^2 & 0 \\ 0 & 0 & \sigma_{gps,z}^2 \end{bmatrix}$$

GPS/IMU INTEGRATION

GPS NOISE

Carrierphase
2-Frequency
Differential
Kinematic
Post-Processing

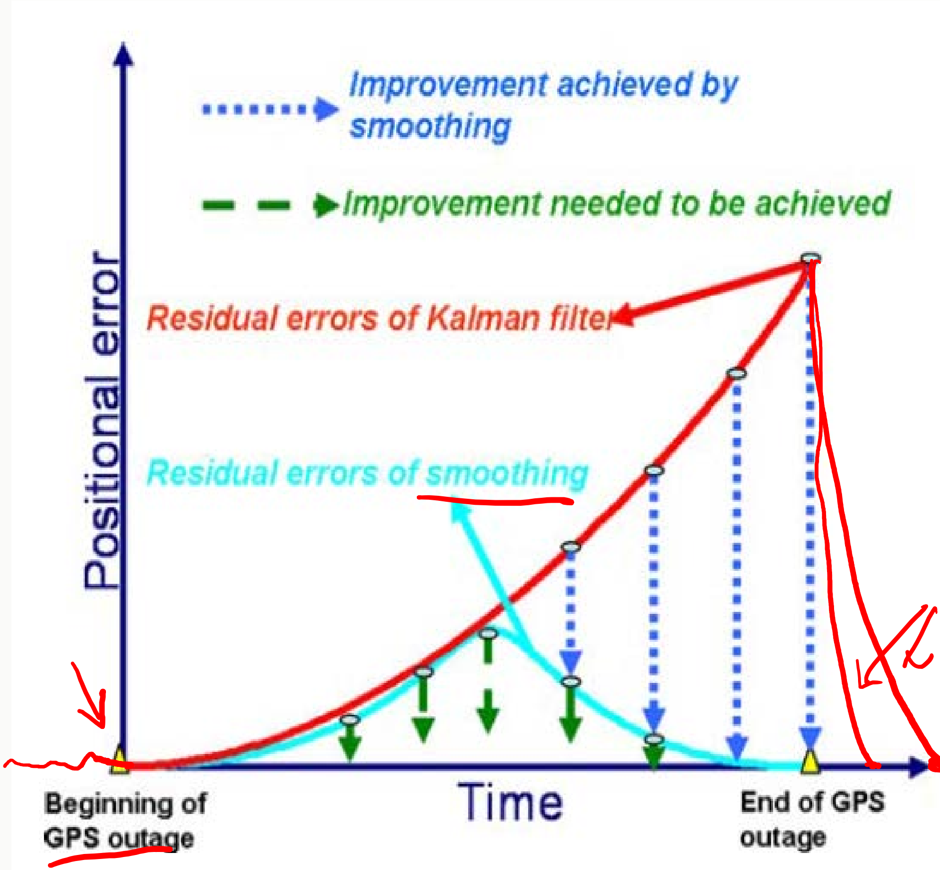


Remember

GPS noise is not white!

$$\mathbf{R} = \begin{bmatrix} \sigma^2_{gps,x} & 0 & 0 \\ 0 & \sigma^2_{gps,y} & 0 \\ 0 & 0 & \sigma^2_{gps,z} \end{bmatrix}$$

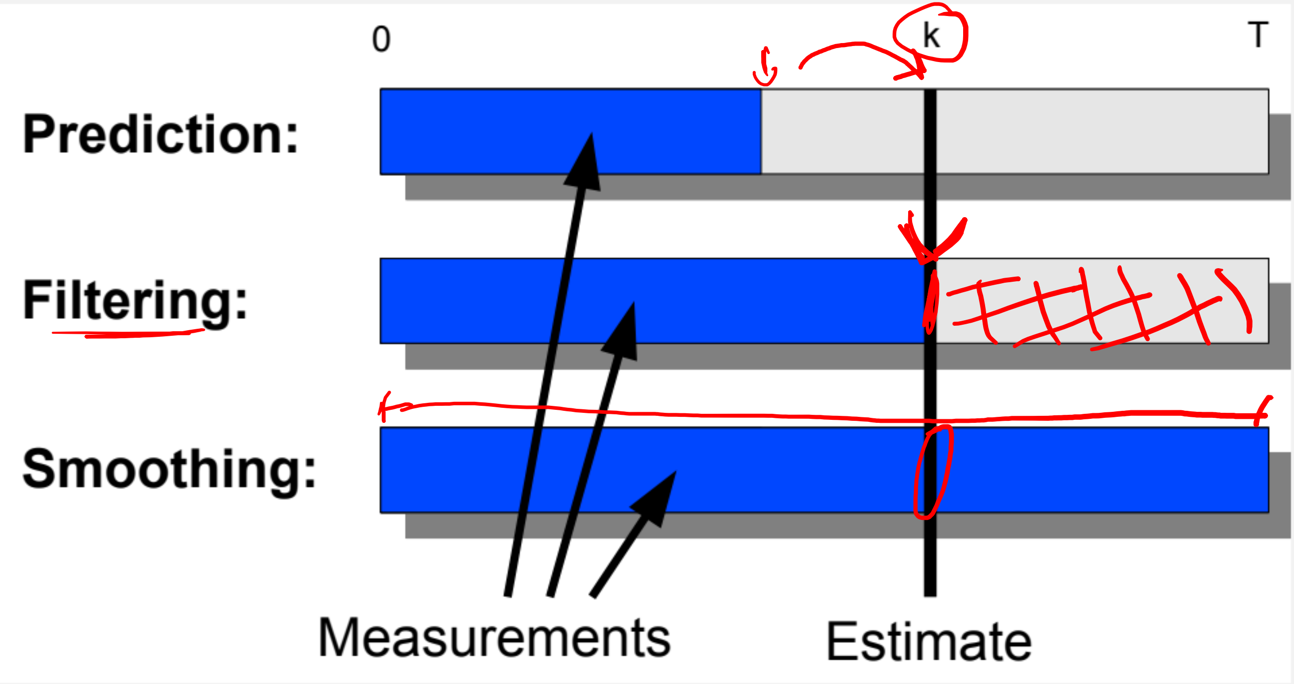
wrong



~~moiset~~
touin

From: Chiang, K., Lin, Y., Huang, Y. et al. An ANN–RTS smoother scheme for accurate INS/GPS integrated attitude determination. GPS Solut 13, 199–208 (2009). https://doi.org/10.1007/s10291-008-0113-0

