Uncertainty of Point Clouds
Part 1/2

Advanced Techniques for
Mobile Sensing and Robotics
(Geodesy Track)

Dr.-Ing. Christoph Holst

SS 2020

Agenda

6 Uncertainty of point clouds

Learning objectives

- Which error sources make a point cloud uncertain?
- How do these errors contribute to the uncertainty?
- How can they be minimized?

Agenda

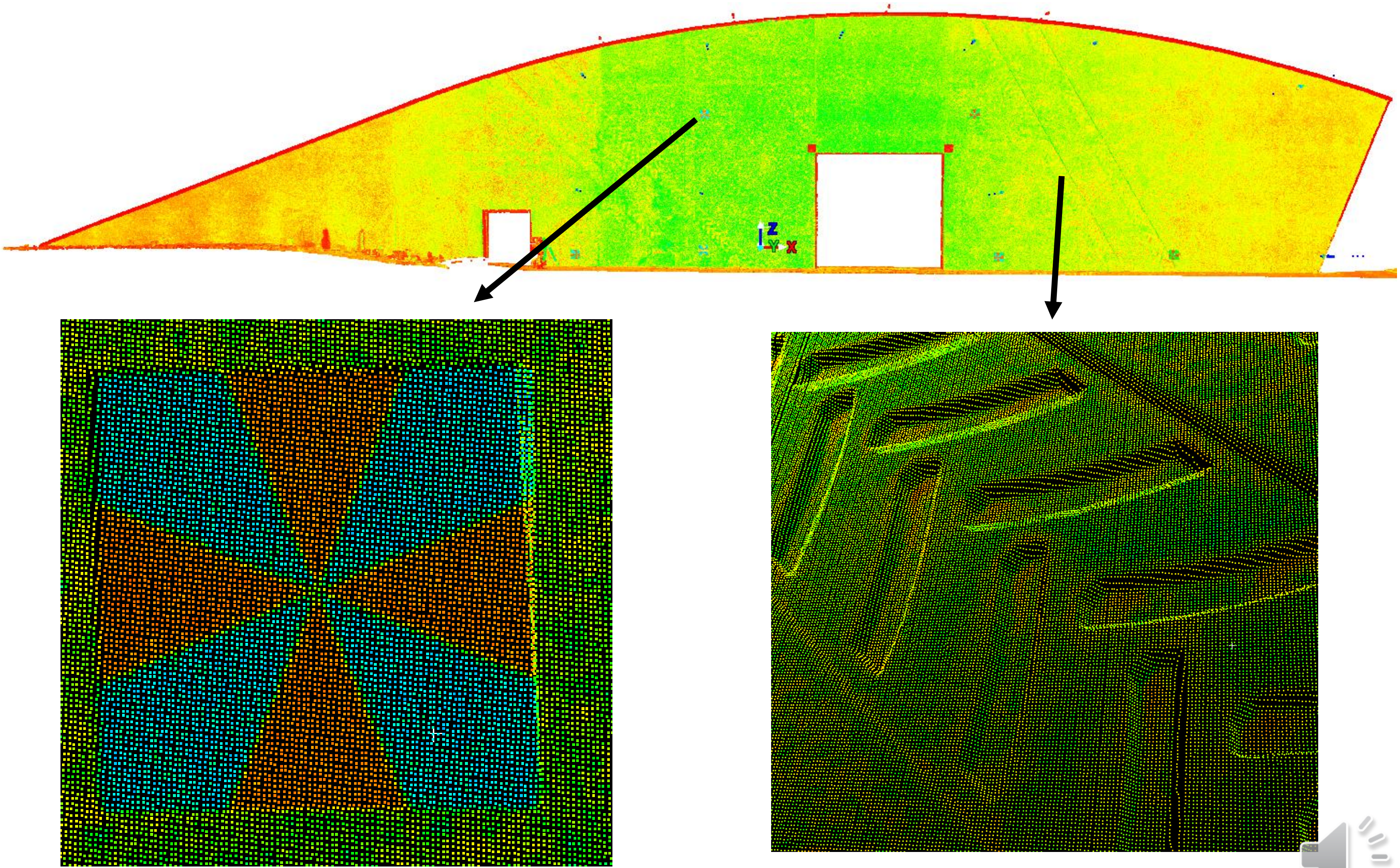
6 Uncertainty of point clouds

High-end terrestrial laser scanners

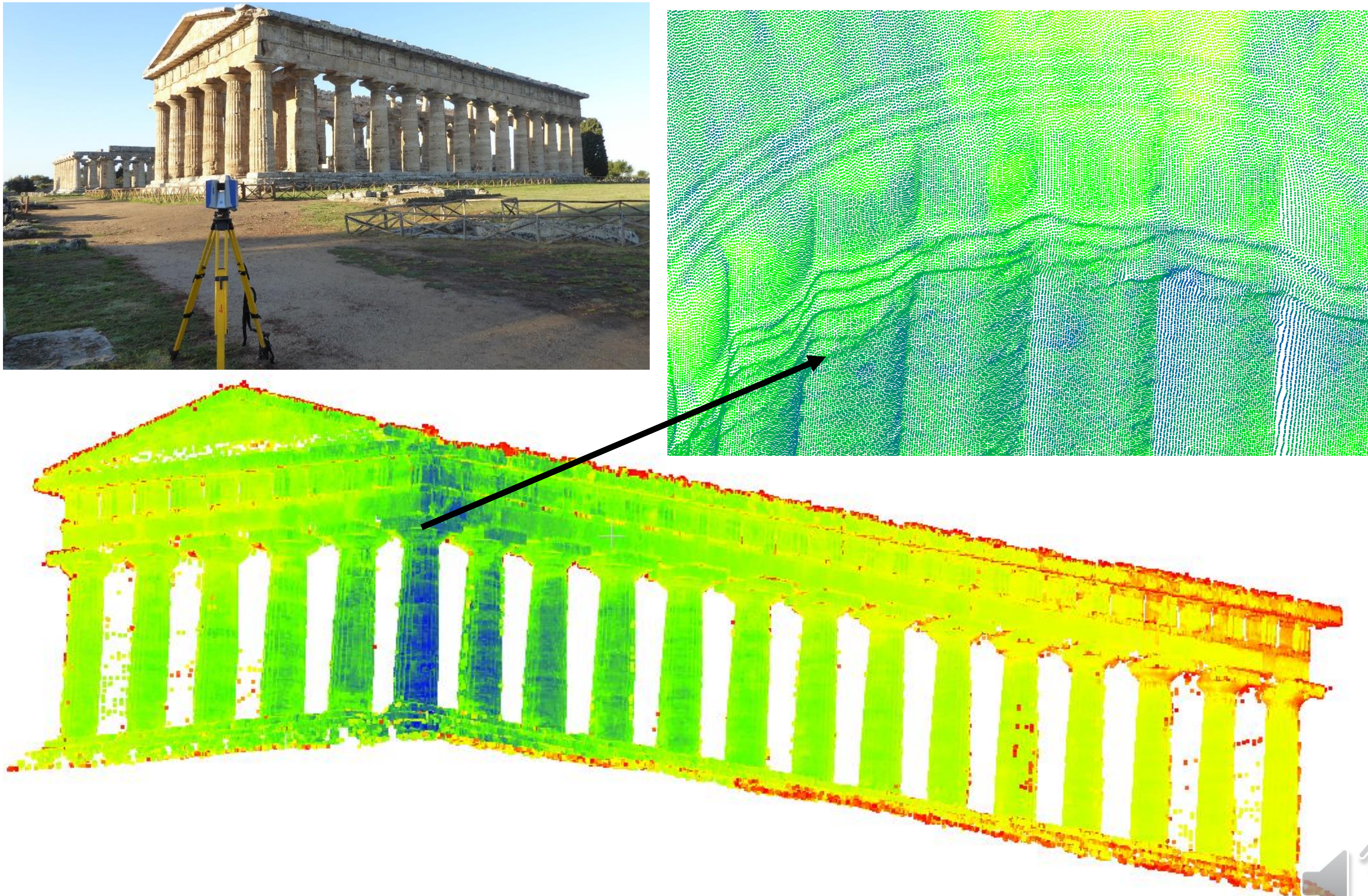
www.leica-geosystems.com

www.scancric.de

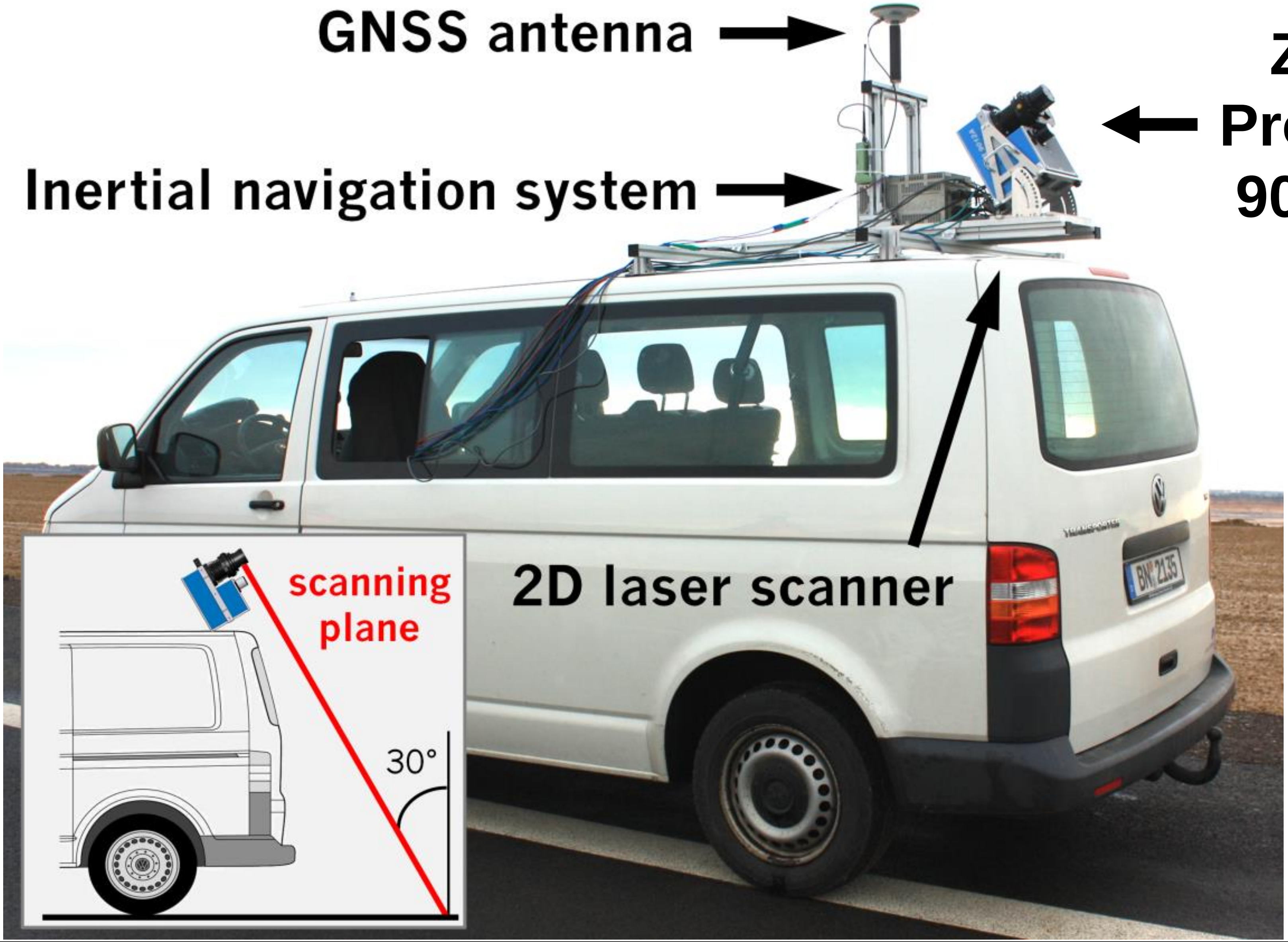
Smoothness (Leica P50)



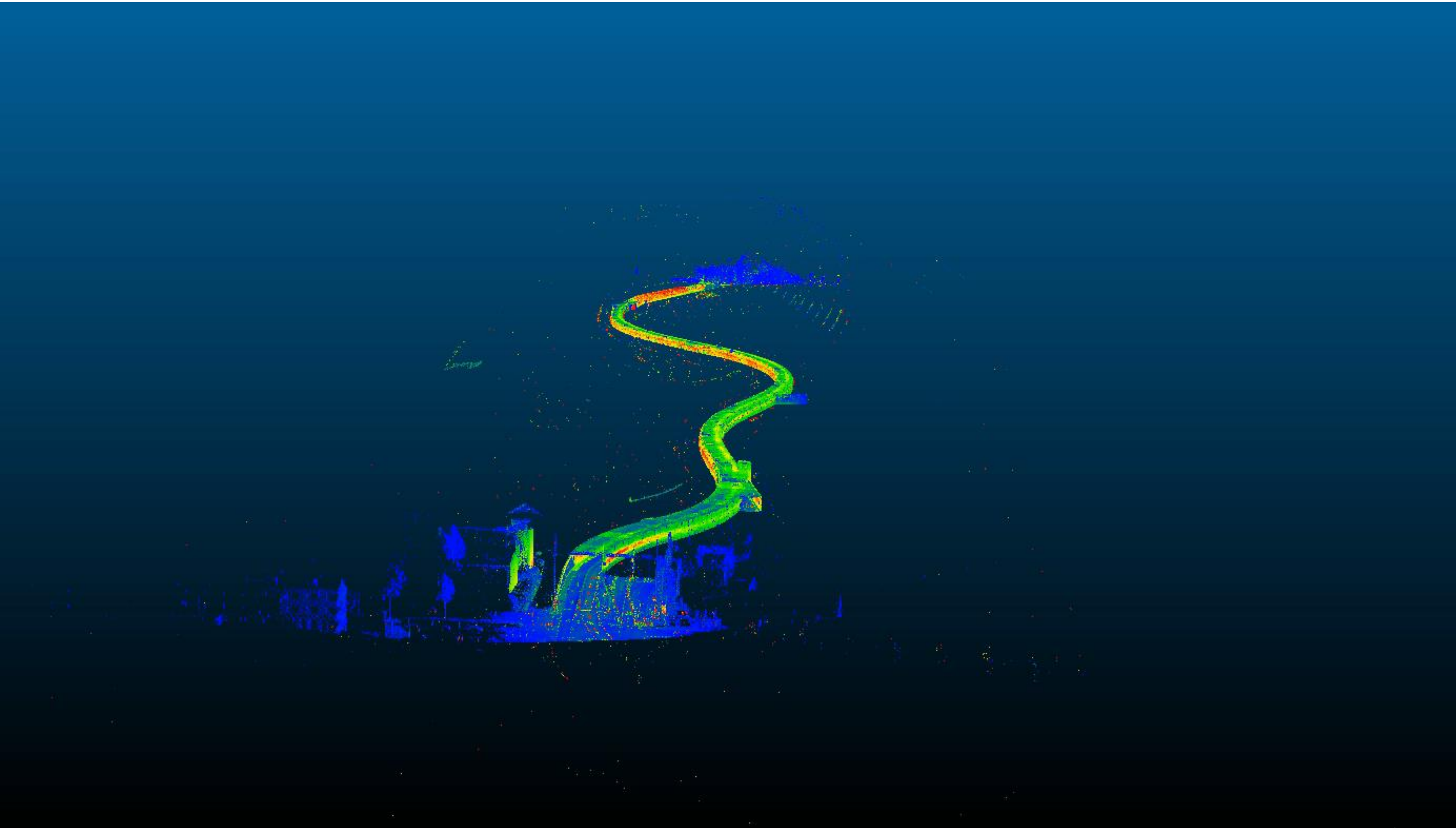
Smoothness (Z+F Imager 5016)