FILTERING/SMOOTHING



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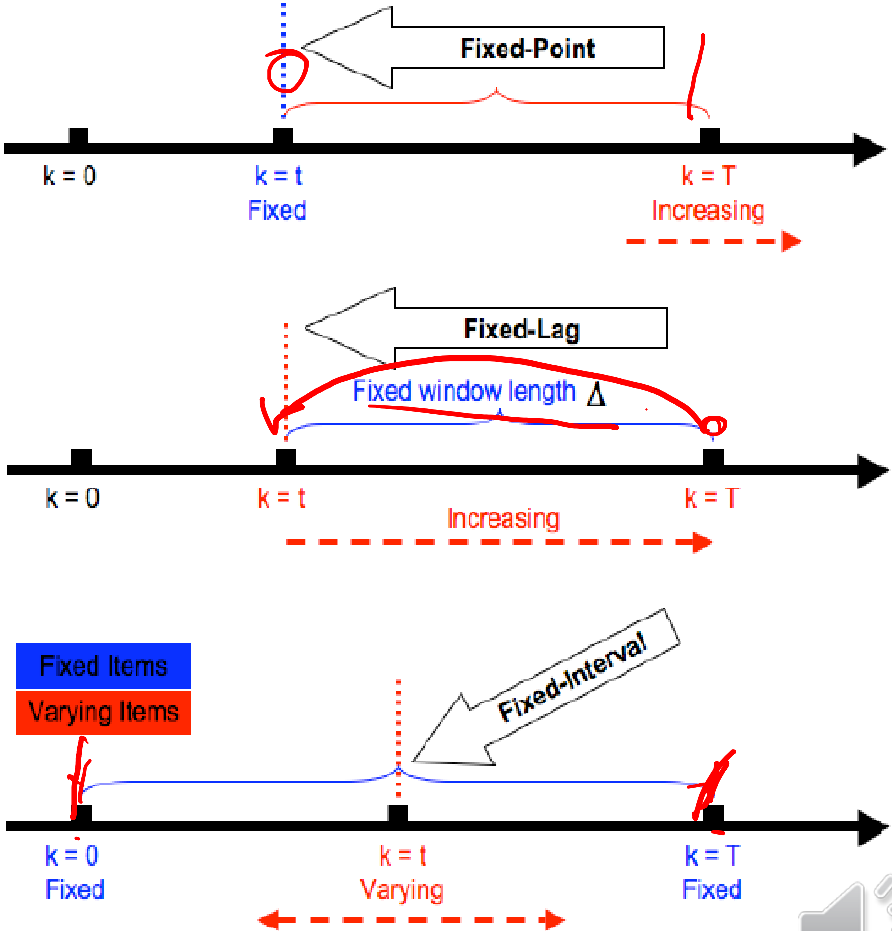
SMOOTHING



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KALMAN FILTER

Predicted covariance

Predicted estimate

System noise

$$\mathbf{x}_k^- = \Phi \mathbf{x}_{k-1}$$

$$P_k^- = \Phi P_{k-1} \Phi^T + \mathbf{Q}$$

Prediction

measurement noise

Kalman Gain

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

$$\mathbf{x}_k = \mathbf{x}_k^- + K_k (z_k - H \mathbf{x}_k^-)$$

$$P_k = (I - K_k H) P_k^-$$

Correction

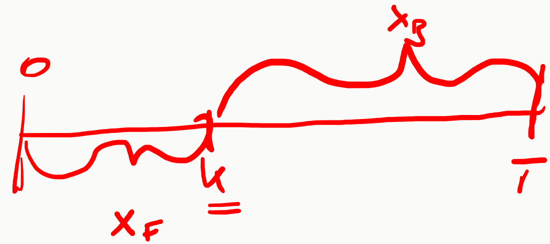
measurement

Predicted measurement

Updated estimate

Updated covariance

FORWARD BACKWARD FILTER



$$\hat{x}_k = K_{F_k}\hat{x}_{F_k} + K_{B_k}\hat{x}^-_{B_k}$$

$$= K_F x_F + (1-K_F)x_B^-$$

$$P(x) = E\left[(x-\hat{x})(x-\hat{x})^T\right]$$

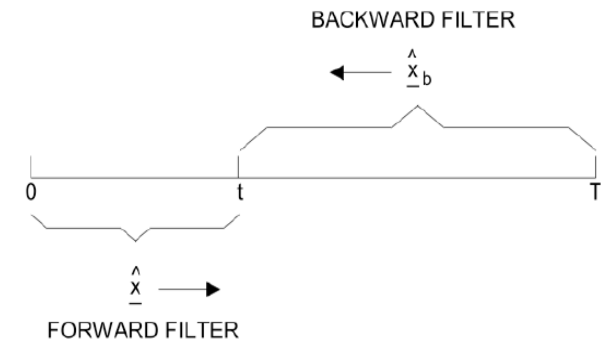
minimize P

$$\Rightarrow \quad K_F = \frac{P_B^-}{P_F + P_B^-} \qquad K_B = \frac{P_F}{P_F + P_B^-}$$

$$P = \left(P_F^{-1} + P_B^{-1}\right)^{-1}$$

weighted mean

FORWARD BACKWARD FILTER



$$
\begin{aligned}
\mathbf{x}_{S,k} &= K_{F,k}\mathbf{x}_{F,k} + K_{B,k}\mathbf{x}^{-}_{B,k} \\
&= K_{F,k}\mathbf{x}_{F,k} + (1 - K_{F,k})\mathbf{x}^{-}_{B,k} \\
K_{F,k} &= P^{-}_{R,k}(P_{F,k} + P^{-}_{B,k})^{-1} \\
P_{S,k} &= \left[(P_{F,k})^{-1} + (P^{-}_{B,k})^{-1}\right]^{-1}
\end{aligned}
$$

Details on the derivation in:
Dan Simon: Optimal State Estimation, Wiley Interscience

RAUCH-TUNG-STRIEBEL (RTS) SMOOTHER

1) $\hat{x}_F$, $\hat{P}_F$ $\quad 0 \longrightarrow T$

2) $T \longrightarrow 0$

$$\hat{x}_k = \hat{x}_{F_k} + A\left(\hat{x}_{k+1} - \hat{x}_{F_{k+1}}\right)$$

$$A = \frac{\hat{P}_{F_k} \cdot \Phi_k^T}{P^-_{F_{k+1}}}$$

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RAUCH-TUNG-STRIEBEL (RTS) SMOOTHER

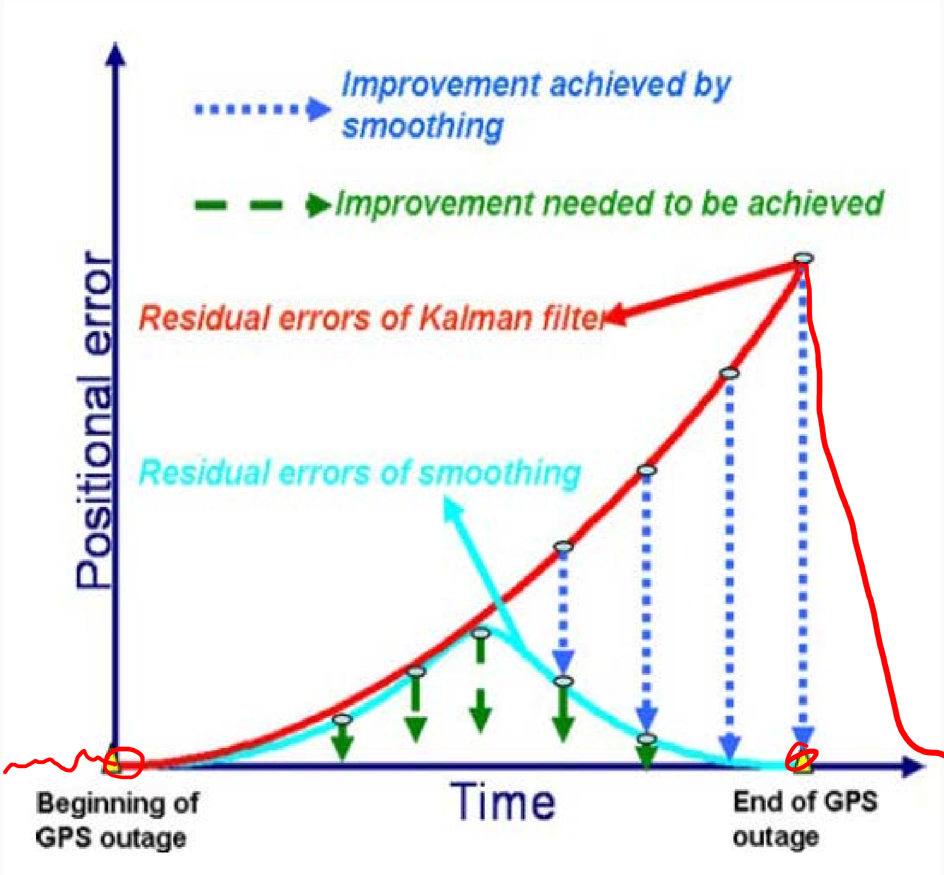
Runs from k to 1

$$\mathbf{x}_{S,k} = \mathbf{x}_{F,k} + A_k(\mathbf{x}_{S,k+1} - \mathbf{x}^-_{F,k+1})$$

$$A_k = P_{F,k}\Phi_k^T \left[P^-_{F,k+1}\right]^{-1}$$

$$P_{S,k} = P_{F,k} + A_k(P_{S,k+1} - P^-_{F,k+1})$$

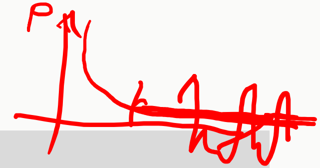
Details on the derivation in:
Dan Simon: Optimal State Estimation, Wiley Interscience



From: Chiang, K., Lin, Y., Huang, Y. et al. An ANN–RTS smoother scheme for accurate INS/GPS integrated attitude determination. GPS Solut 13, 199–208 (2009). https://doi.org/10.1007/s10291-008-0113-0

HOW TO EVALUATE THE QUALITY OF THE ESTIMATION?

A) LOOK AT THE COVARIANCE OF THE ESTIMATE



Prediction

$$\mathbf{x}_k^- = \mathbf{\Phi}\mathbf{x}_{k-1}$$

$$P_k^- = \mathbf{\Phi} P_{k-1} \mathbf{\Phi}^T + \mathbf{Q}$$

Correction

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

$$\mathbf{x}_k = \mathbf{x}_k^- + K_k (z_k - H\mathbf{x}_k^-)$$

$$P_k = (I - K_k H) P_k^-$$

- Does not depend on measurements
- Only ‚valid‘, if all models and uncertaincies are ‚correct‘
- No systematic errors included

HOW TO EVALUATE THE QUALITY OF THE ESTIMATION?

B) LOOK AT THE INNOVATION

Prediction

$$\mathbf{x}_k^- = \mathbf{\Phi}\mathbf{x}_{k-1}$$
$$P_k^- = \mathbf{\Phi} P_{k-1} \mathbf{\Phi}^T + \mathbf{Q}$$

Correction

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$
$$\mathbf{x}_k = \mathbf{x}_k^- + K_k(\underbrace{z_k - H\mathbf{x}_k^-}_{\text{Innovation}})$$
$$P_k = (I - K_k H)P_k^-$$

- Gives an idea about the consistency between models and observations
- It is not clear if iconsistencies come from wrong measurements or predictions

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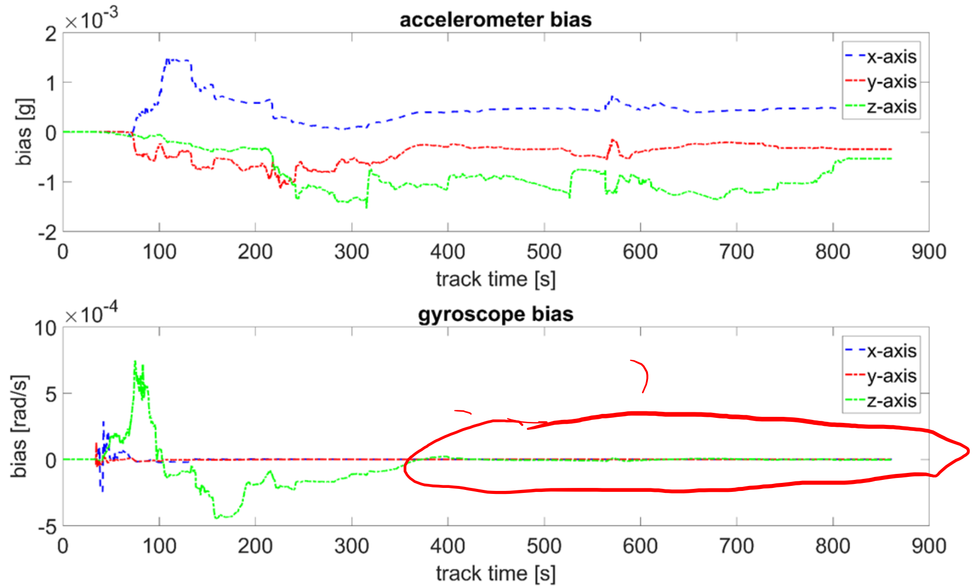
HOW TO EVALUATE THE QUALITY OF THE ESTIMATION?

C) CAREFULLY LOOK AT ALL ESTIMATED STATES



- A nice looking position result can also appear if your process model is completely wrong and your measurements are good
- E.g. Estimated bias values show ‚consistency‘ of models

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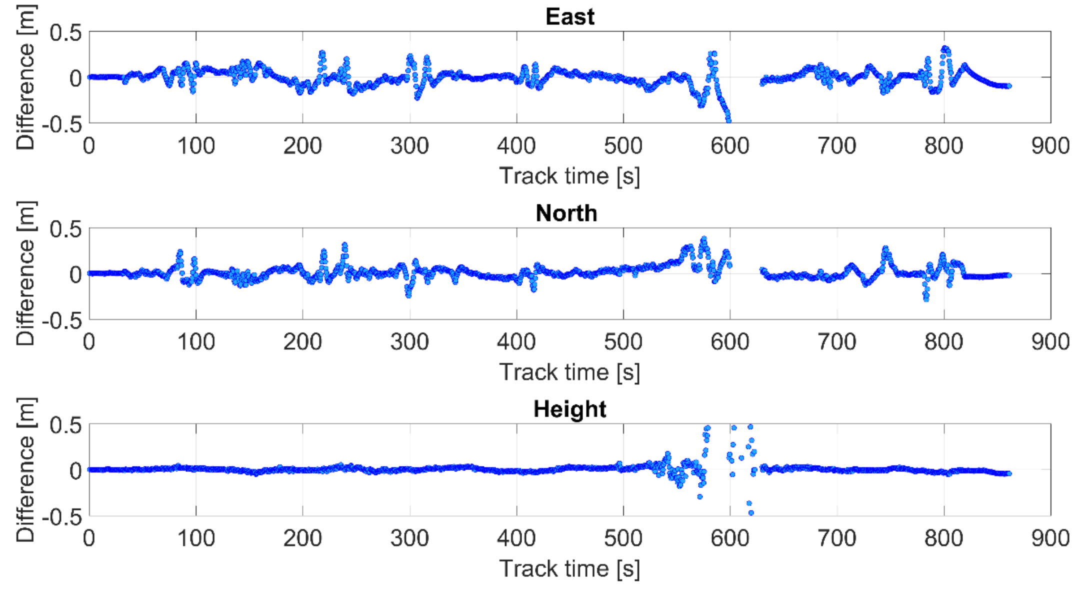
HOW TO EVALUATE THE QUALITY OF THE ESTIMATION?