High-end mobile mapping system

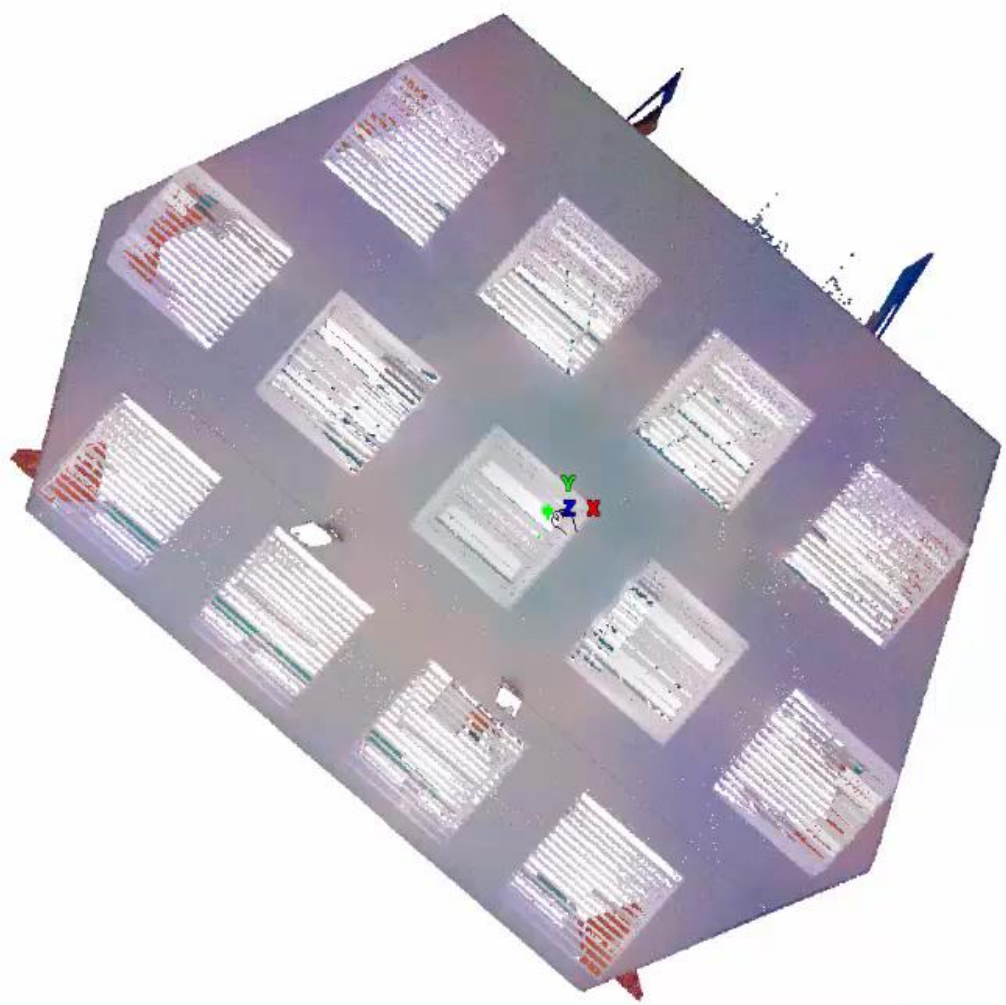


Smoothness (Z+F Profiler 9012A)

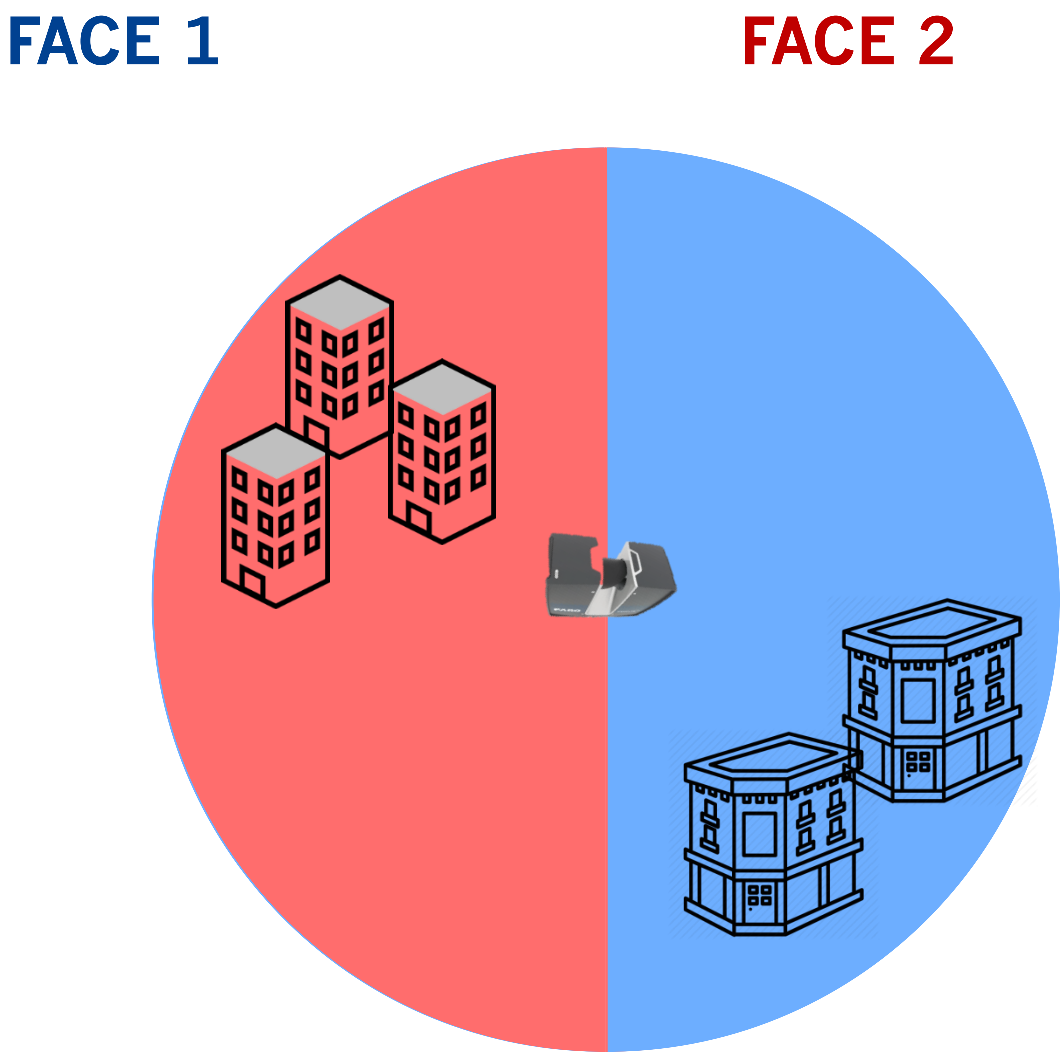


Courtesy: Lasse Klingbeil & Erik Heinz

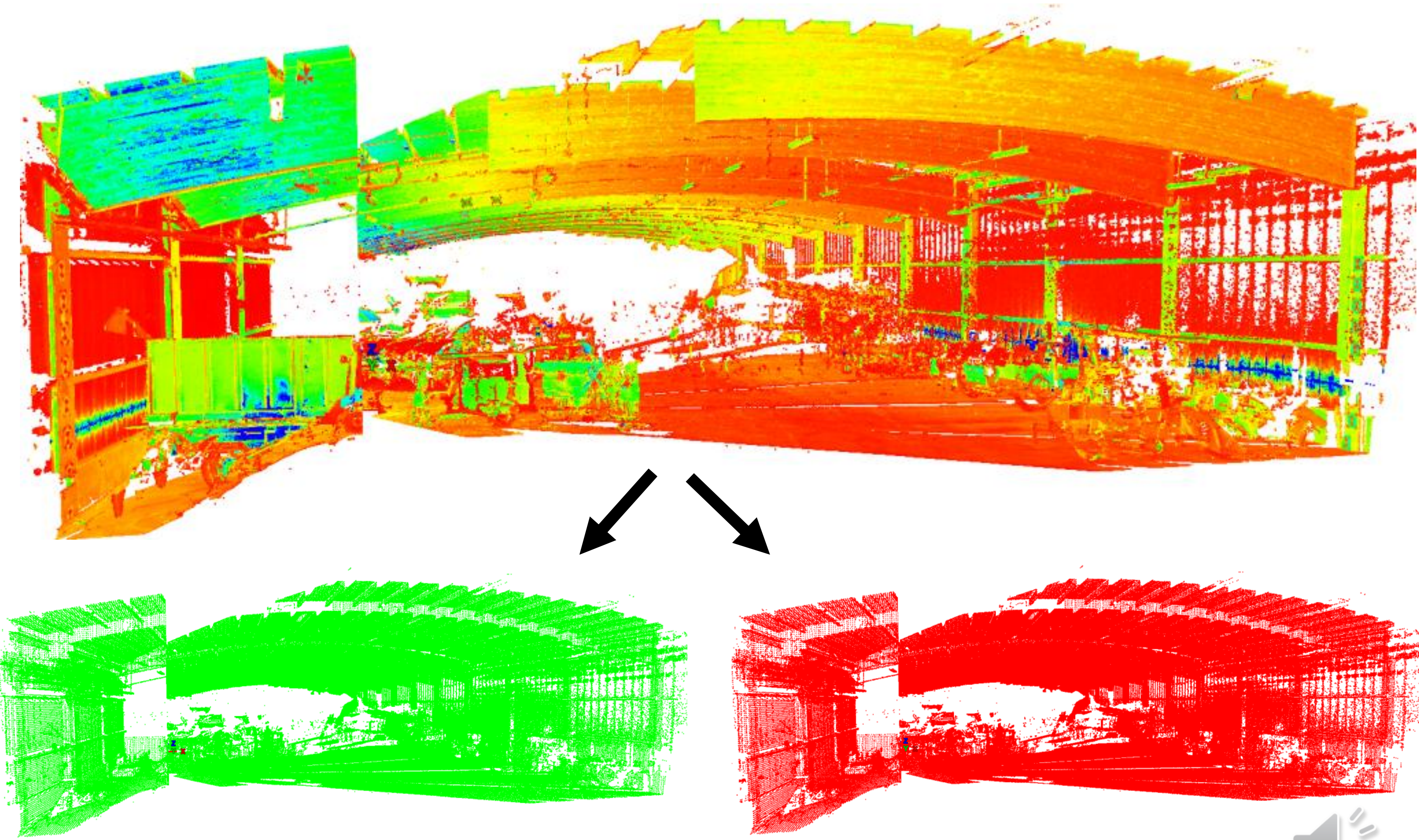
Uncertainty?



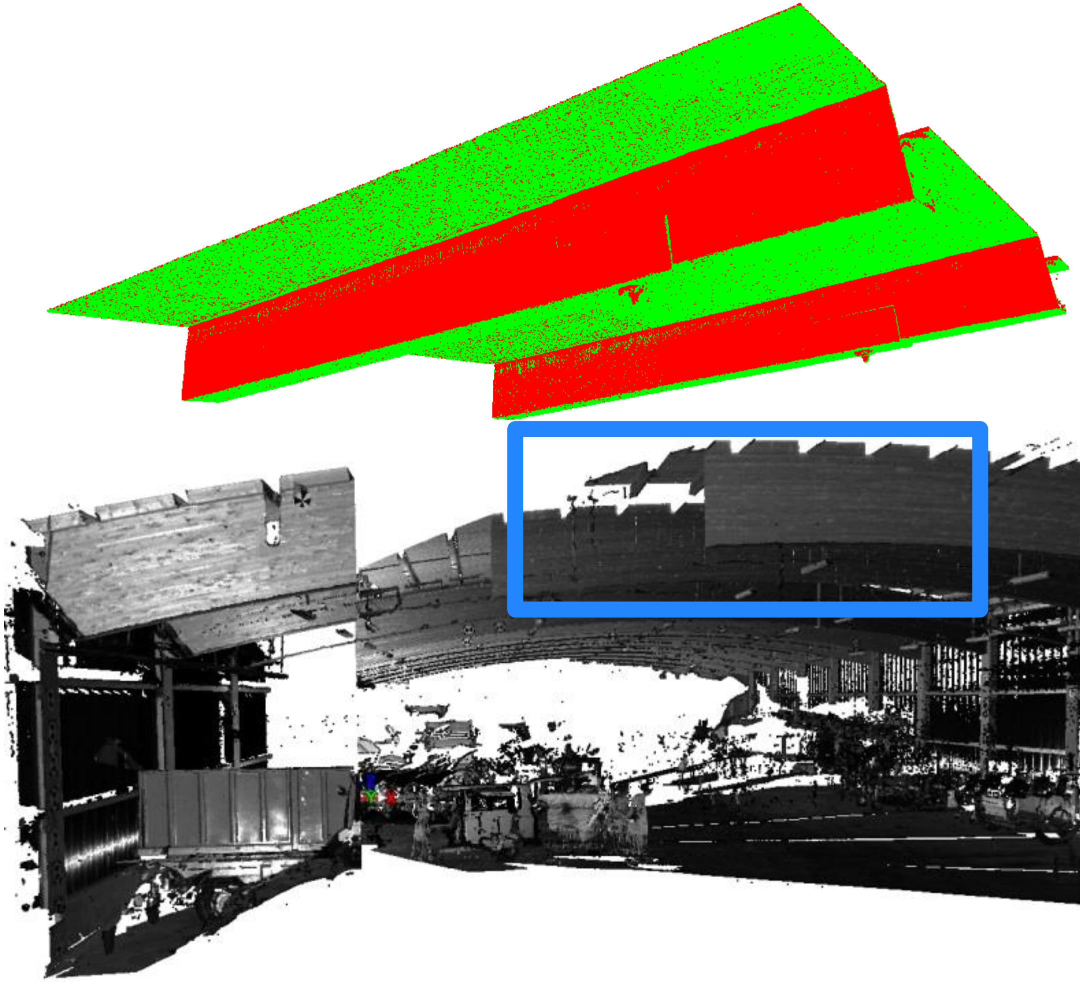
2-Face-Scan

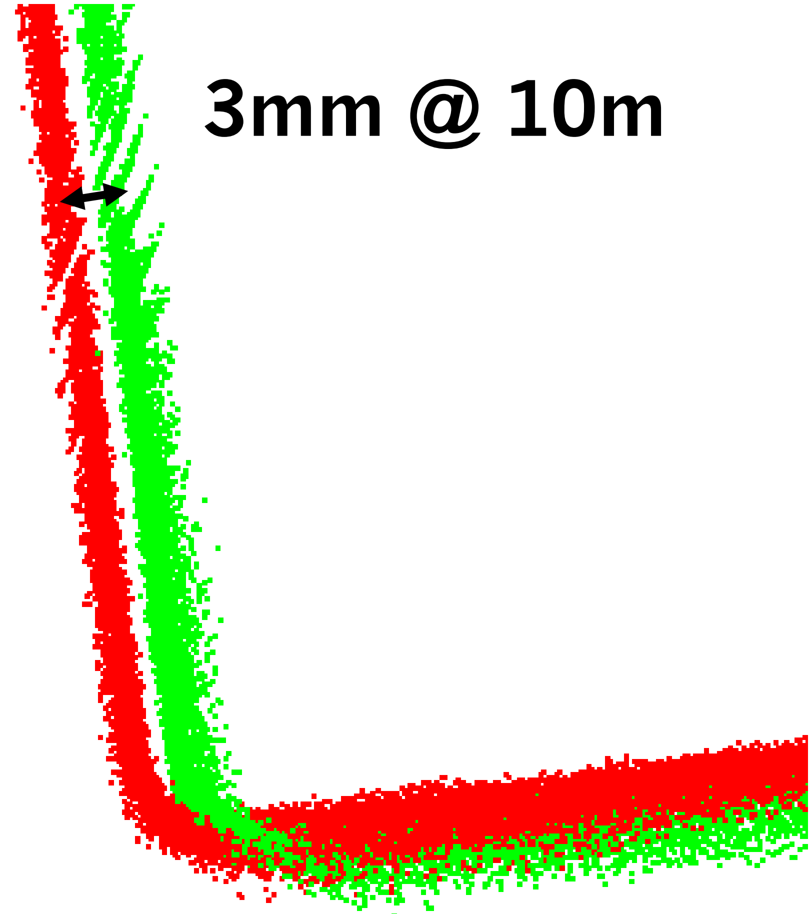


Separation of two scans



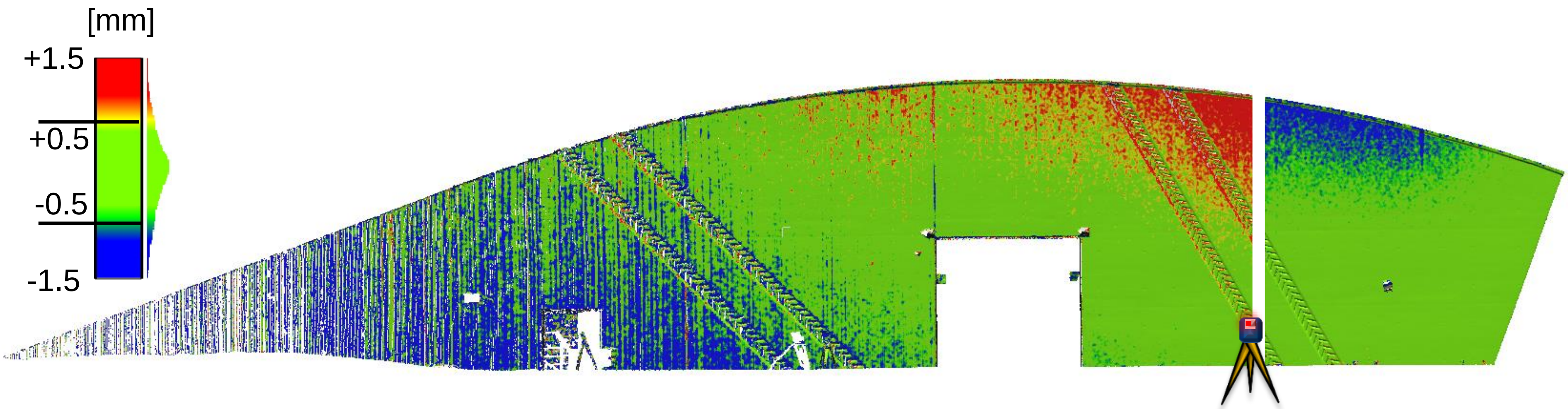
Face 1 vs. Face 2





- Quantity: millimeters, even at short distances
- Visible in smooth point cloud

Face 1 minus Face 2



- Errors of up to +/- 1.5 mm at 10m distance
- Dependend on vertical angle and horizontal angle
- Consequence: offset between face 1 and face 2
- Other probable consequences: innere deformation of scanned objects, tilt or bending of object

=> Errors exist in any case, maybe just visually hidden!

6 Uncertainty of point clouds

6.1 Basics

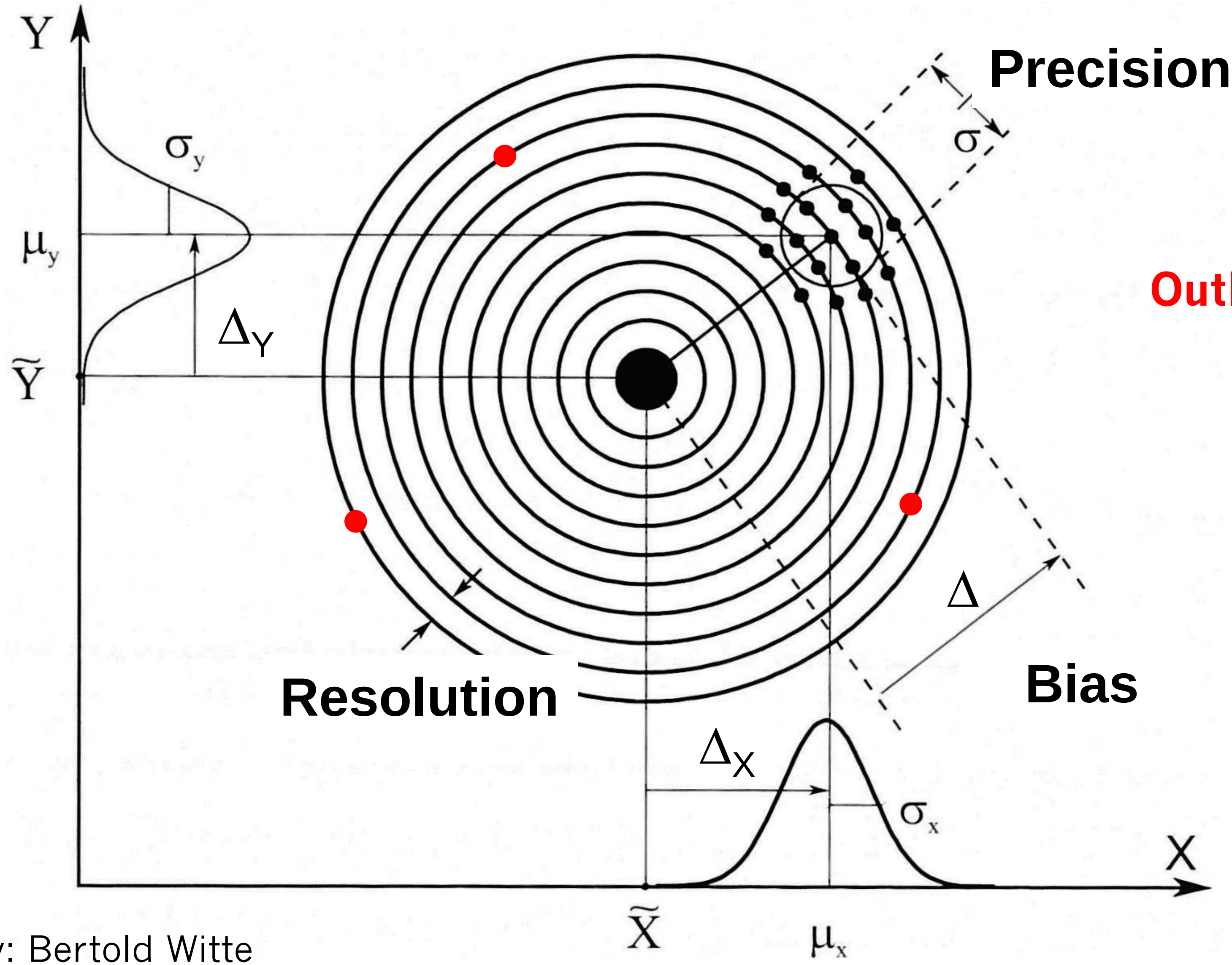
6.1.1 Visual inspection of uncertainty

6.1.2 Error types and their treatment

6.2 Error sources at laser scanning

6.3 Strategies for minimizing the uncertainty

Separation into error types

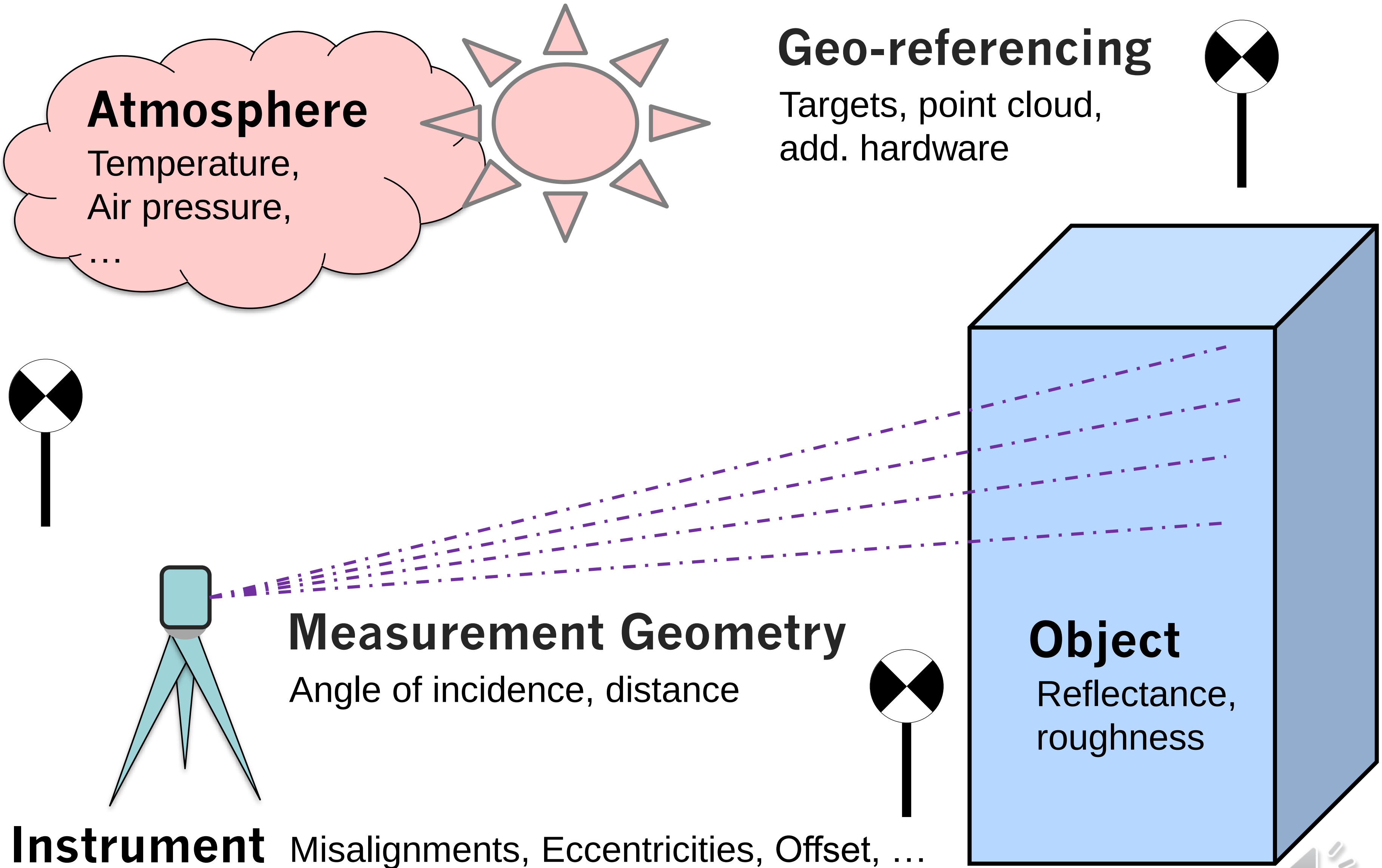


Courtesy: Bertold Witte

Characterization of error types

	Outliers	Systematic errors	Random errors
Appear-ance	Extremely erroneous individual measurements	One-sided effects	Gaußian distributed (positive and negative, more often small than large)
Source	Mistakes of operator, Inappropriate used of instrument	Imperfection of measurement procedure, insufficient calibration of instrument	Fluctuations not trackable by calibration, uncontrolled changes of measured object and environment
Minimi-zing	Avoidance by care and control	Calibration, math. compensation, measurem. strategies	Averaging by multiple measurements
Quanti-tative measure	–	**Bias / absolute accuracy**	**Precision / relative accuracy**
	Uncertainty		

Error sources at laser scanning



6 Uncertainty of point clouds

6.1 Basics

6.2 Error sources at laser scanning

6.2.1 Instrument

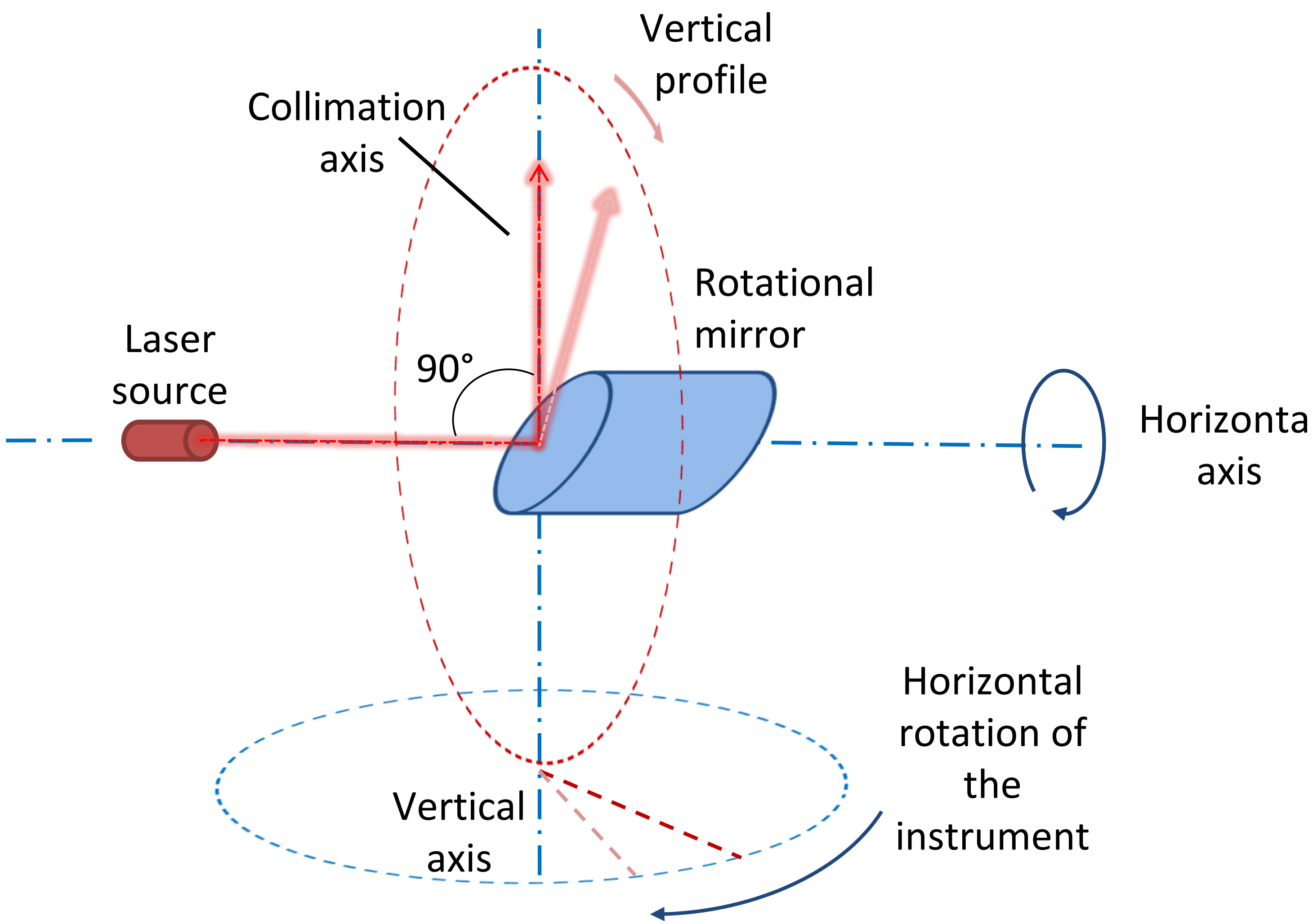
6.2.2 Atmosphere

6.2.3 Geometry / Object

6.2.4 Geo-referencing

6.3 Strategies for minimizing the uncertainty

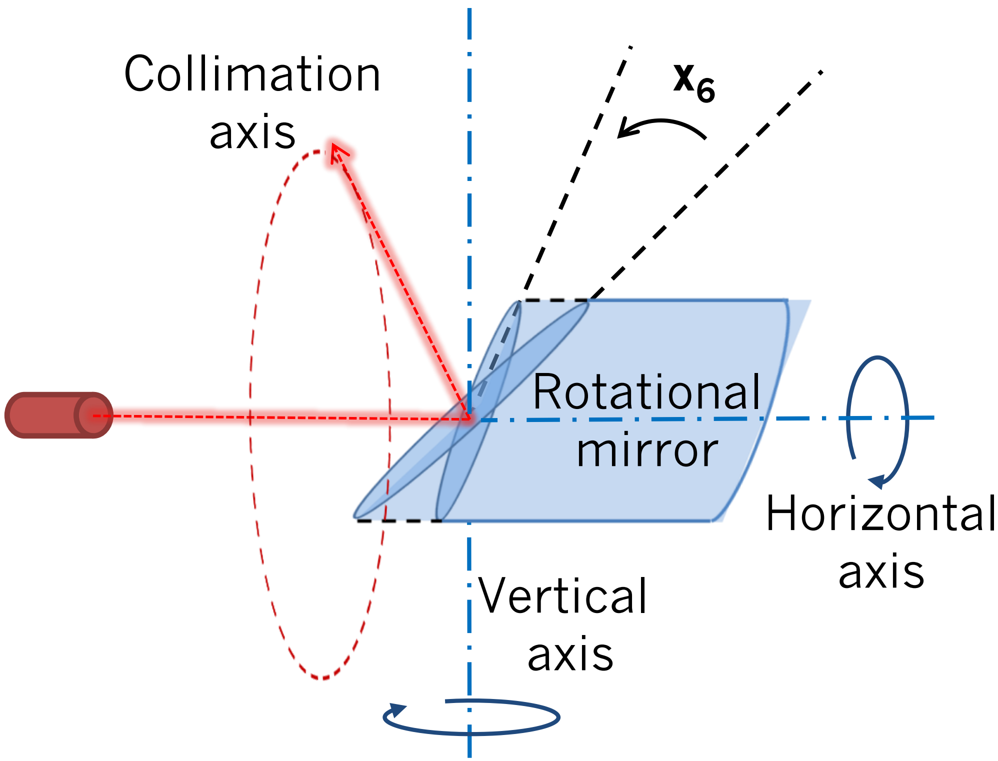
Construction of 3D laser scanner



Courtesy: Tomislav Medic

Mirror tilt

Courtesy:
Tomislav Medic



Error in Face 1:

$$\Delta\varphi_j^i = +\frac{2x_6}{\sin(\theta_j^i)}$$

Error in Face 2:

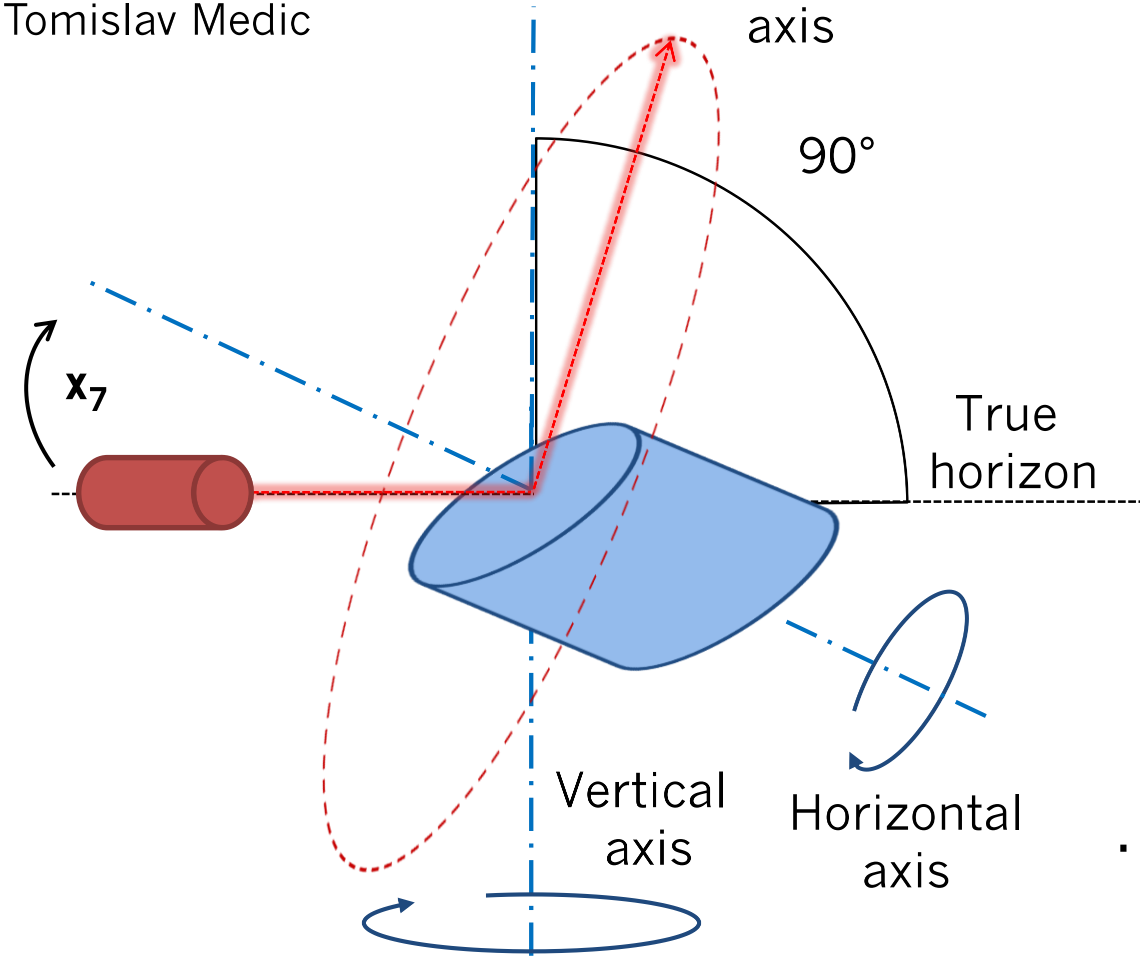
$$\Delta\varphi_j^i = -\frac{2x_6}{\sin(\theta_j^i)}$$

... with θ = vertical angle

=> Equals collimation axis error at total stations

Horizontal axis error

Courtesy:
Tomislav Medic



Error in Face 1:

$$\Delta\varphi_j^i = +\frac{x_7}{\tan(\theta_j^i)}$$

Error in Face 2:

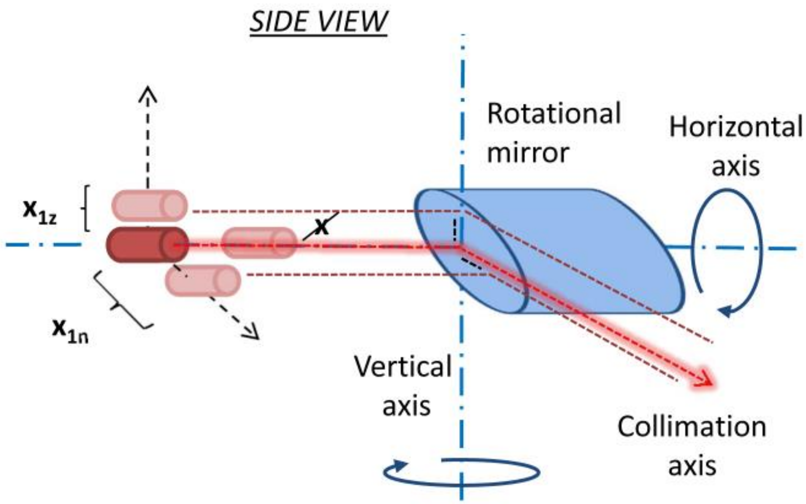
$$\Delta\varphi_j^i = -\frac{x_7}{\tan(\theta_j^i)}$$

... with θ = vertical angle

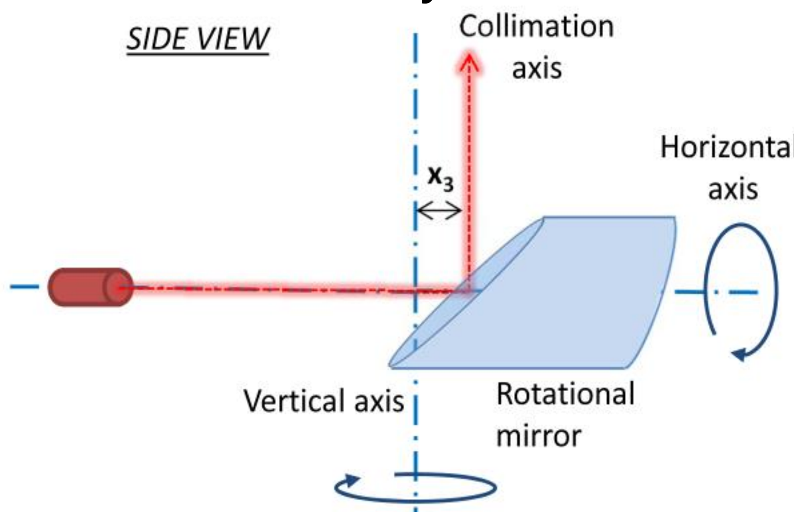
=> Equals horizontal axis errors at total stations

Further misalignments

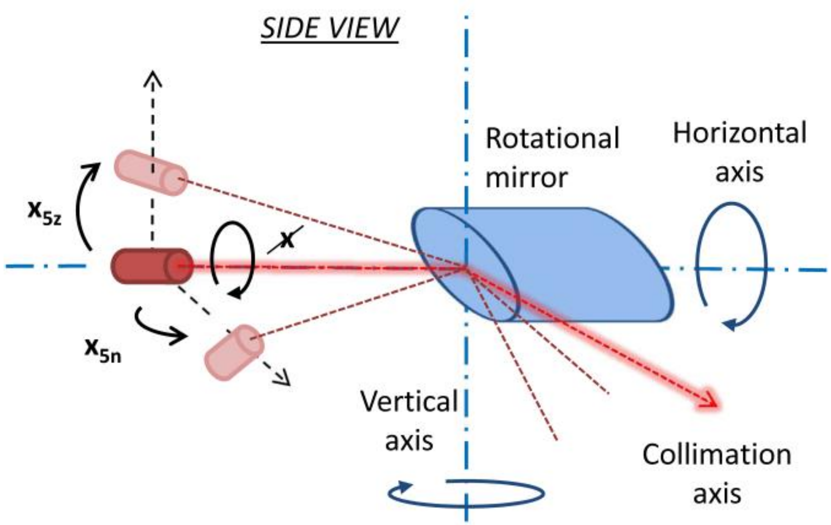
Eccentricity of laser



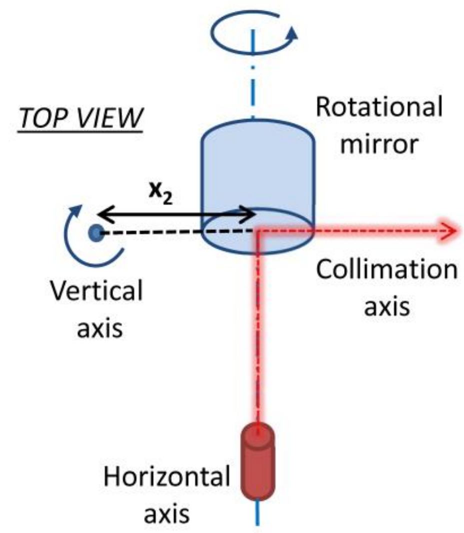
Eccentricity of mirror



Alignment of laser



Eccentricity of axes



Courtesy: Tomislav Medic

Calibration parameters

Description	Parameter	TS	2-faces	high-end
Horizontal beam offset	x_{1n}	similar	no	yes
Vertical beam offset	x_{1z}	no	yes	yes
Horizontal axis offset	x_2	similar	yes	yes
Mirror offset	x_3	no	yes	yes
Vertical index offset	x_4	yes	yes	yes
Horizontal beam tilt	x_{5n}	similar	yes	yes
Vertical beam tilt	x_{5z}	similar	no	yes
Mirror tilt	x_6	yes	yes	yes
Horizontal axis error (tilt)	x_7	yes	yes	yes
Horizontal angle encoder eccentricity	x_{8x}	yes	yes	no
Horizontal angle encoder eccentricity	x_{8y}	yes	yes	no
Vertical angle encoder eccentricity	x_{9n}	yes	yes	no
Vertical angle encoder eccentricity	x_{9z}	yes	no	no
Second order scale error in the horizontal angle encoder	x_{11a}	yes	no	no
Second order scale error in the horizontal angle encoder	x_{11b}	yes	no	no
Second order scale error in the vertical angle encoder	x_{12a}	yes	yes	no
Second order scale error in the vertical angle encoder	x_{12b}	yes	no	no
Rangefinder offset	x_{10}	yes	no	yes
Rangefinder scale error	/	yes	no	yes
Rangefinder cyclic error	/	yes	no	yes*

* existing only in TLSs using phase-shift distance measuring principle

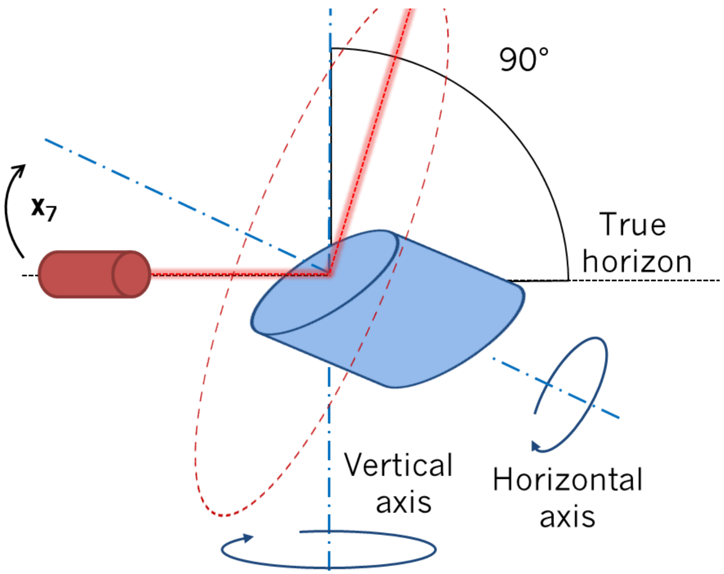
Functional model

$$\Delta r_j^i = x_2 \sin(\theta_j^i) + x_{10} + v_{r_j^i}$$

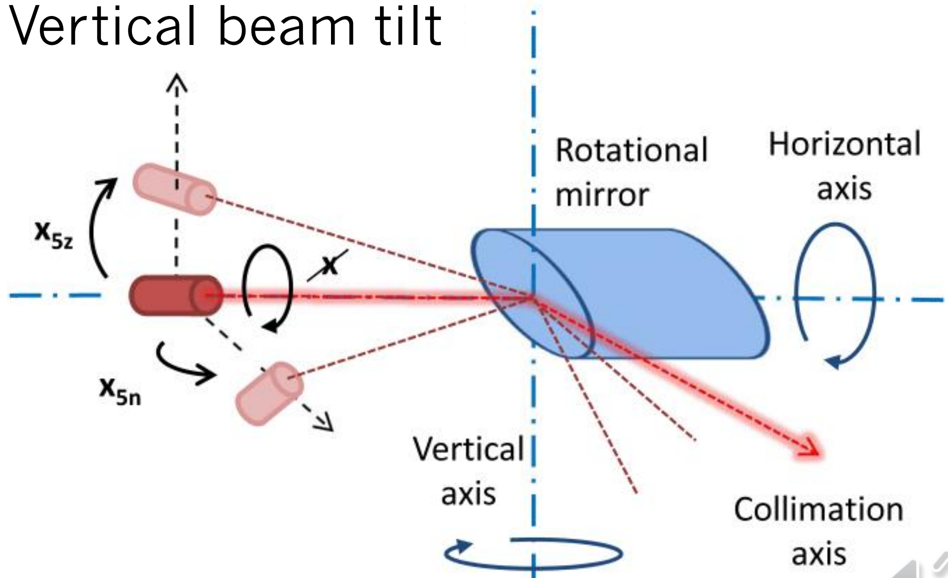
$$\Delta \varphi_j^i = \frac{x_{1z}}{r_j^i \tan(\theta_j^i)} + \frac{x_3}{r_j^i \sin(\theta_j^i)} + \frac{x_{5z-7}}{\tan(\theta_j^i)} + \frac{2x_6}{sin(\theta_j^i)} + \frac{x_{1n}}{r_j^i} + v_{\varphi_j^i}$$

$$\Delta \theta_j^i = \frac{x_{1n+2} \cos(\theta_j^i)}{r_j^i} + x_4 + x_{5n} \cos(\theta_j^i) - \frac{x_{1z}\, sin(\theta_j^i)}{r_j^i} - x_{5z}\, sin(\theta_j^i) + v_{\theta_j^i}$$

Horizontal axis error



Vertical beam tilt



Interim summary

- Systematic and random errors of angular and range measurements
- Sources similar to total station, but more complex
- Impacts on point cloud large (larger than smoothnes / random errors)

- Here: 3D panoramic scanner
- 2D profile scanner analogous, but with less error sources

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6 Uncertainty of point clouds

Propagation delay of signal

Time of flight principle

$$r = \frac{1}{2} \cdot c \cdot t$$

- c: speed of light in atmosphere
- t: time of flight

$$c = \frac{c_0}{n}$$

- c_0: speed of light in vacuum (299 792 458 m/s)
- n: refraction index in atmosphere

$$\boldsymbol{r = \frac{1}{2} \cdot \frac{c_0}{n} \cdot t}$$

Phase shift principle

$$r = \frac{1}{2} \cdot \left(N \cdot \lambda + \frac{\Delta\varphi}{2\pi} \cdot \lambda \right)$$

- N: number of waves
- λ: wavelength
- $\Delta\varphi$: phase shift
- f: frequency

$$\lambda = \frac{c}{f}$$

$$\boldsymbol{r = \frac{1}{2} \cdot \left(N + \frac{\Delta\varphi}{2\pi} \right) \cdot \frac{c_0}{n \cdot f}}$$

Propagation delay of signal

$$\boldsymbol{\Delta n \cdot 10^{-6} = -0,99 \cdot \Delta t + 0,28 \cdot \Delta p - 0,04 \cdot \Delta e}$$

- Δn: Change of refraction index…
- Δt: Change of temperature…
- Δp: Change of air pressure…
- Δe: Change of air humidity…