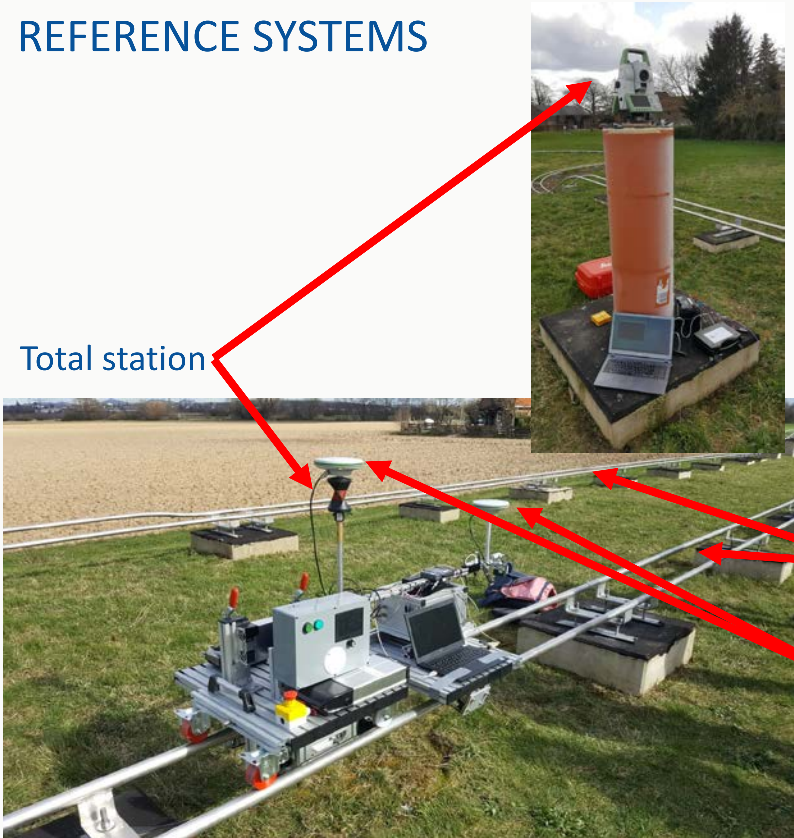
D) COMPARE WITH REFERENCE SOLUTION



- Needs synchronization between reference and estimation results
- Difficult to realize in outdoor scenarios (especially for orientation)
- How to ‚parametrize' the error? Depends on motion, environment, time …

HOW TO EVALUATE THE QUALITY OF THE ESTIMATION?

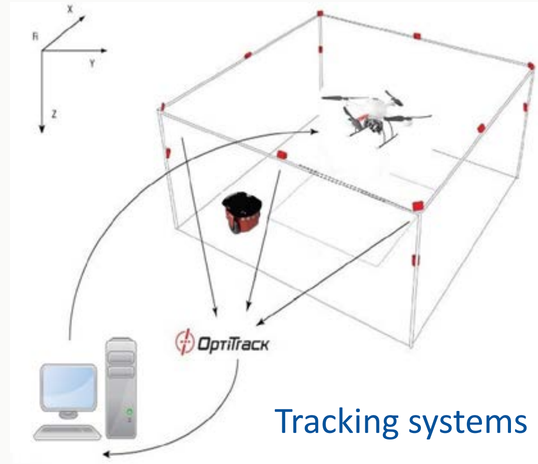
REFERENCE SYSTEMS



Total station

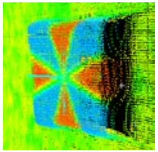
Reference trajectory

RTK GNSS



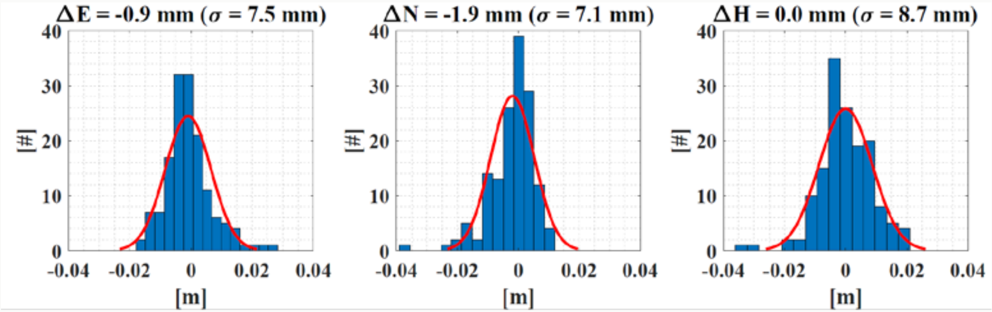
HOW TO EVALUATE THE QUALITY OF THE ESTIMATION?

E) INDIRECT EVALUATION



$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \mathbf{R}_n^e(L,B)\,\mathbf{R}_b^n(\phi,\theta,\psi)\cdot\left[\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \mathbf{R}_s^b(\alpha,\beta,\gamma)\cdot\begin{bmatrix} 0 \\ d\cdot\sin b \\ d\cdot\cos b \end{bmatrix}\right]$$

Compare with known reference



- Includes many other errors
- Assignement of error to specific sources is difficult

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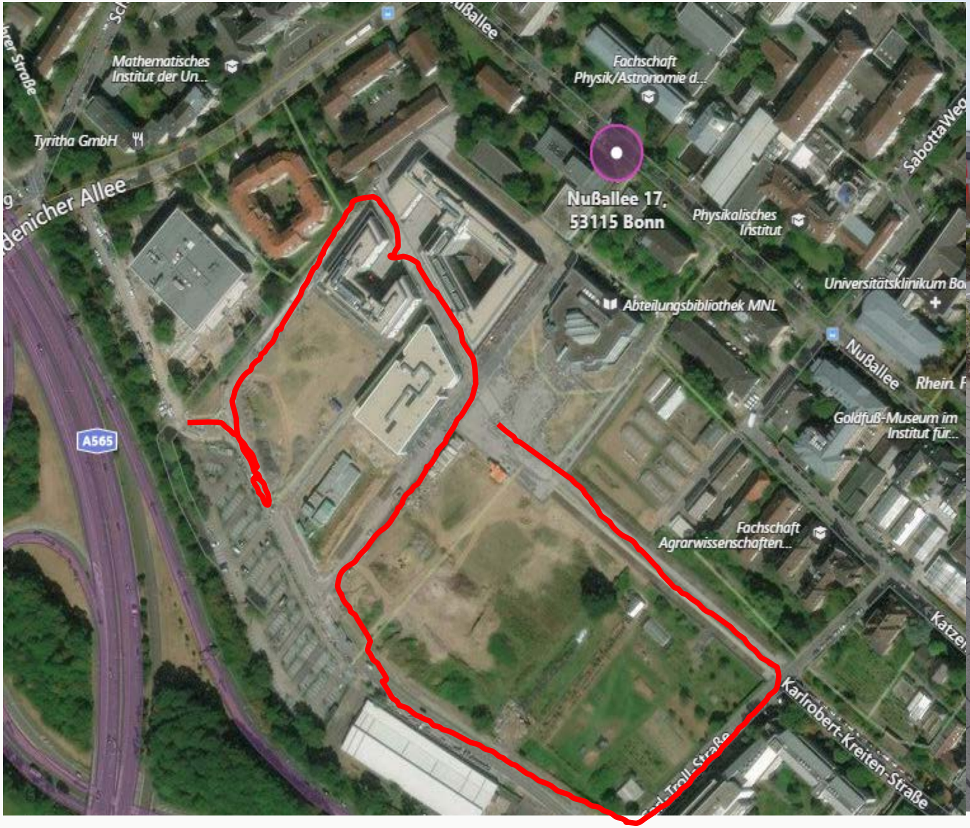
EXAMPLE



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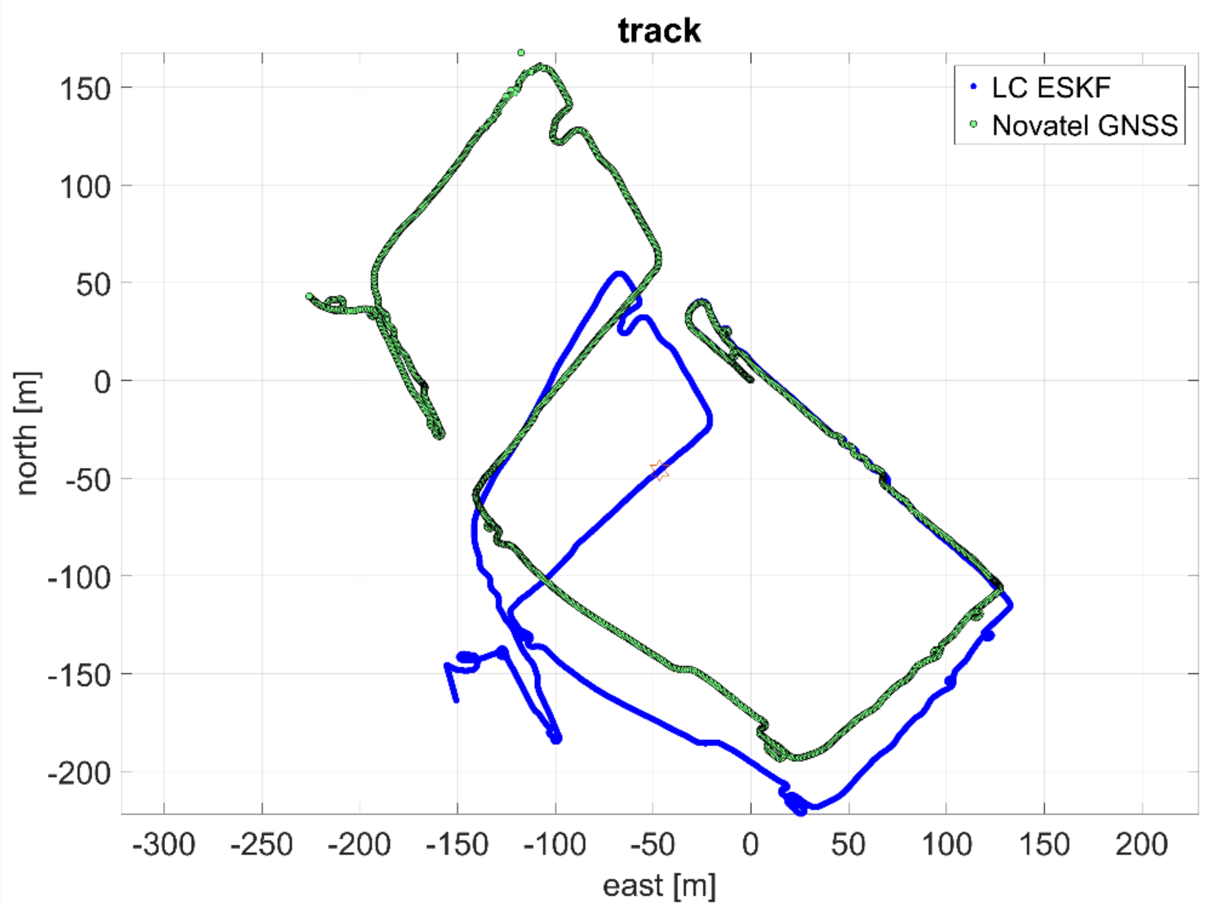
STRAPDOWN ONLY



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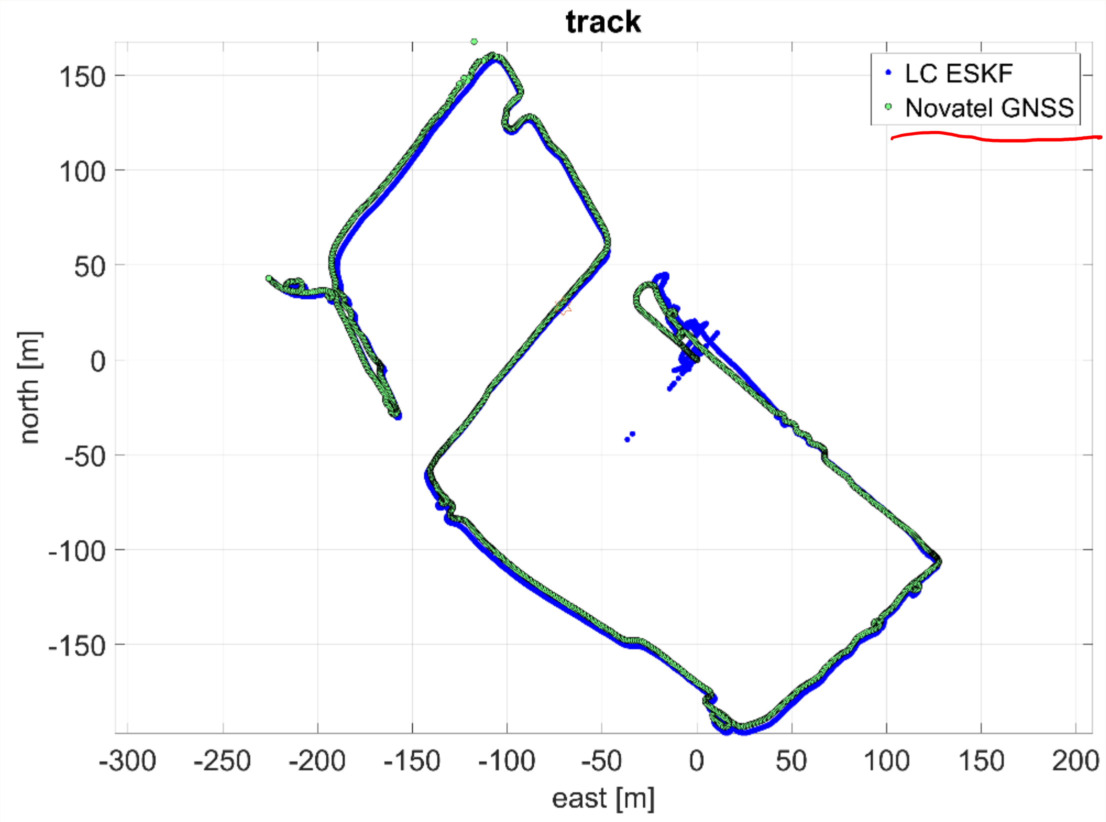
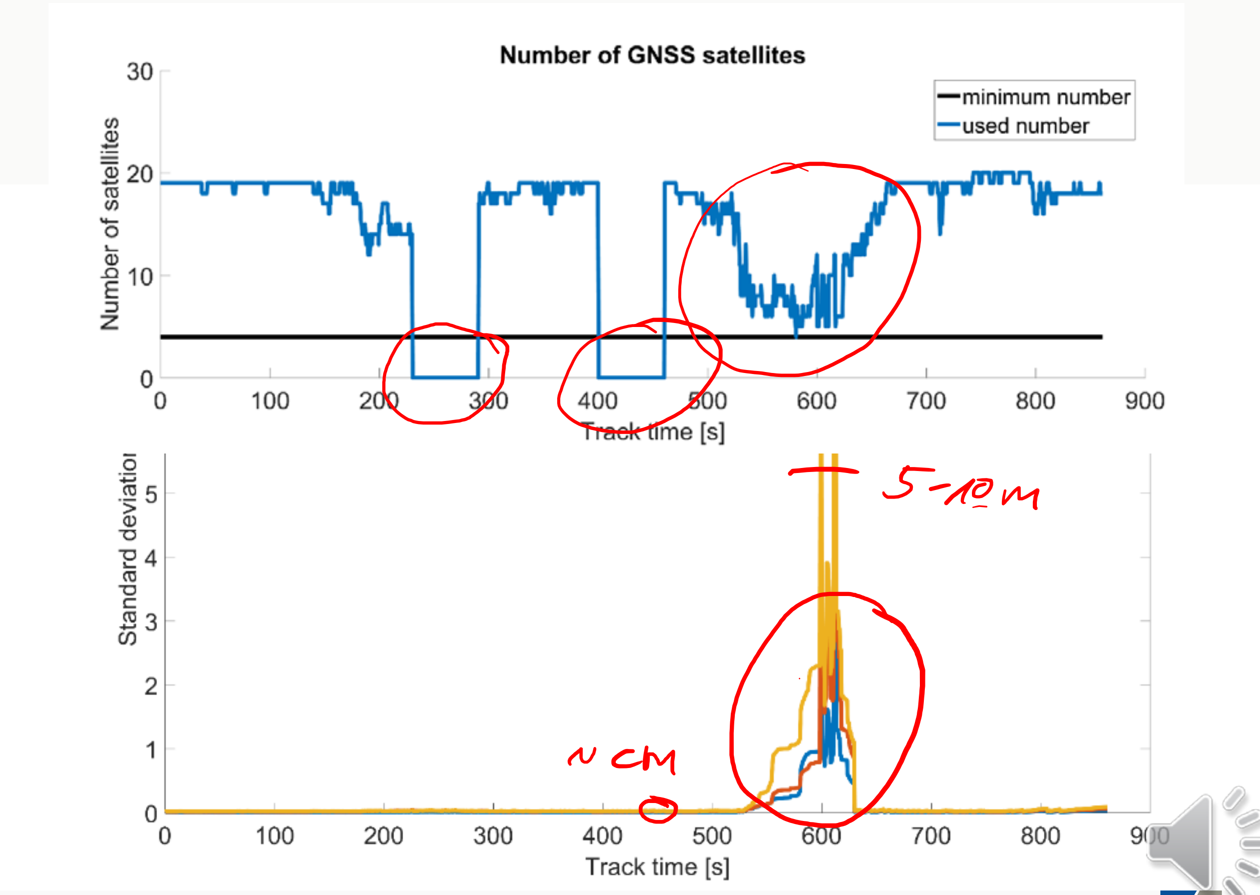
KF WITH ODOMETER. NO GPS!