… in relation to reference atmosphere

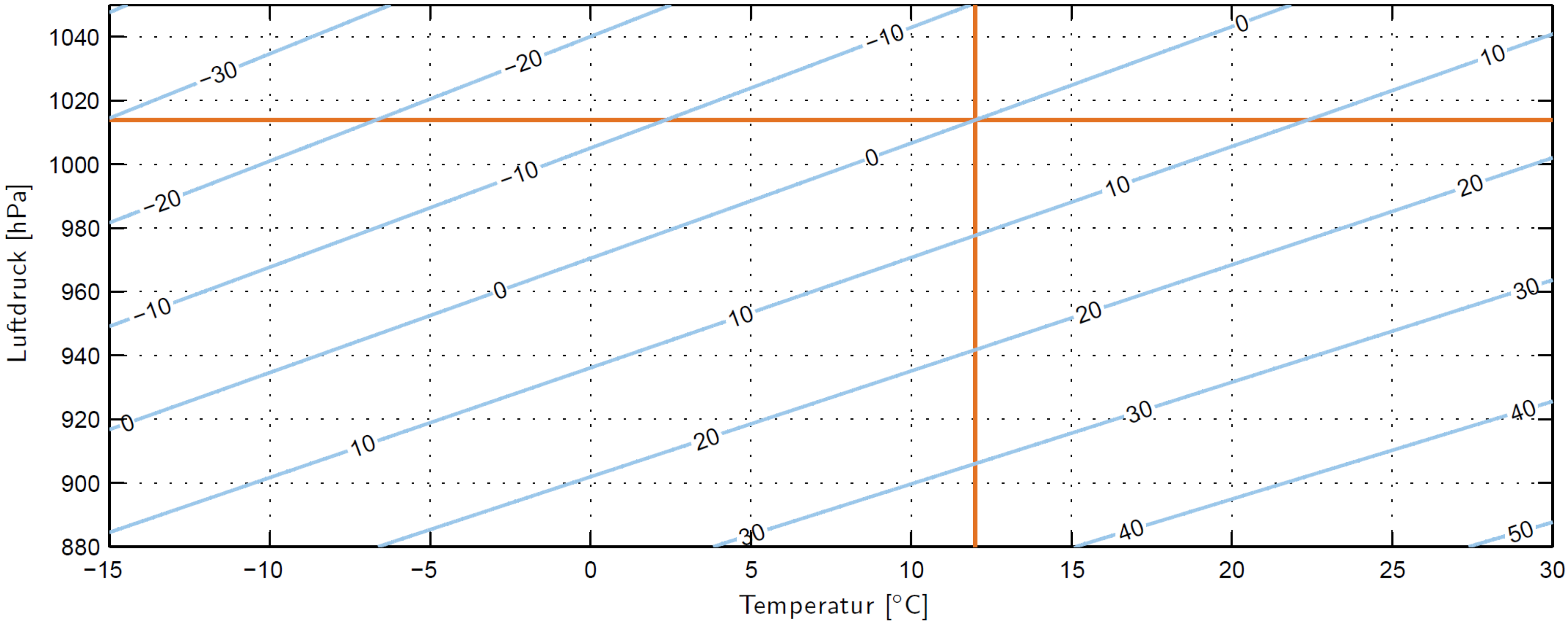
Temperature: 1°C = - 1 ppm

Air pressure: 3 hPa = + 1 ppm

Air humidity: 25 hPa = - 1 ppm (neglectable)

- Neglectable in close range applications
- At larger distances (> 100m) presumably important

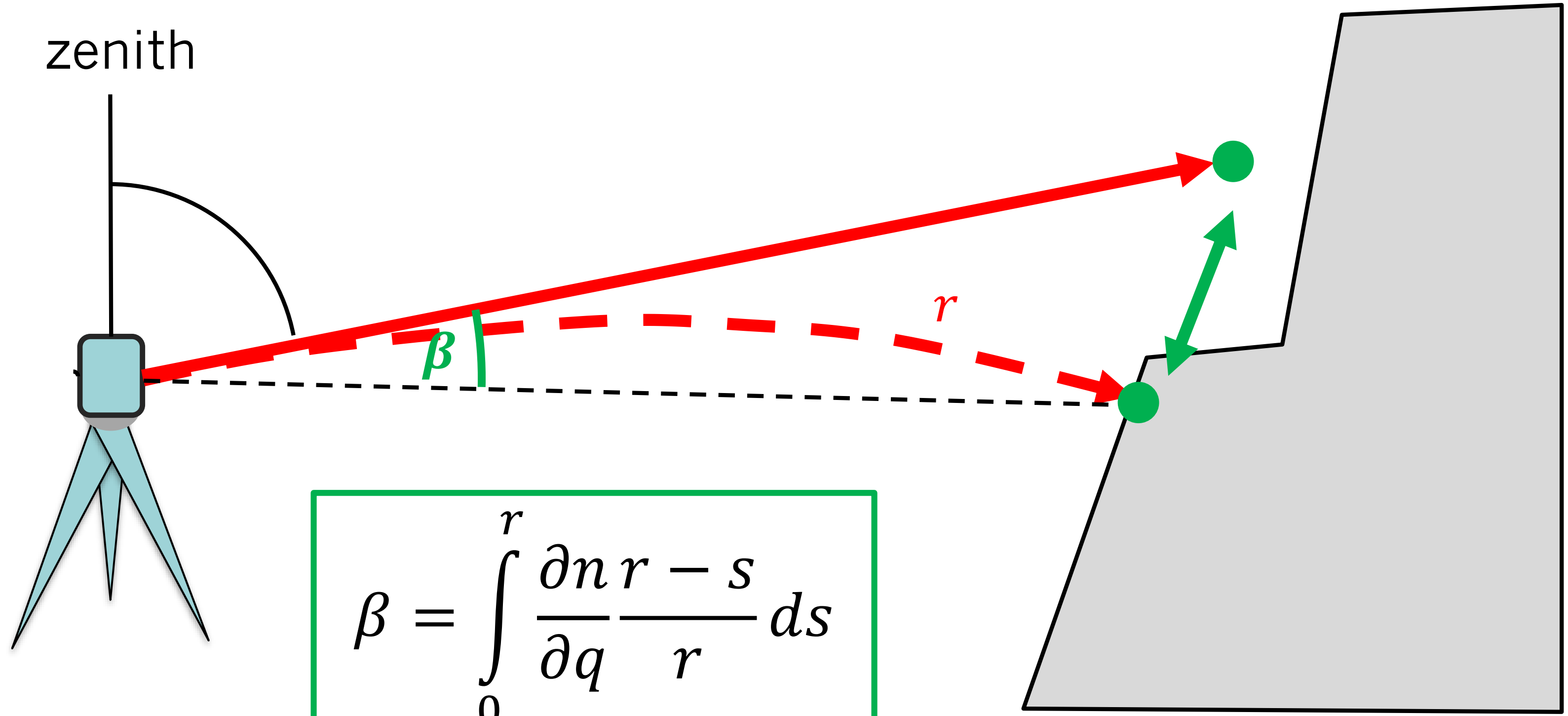
Numerical values



- Atmospheric correction in ppm
- Reference atmosphere: 12°C, 1013.25 hPa, 60%
- Here: carrier wavelength of 700 nm

Courtesy: Thomas Schäfer

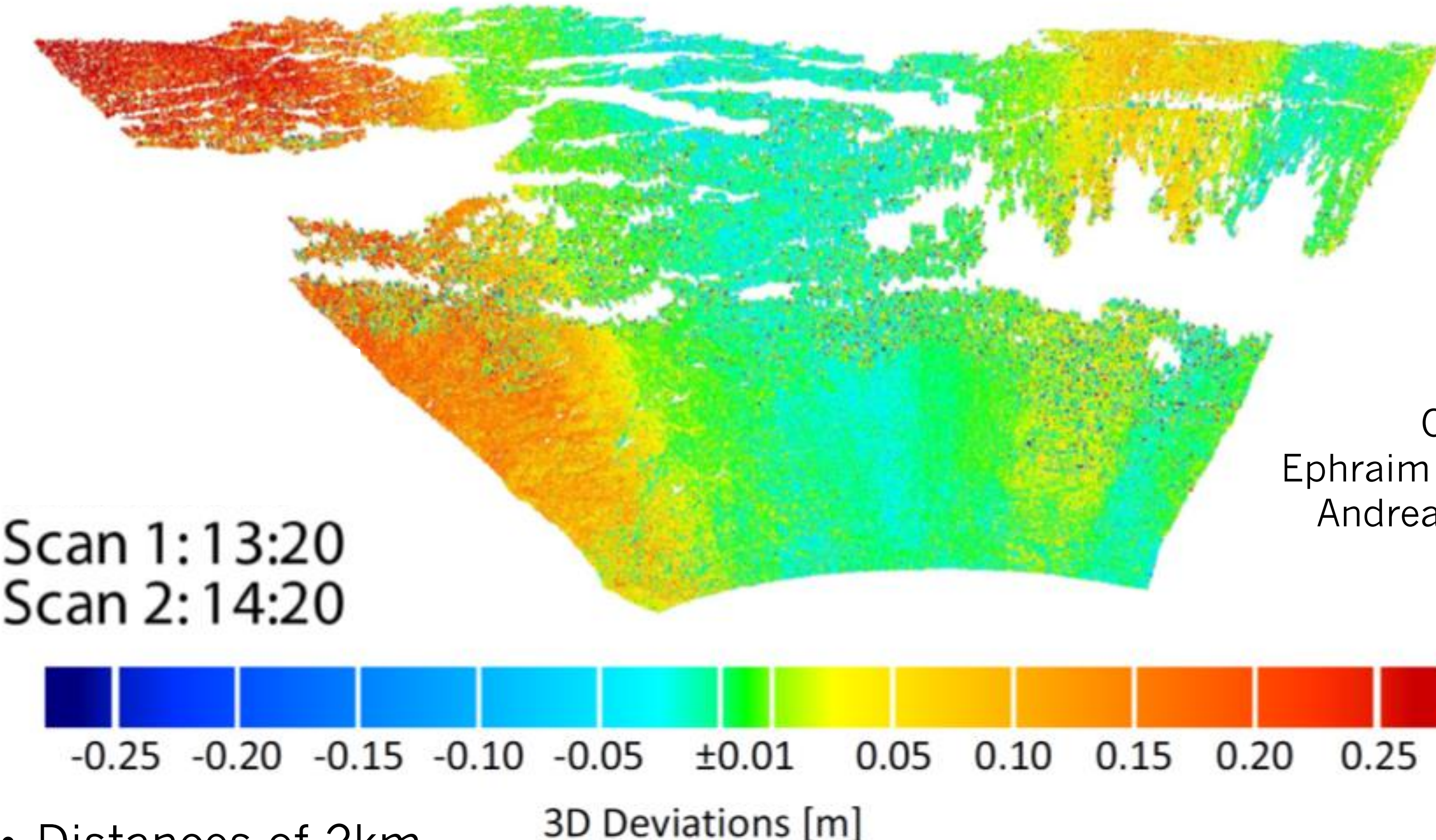
Refraction: theory



$$\beta = \int_0^r \frac{\partial n}{\partial q} \frac{r - s}{r} ds$$

- Atmosphere composes of horizontal layers with varying density
- Vertical gradient of refraction index leads to curved beam
- Measured distance is assigned to biased angle (refraction angle β)
- Hard to compensate due to unknown layering of atmosphere
- Avoiding by measuring during low atmospheric turbulence

Refraction: empirical values



Courtesy:
Ephraim Friedli &
Andreas Wieser

- Distances of 2km
- Variation of 10 mgon during day (see fig.), 2 mgon during night

Interim summary

- Systematic errors due to atmospheric effects
- Important for longer distances and at changing atmospheric conditions
- In any case: mathematical correction of propagation delay

Agenda

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6.1 Basics

6.2 Error sources at laser scanning

6.2.1 Instrument

6.2.2 Atmosphere

6.2.3 Geometry / Object

6.2.4 Geo-referencing

6.3 Strategies for minimizing the uncertainty

Laser Radar Equation

Receiver Aperture Diameter

System Transmission Factor

Transmitted Power

Angle of Incidence

$$P_r = \frac{P_t D_r^2 \eta_{sys}}{4} \eta_{atm} \frac{\cos\alpha}{r^2} \rho$$

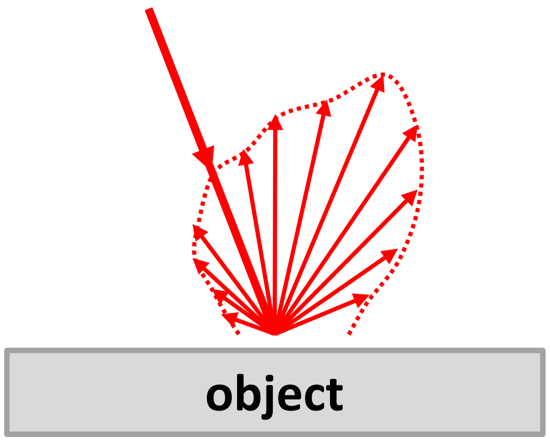
Received Power

Reflectance of Surface

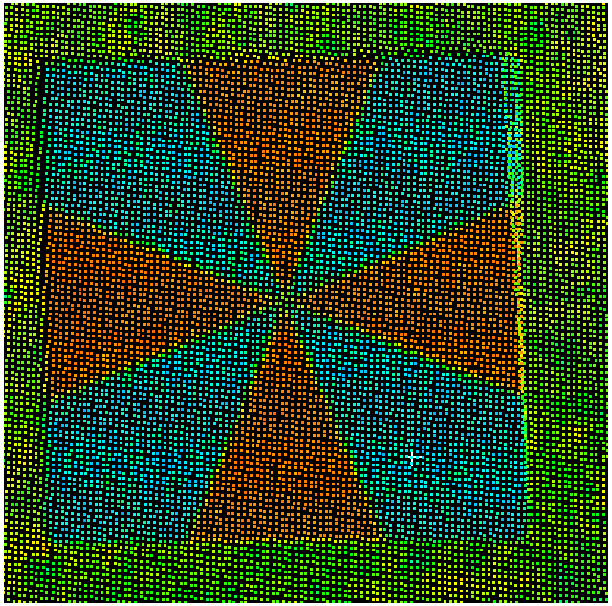
Atmospheric Transmission Factor

Measured Range

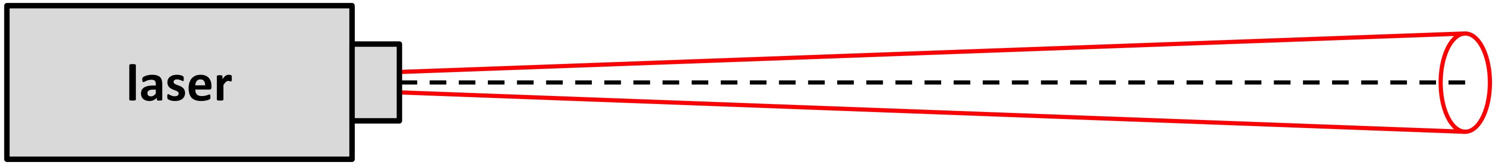
Angle of incidence



Reflectance



Distance



Precision ∝ Intensity

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An intensity-based stochastic model for terrestrial laser scanners

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ABSTRACT

Up until now no appropriate models have been proposed that are capable to describe the stochastic characteristics of reflectorless rangefinders - the key component of terrestrial laser scanners. This state has to be rated as unsatisfactory especially from the perception of Geodesy where comprehensive knowledge about the precision of measurements is of vital importance, for instance to weigh individual observations or to reveal outliers. In order to tackle this problem, a novel intensity-based stochastic model for the reflectorless rangefinder of a Zoller + Fröhlich Imager 5006 h is experimentally derived. This model accommodates the influence of the interaction between the emitted signal and object surface as well as the acquisition configuration on distance measurements. Based on two different experiments the stochastic model has been successfully verified for three chosen sampling rates.

1. Introduction

Terrestrial laser scanners (TLS) have reached a high level of acceptance in the field of Geodesy and are consequently used in various fields of applications, for instance kinematic laser scanning (Böder et al., 2010), deformation monitoring (Lindenbergh and Pietrzyk, 2015), cultural heritage (Böhler and Marbs, 2004) and bio mass estimation (Tilly et al., 2013). As for all other surveying instruments, the achievable precision or measurement noise is used, among other characteristics, to decide whether a specific sensor is suitable to fulfil a particular task or not. Furthermore, this information is also vital for identification of outliers, statistically significant identification of deformation, comparison of different laser scanners, and weighting of individual observations in an adjustment. For these reasons, the assumed precision of observations is gathered within the stochastic model.

Regarding the noise of TLS observations it is emphasised that after more than a decade of intensive research no appropriate stochastic model has been published (e.g. Böhler et al., 2003; Soudarissanane, 2016) - a circumstance which has to be rated as unsatisfactory from a geodetic point of view. The reason why the stochastic properties are not well understood up to this point can be associated to the component that lead to the development of laser scanners, namely reflectorless rangefinders (RL-RF) that are also referred to as electro-optic distance measurement units (RL-EDM). The advantage of these RL-EDMs is that distances can be determined between an instrument and an object point without the necessity of bringing artificial reflectors, such as prisms or markers, into the object space. Yet, some requirements exist for reflectorless rangefinders namely, that the object is not translucent and that a certain amount of the emitted signal has to be reflected by the object's surface.

Moreover it can be established that the emitted laser signal is subject to various falsifying effects during its way on an uncontrolled optical path. Soudarissanane et al. (2011) declare four main categories that influence the quality of individual points captured by TLS, namely:

- the instrument mechanism;
- the atmospheric conditions;
- the properties of the object's surface; and
- the acquisition configuration.

While various authors do not distinguish between systematic and random errors in their contributions (see next paragraph), this article will strictly focus on stochastic signals in reflectorless rangefinders. Hence, effects are analysed that are caused by the last two categories.

1.1. Related work

The use of appropriate stochastic models is of particular importance for tasks such as sensor calibration of terrestrial laser

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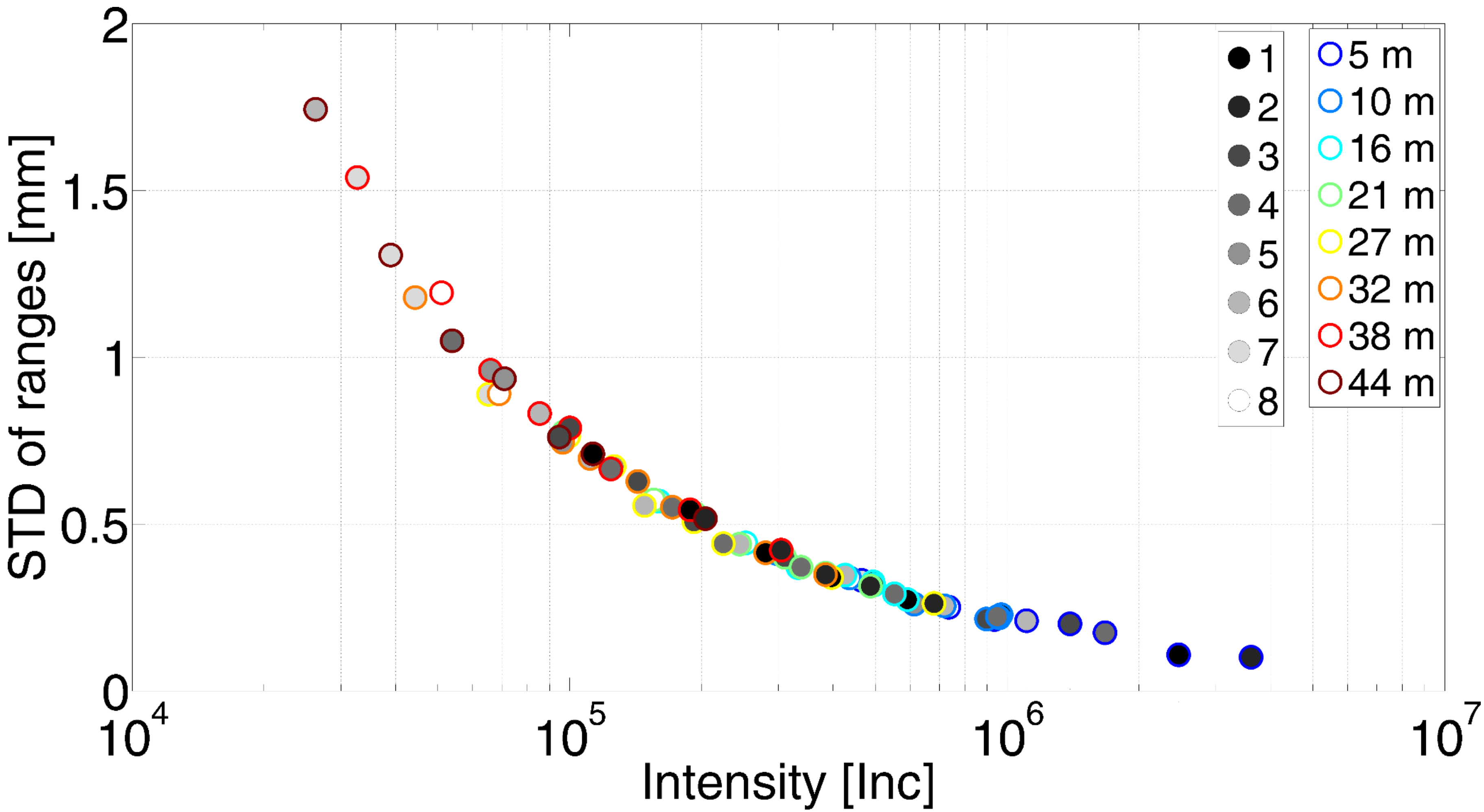
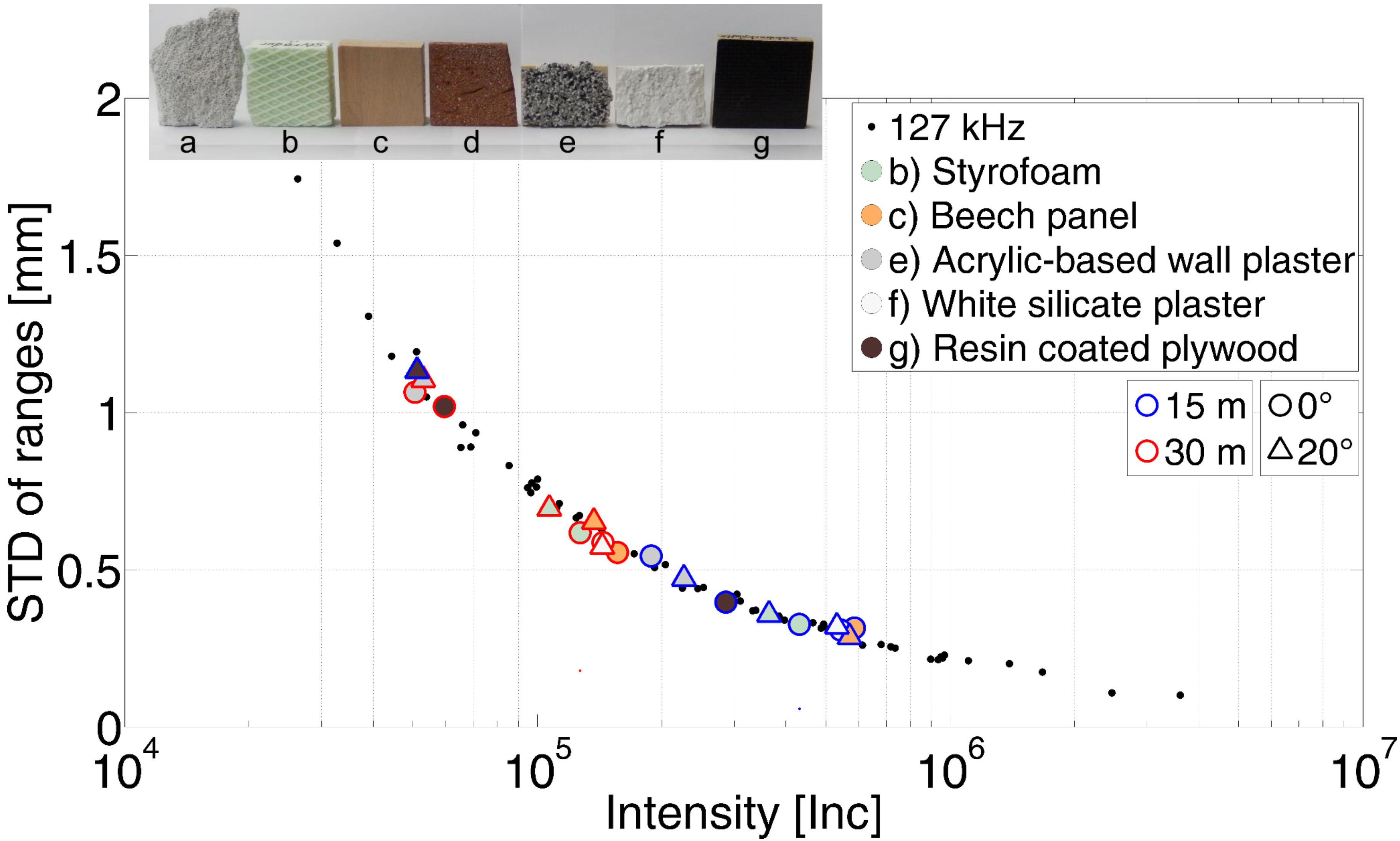
Reflectance (color) and distance



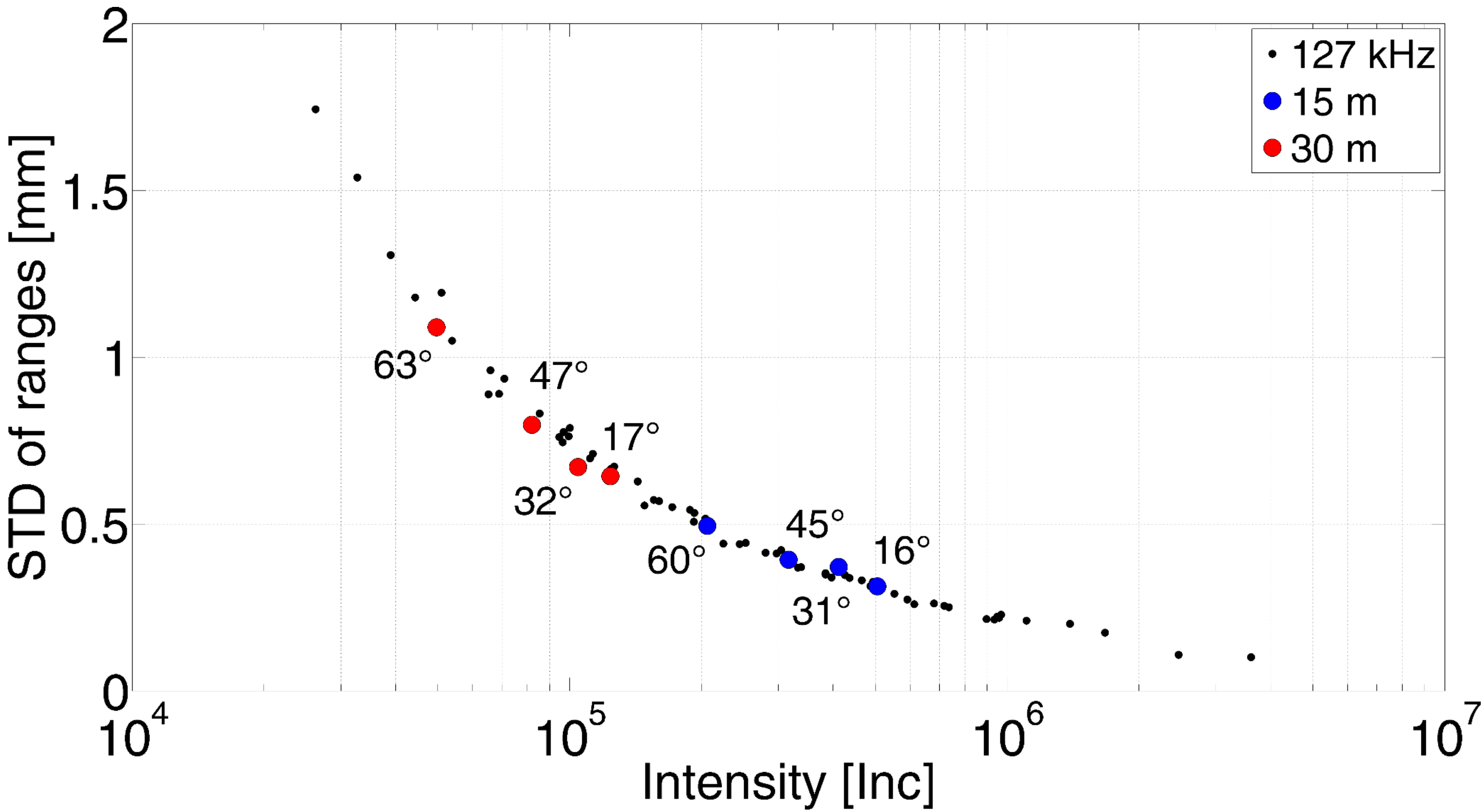
Courtesy: Daniel Wujanz

Reflectance (material)



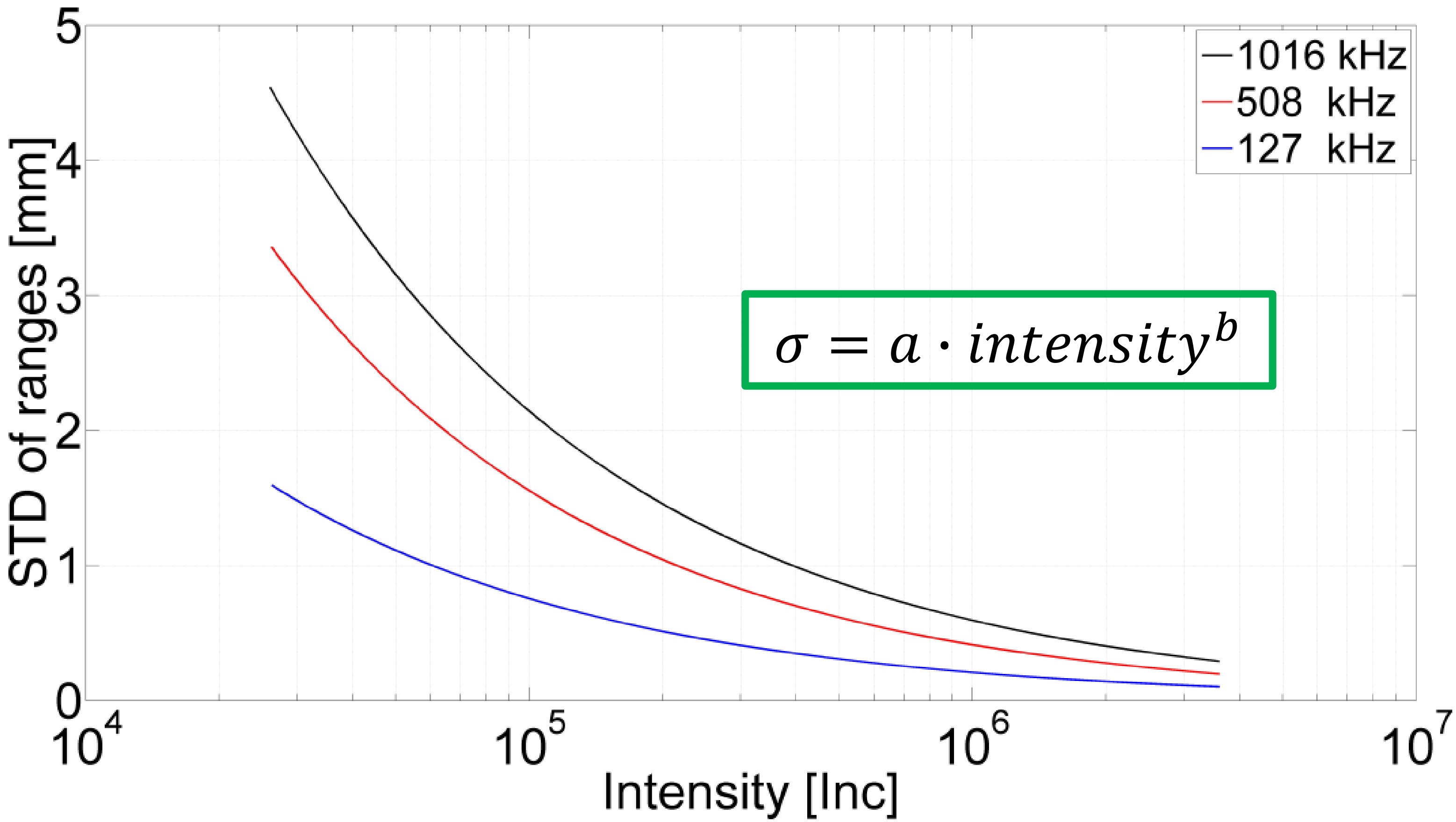
Courtesy: Daniel Wujanz

Angle of incidence and distance



Courtesy: Daniel Wujanz

Rotational speed of laser scanner



$$\sigma = a \cdot intensity^b$$

- a, b to be estimated per scanner model and measurement frequency
- Raw intensities needed, scaled intensities are only limitedly valid

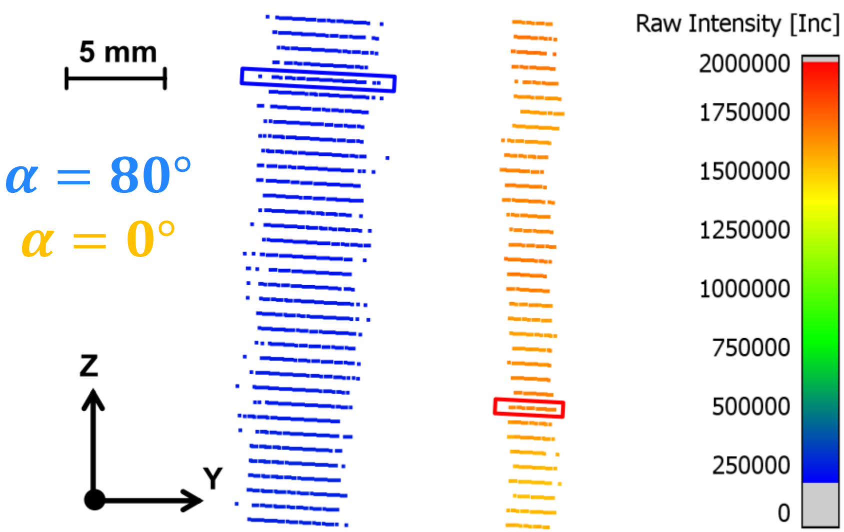
Courtesy: Daniel Wujanz

Estimating the parameters

1D-mode (Wujanz et al. 2017)