GNSS QUALITY



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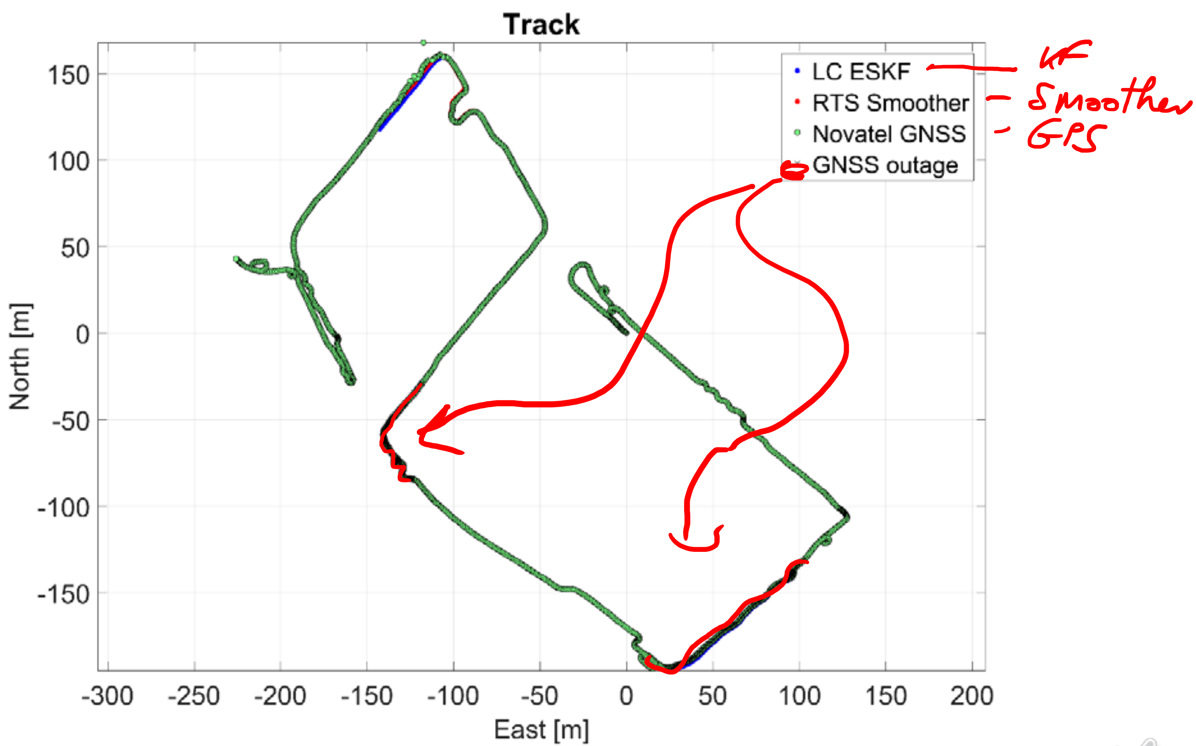
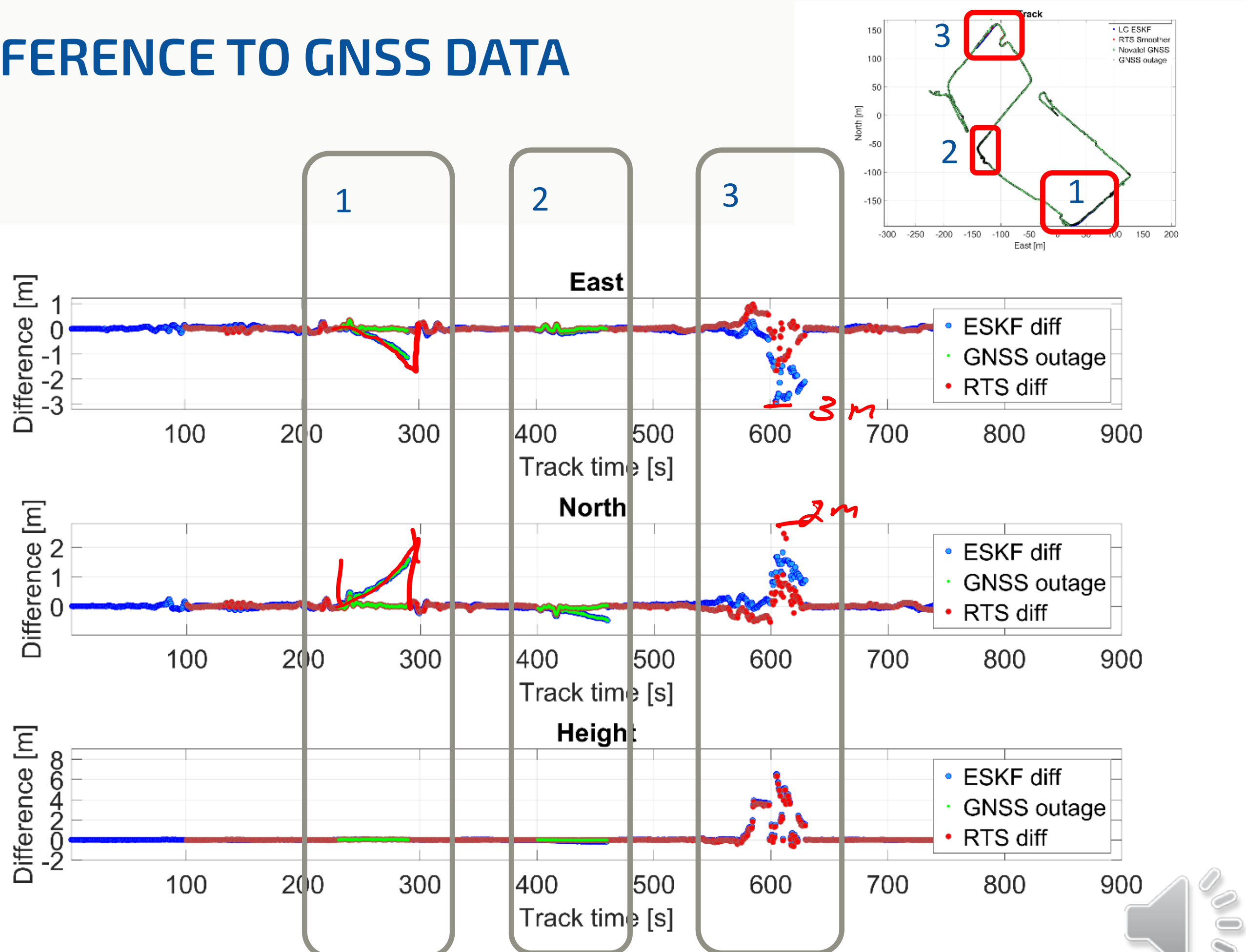
TRACK