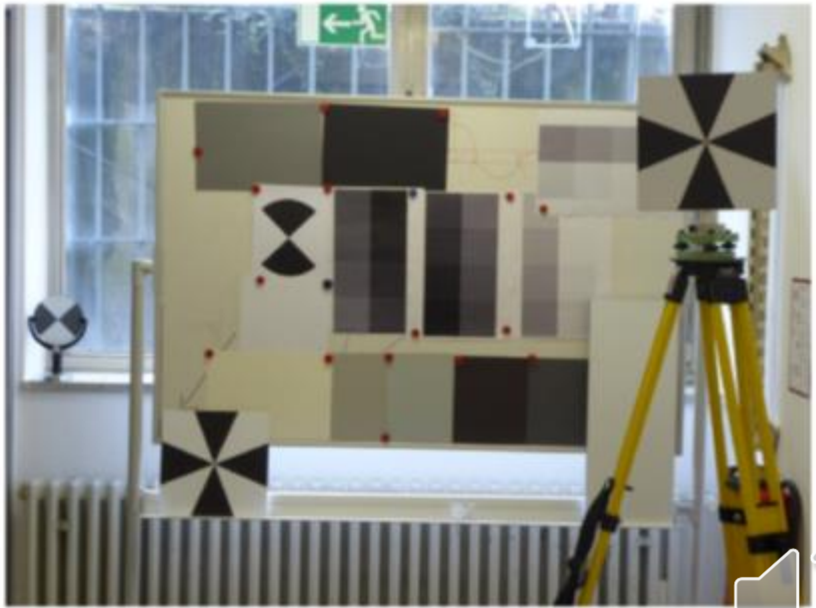
- Repetitive measurements without mirror rotation
- Varying distances, materials, angles of incidence
- Std. dev. + averaged intensity

2D-mode (Heinz et al. 2018)

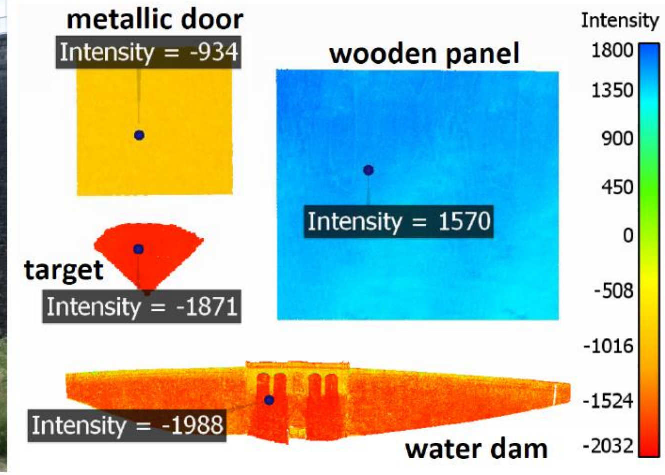


3D-mode (Schmitz et al. 2019)

- Scanning printed pages with varying grey scale
- Fitting of plane throught 3D scans + averaged intensity
- Scaled intensity sometimes also usable



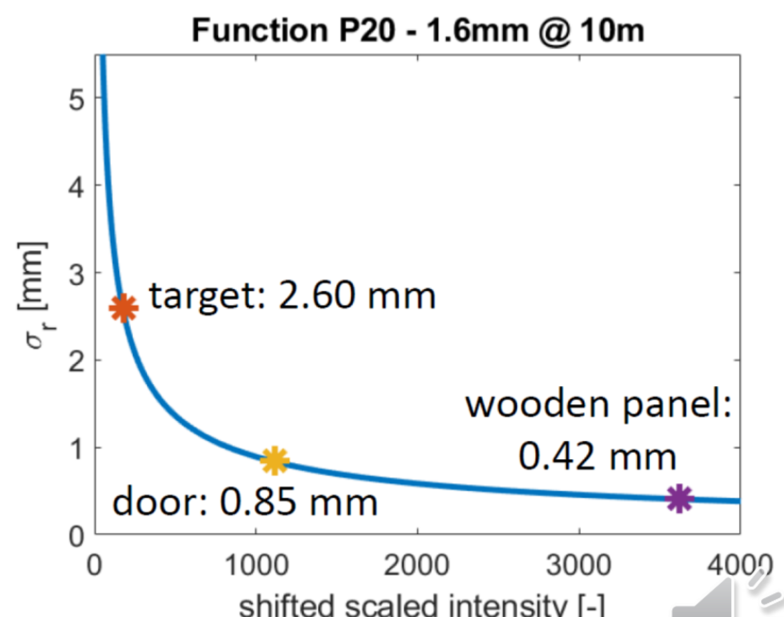
Quantifying random errors



$$\sigma = a \cdot intensity^b$$

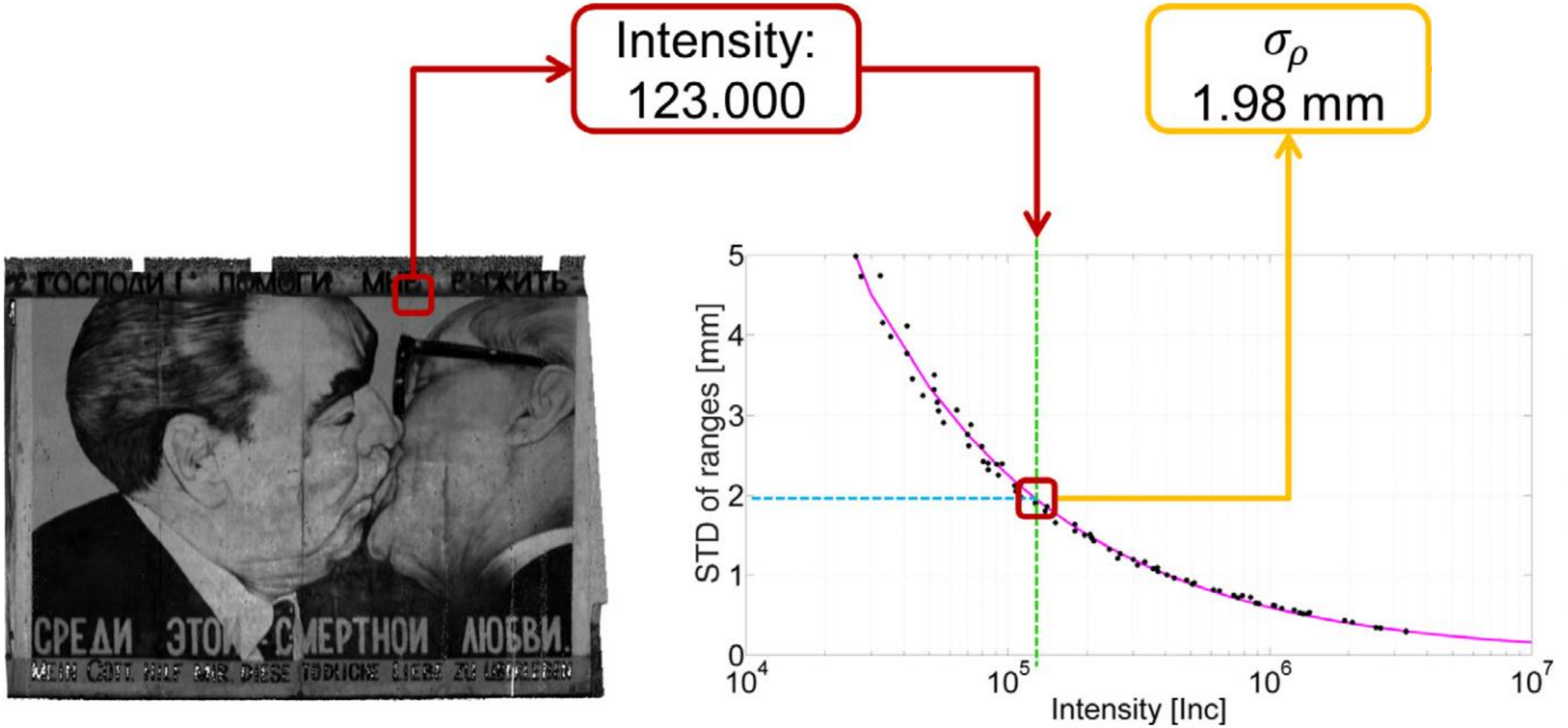
Leica ScanStation P20, 1.6mm @ 10m, Quality level 1

- $a = 60.34 \frac{mm}{inc}, \sigma_a = 3.67 \frac{mm}{inc}$
- $b = -0.61, \sigma_b = 0.01$



Courtesy: Berit Schmitz

Quantifying random errors



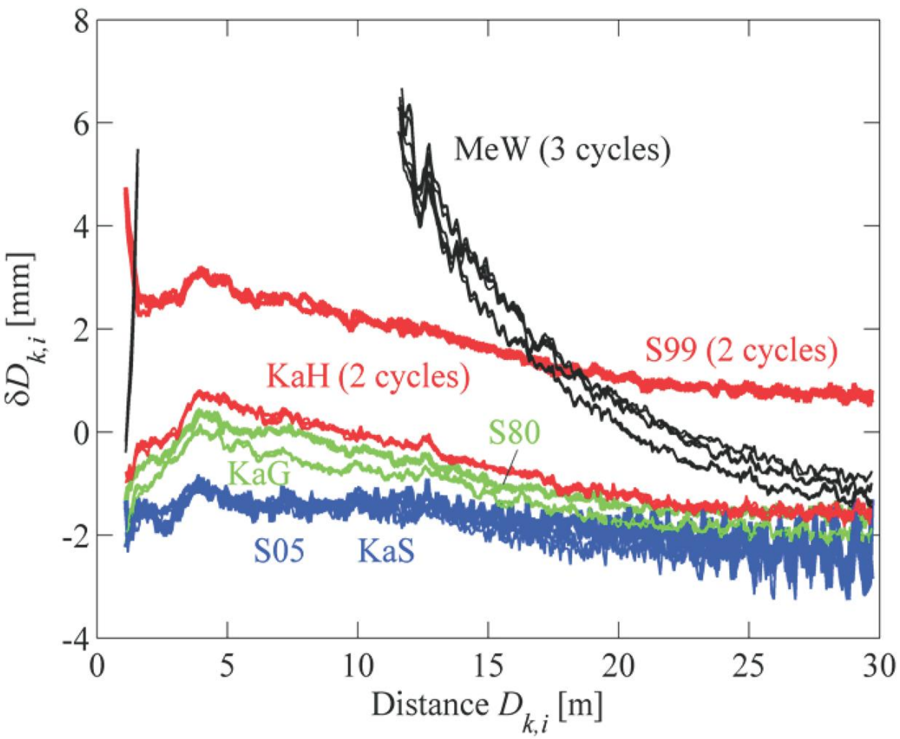
Courtesy: Daniel Wujanz

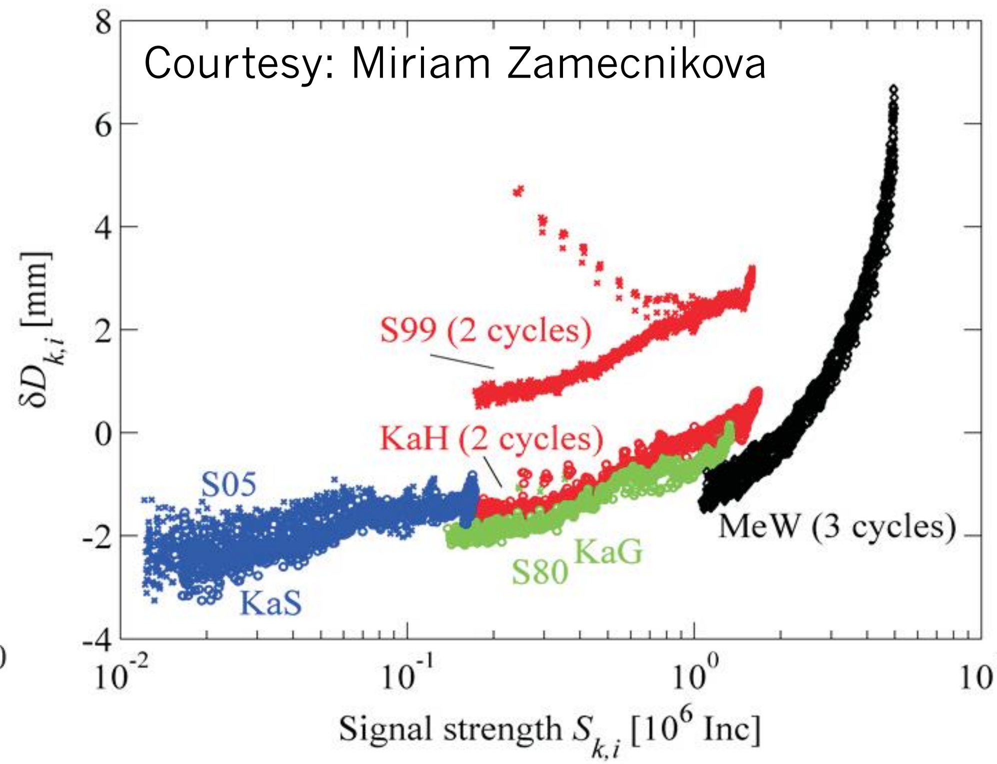
Systematic errors

- Laser Radar Equation only explains random errors
- Further systematic errors exist due to physical and geometric effects

- Errors due to varying **reflectance** of the surface
- Errors due to large **angles of incidence**
- Errors due to transmission / **penetration** of the surface
- Errors due to measurements on **edges**
- Errors due to **mixed pixels**
- Errors due to **total reflection**

Reflectance of the surface



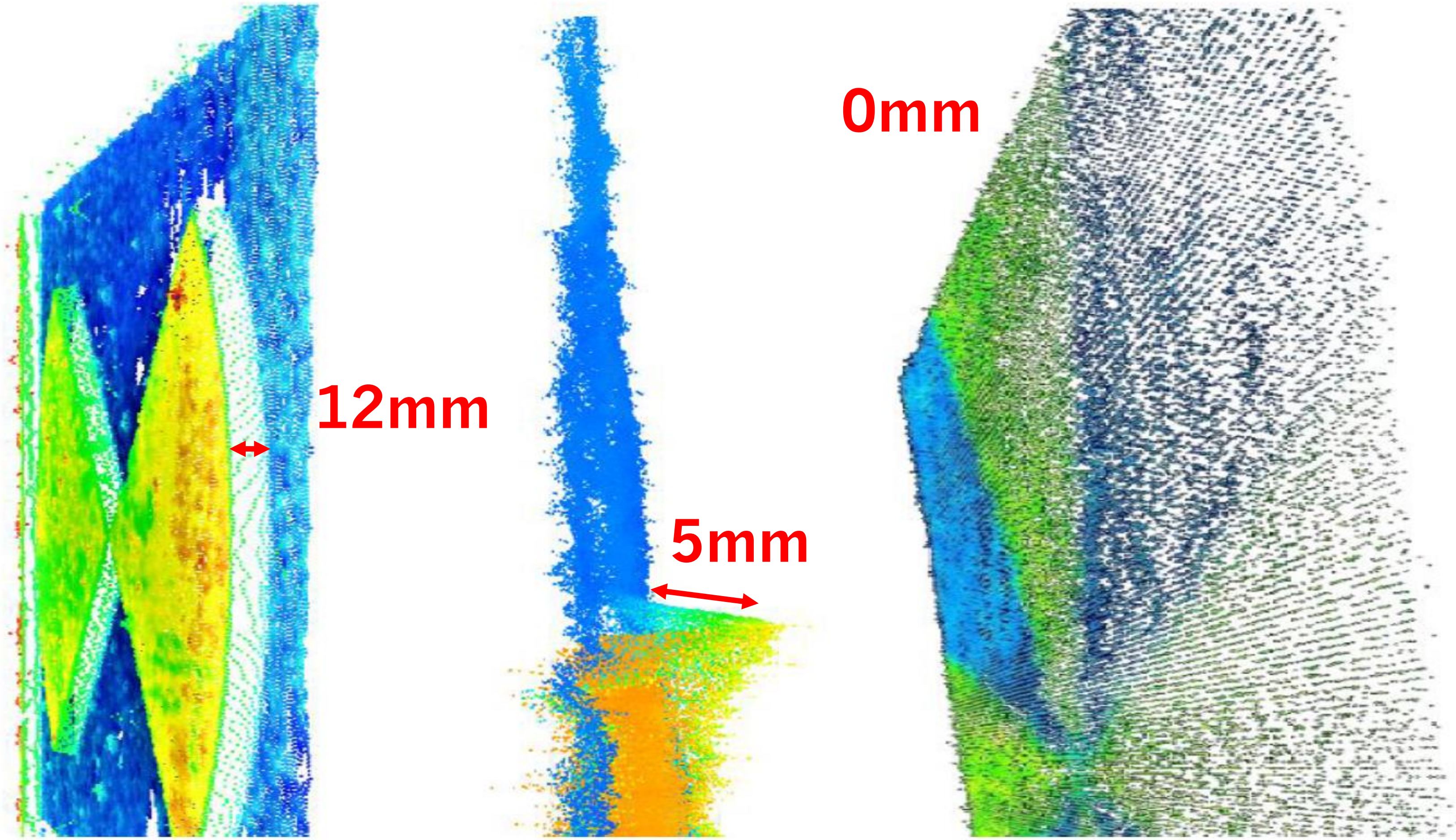


- Up to now no physical parameterization
- Error due to distance and reflectance
- 70% compensatable