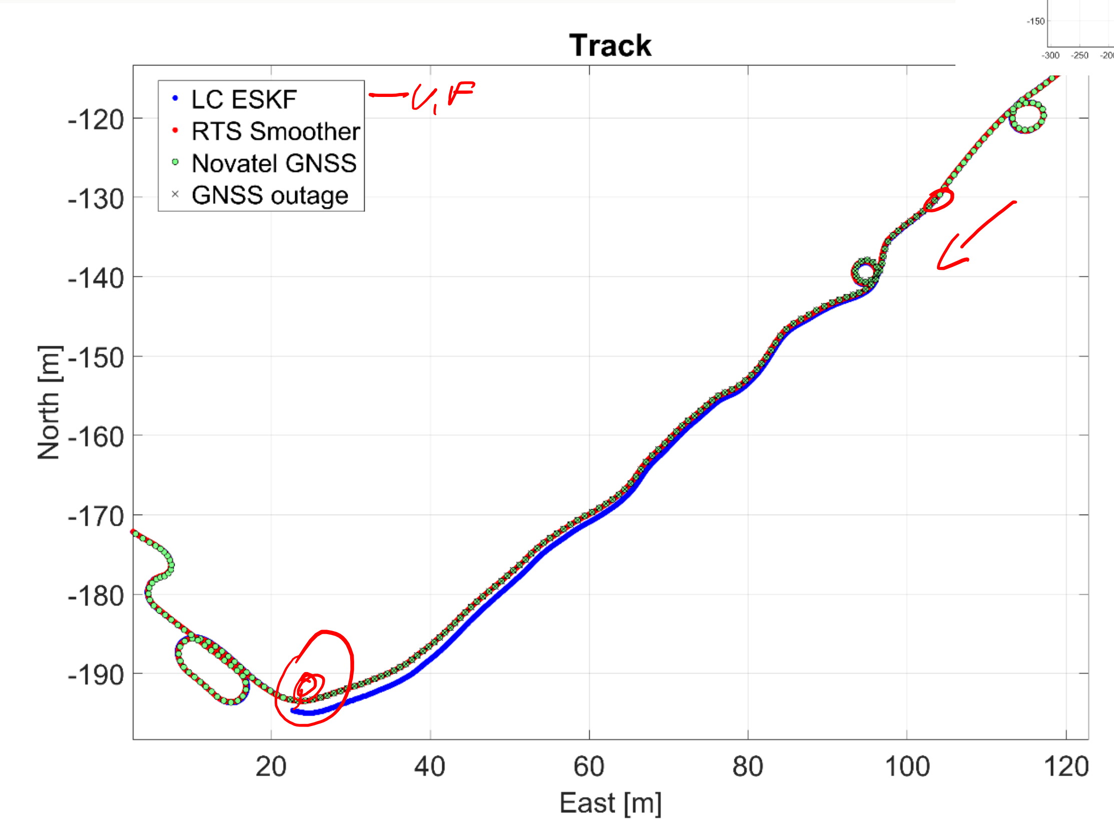


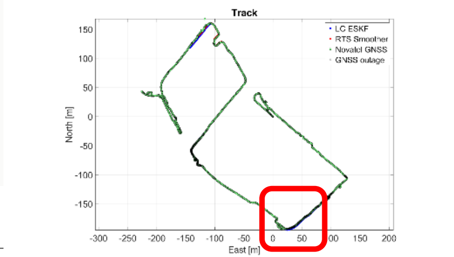
Track comparison: RTS smoother without and with GNSS outages

DIFFERENCE TO GNSS DATA



ZOOM 1





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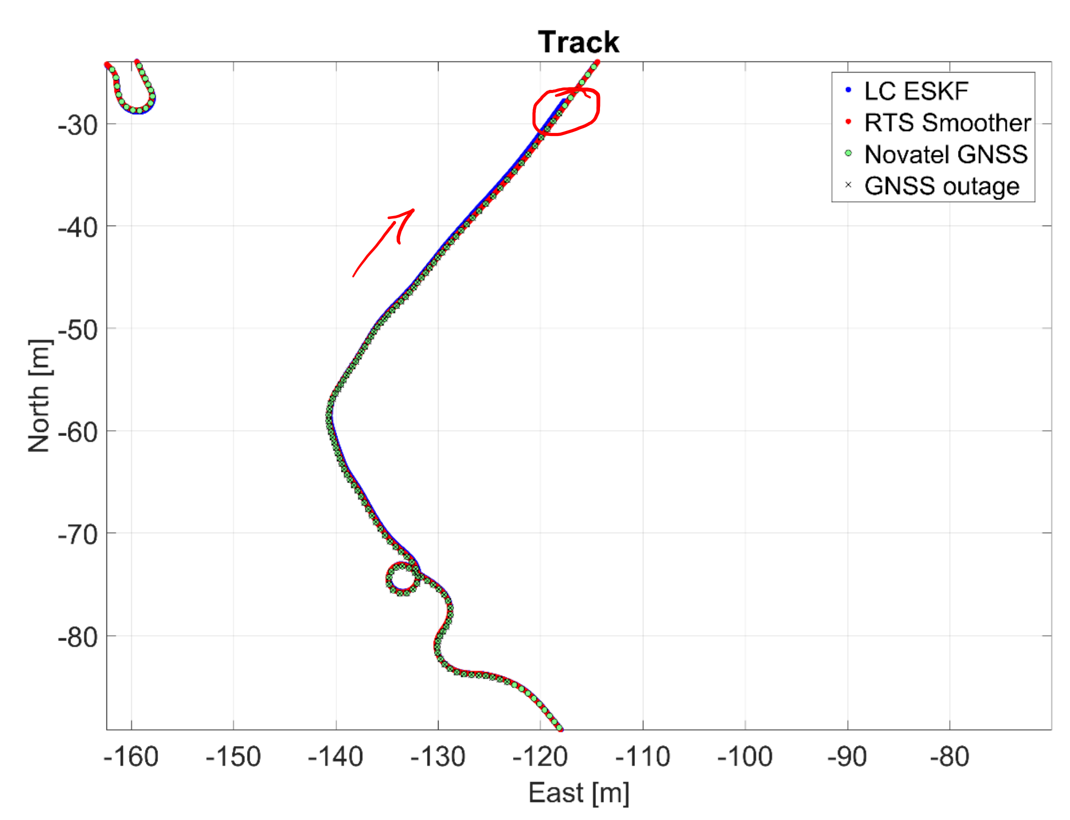
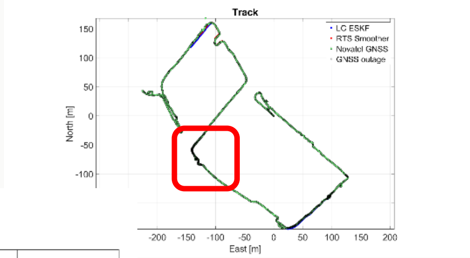
ZOOM 2

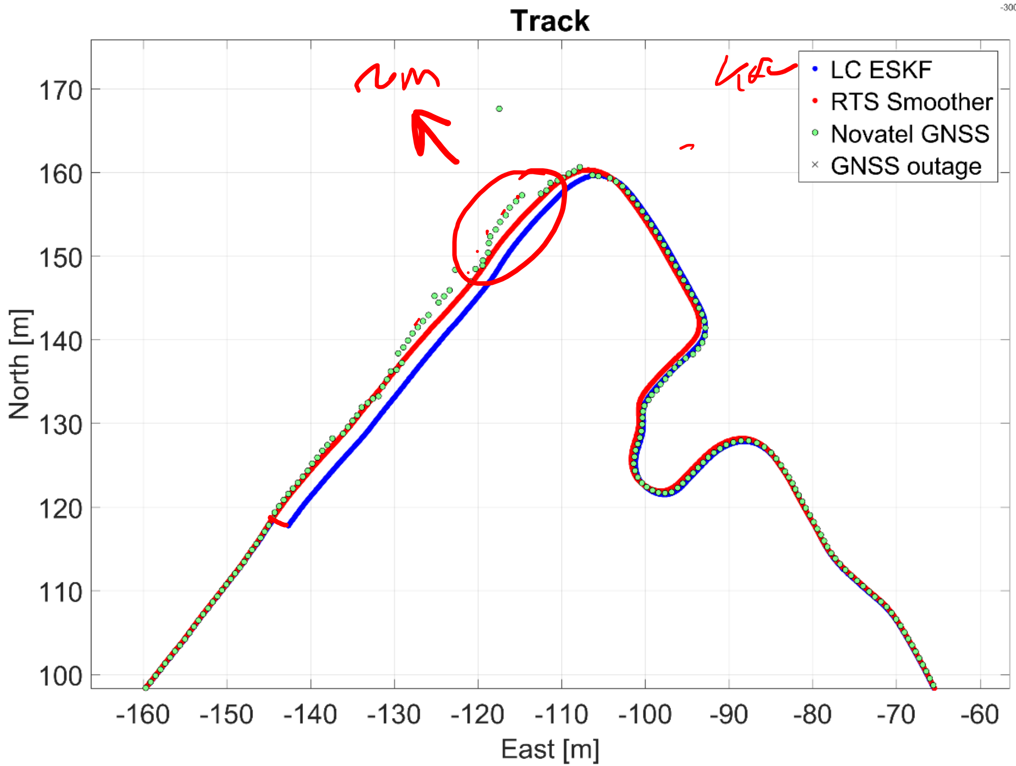


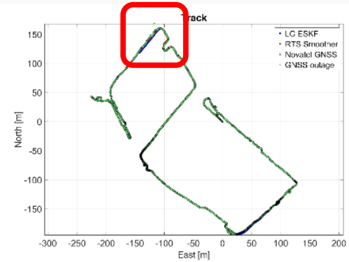
Track
LC ESKF
RTS Smoother
Novatel GNSS
GNSS outage
North [m]
East [m]
-30
-40
-50
-60
-70
-80
-160
-150
-140
-130
-120
-110
-100
-90
-80



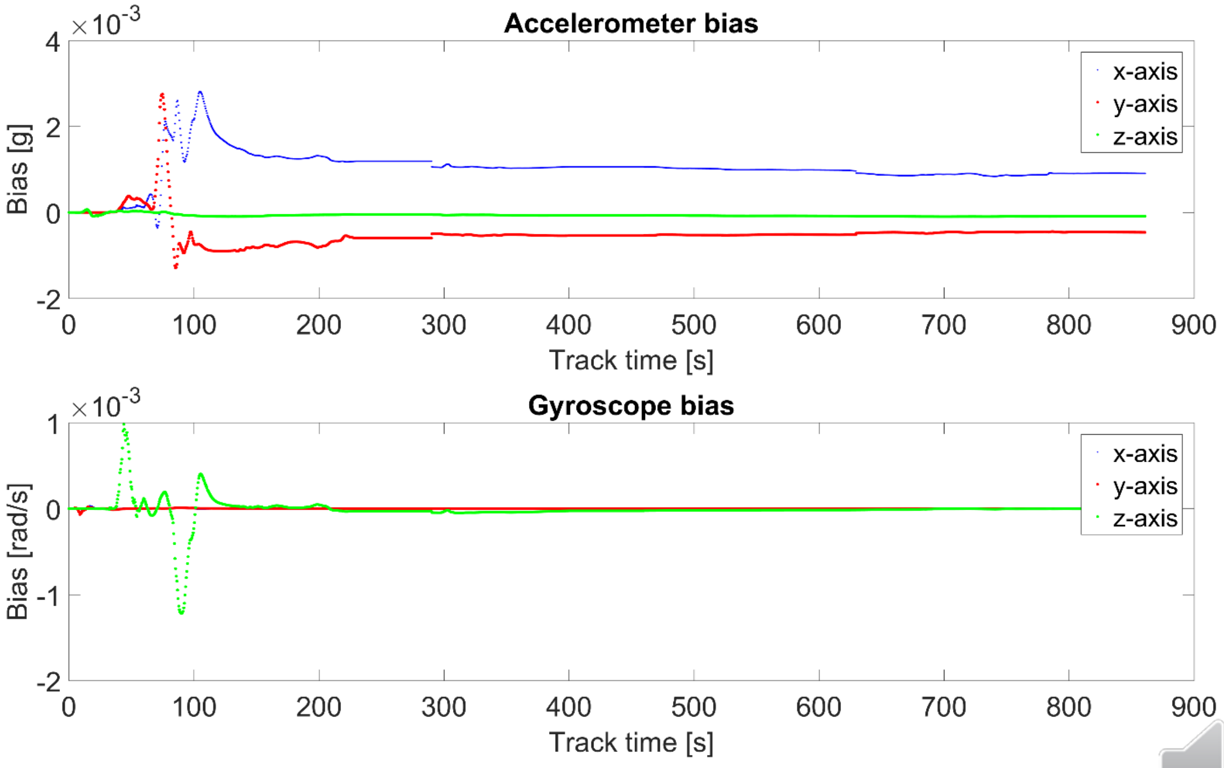
Track
LC ESKF
RTS Smoother
Novatel GNSS
GNSS outage
North [m]
East [m]

ZOOM 3


Track
LC ESKF
RTS Smoother
Novatel GNSS
GNSS outage
North [m]
East [m]


Track
LC ESKF
RTS Smoother
Novatel GNSS
GNSS outage
North [m]
East [m]

BIAS ESTIMATION



WHAT YOU HAVE LEARNED TODAY

- How are position and ortientation calculated using inertial sensors and GNSS?
- How are inertial sensors and GNSS observations combined using filtering algorithms?
- How are inertial sensor errors influencing the estimation?
- What is the difference between filtering and smoothing and when/why is smoothing better?
- What is the RTS smoother?
- How can you evaluate the quality of a trajectory estimation algorithm/system?

THANKS