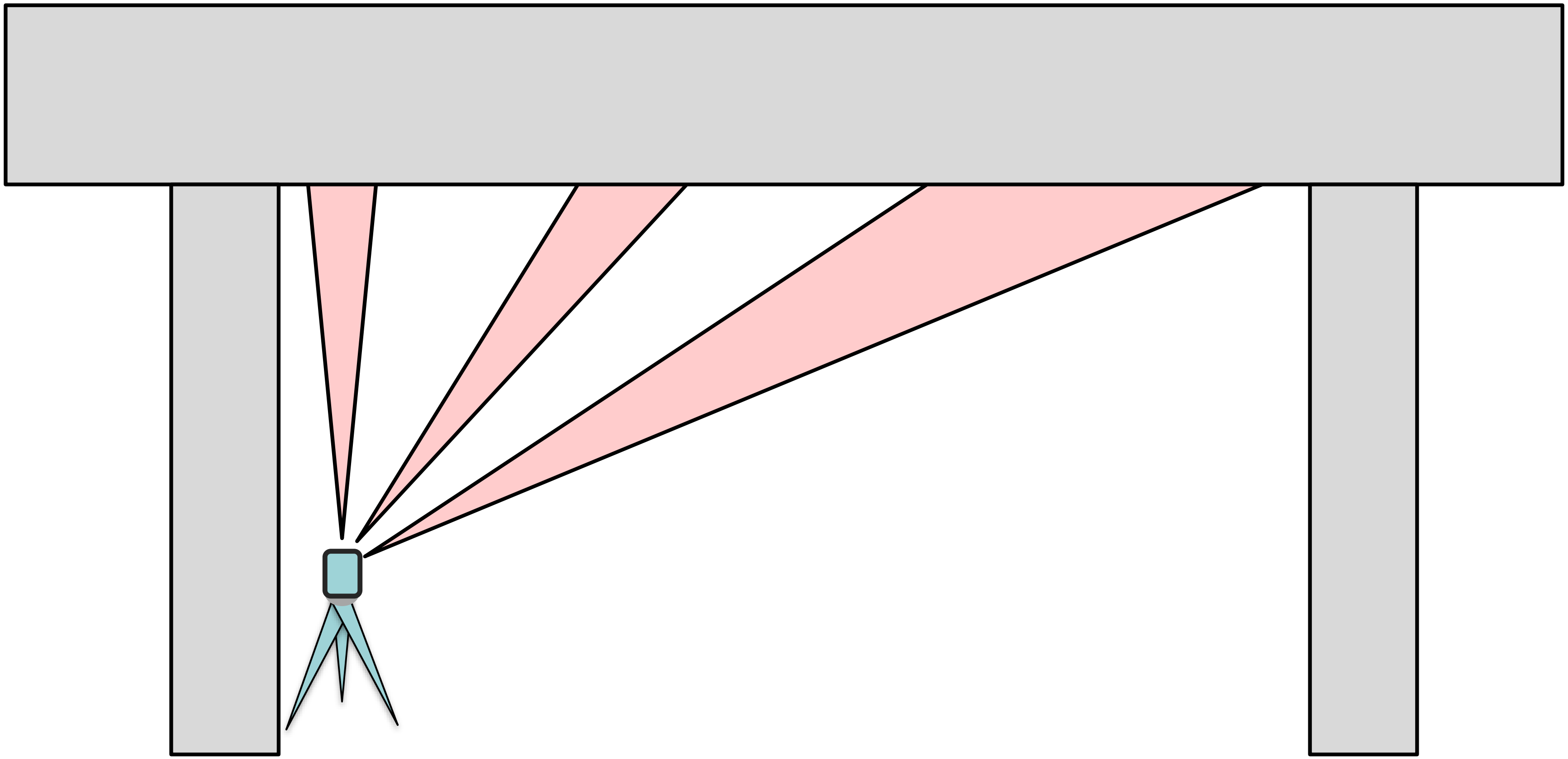
ID	Reflectance†	Material
S99	99 %	Spectralon
S80	87 %	Spectralon
S40	47 %	Spectralon
S05	7 %	Spectralon
KaH	102 %	white cardboard
KaG	81 %	grey cardboard
KaS	10 %	black cardboard
MeW	650 %	aluminum

Reflectance of the surface

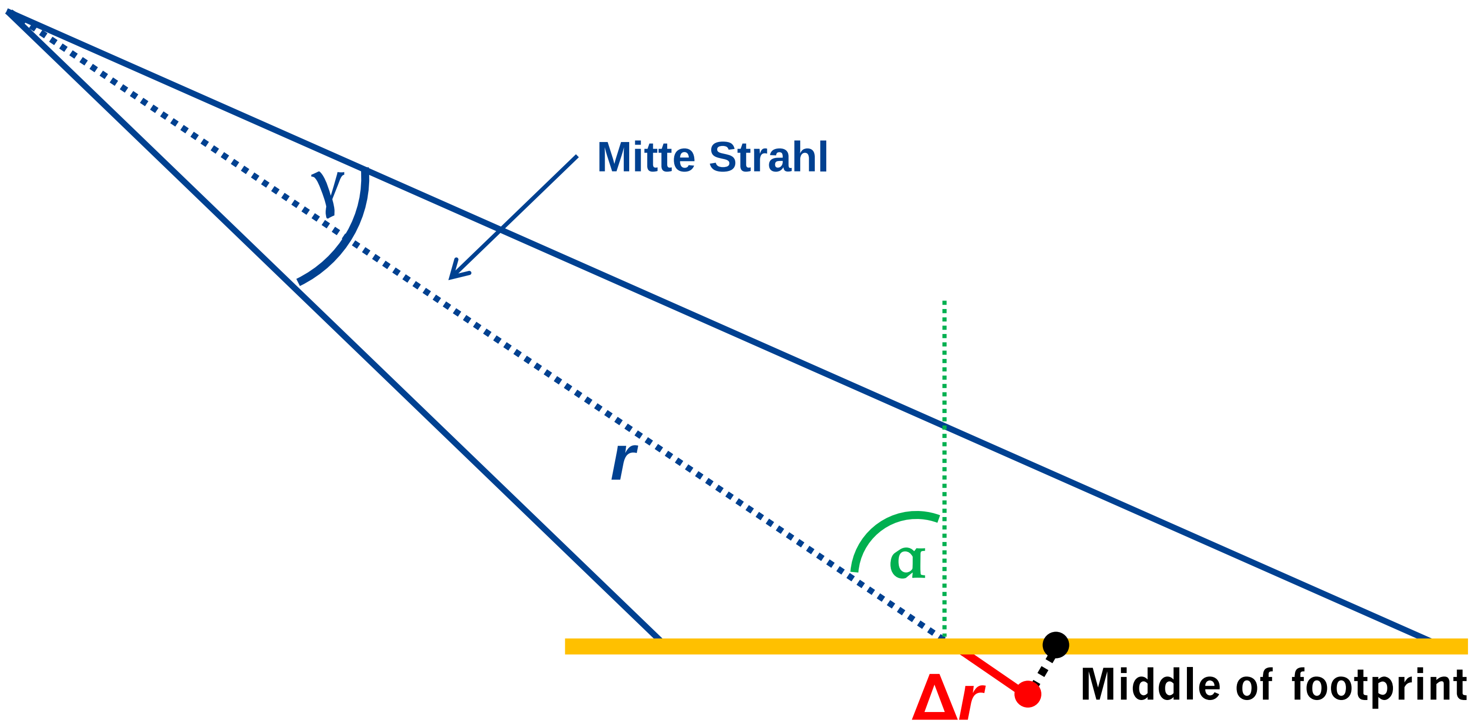
Surphaser 25HSX-IR | Leica P20 | Leica P50



Angle of incidence: geometry



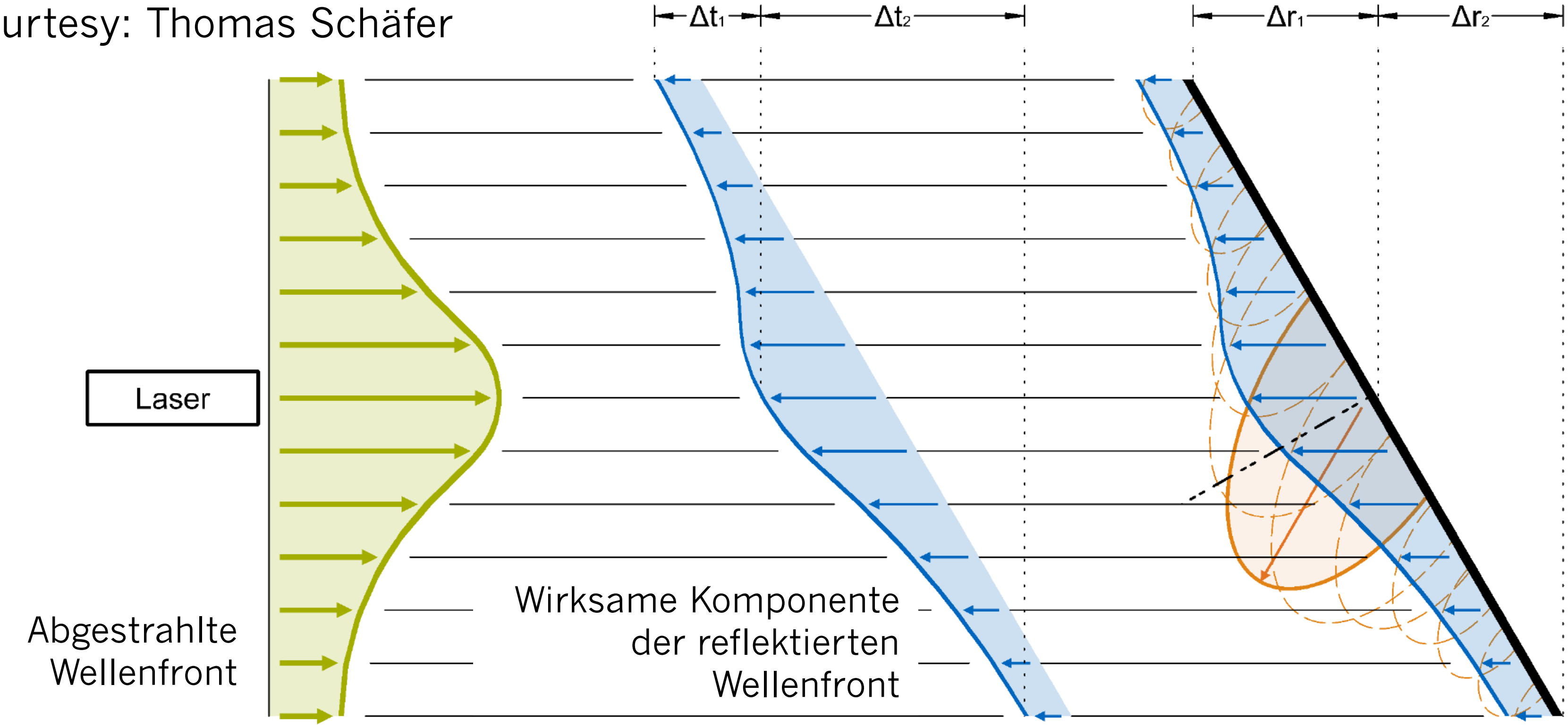
Angle of incidence: geometry



α / r	3,5 m	10 m	20 m	30 m	50 m	70 m	100 m
15°							
30°							
45°					+10 µm	+10 µm	+10 µm
60°			+10 µm	+10 µm	+20 µm	+20 µm	+30 µm

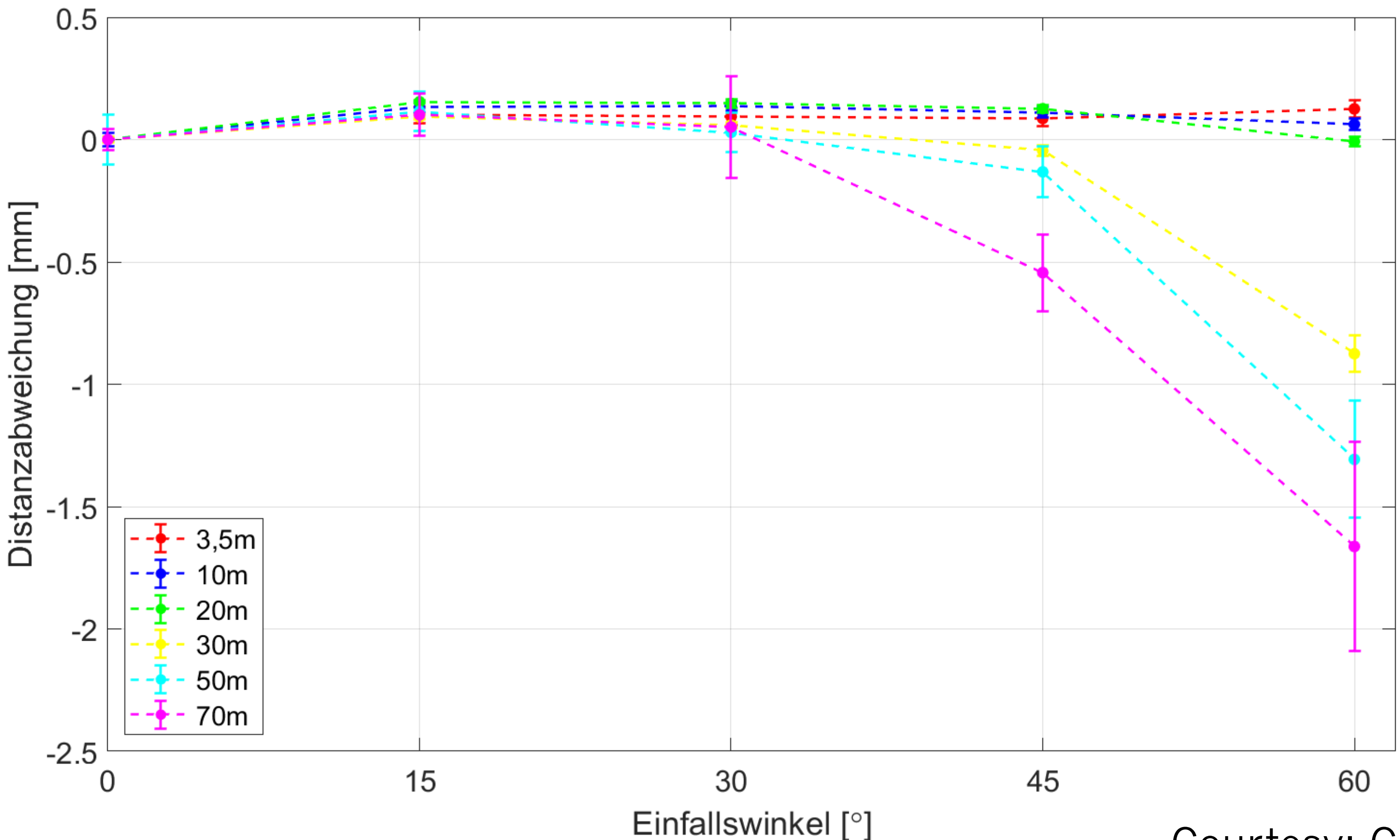
Angle of incidence: physics

Courtesy: Thomas Schäfer



- Deformation of energy wave front if reflected with non-zero angle of incidence
- Intensities are pulled together at Δr_1 and pulled apart at Δr_2
- Intensity distribution at photo detector shifted towards Δr_1 by time shift Δt_1
- Consequence is shortened distance measurement

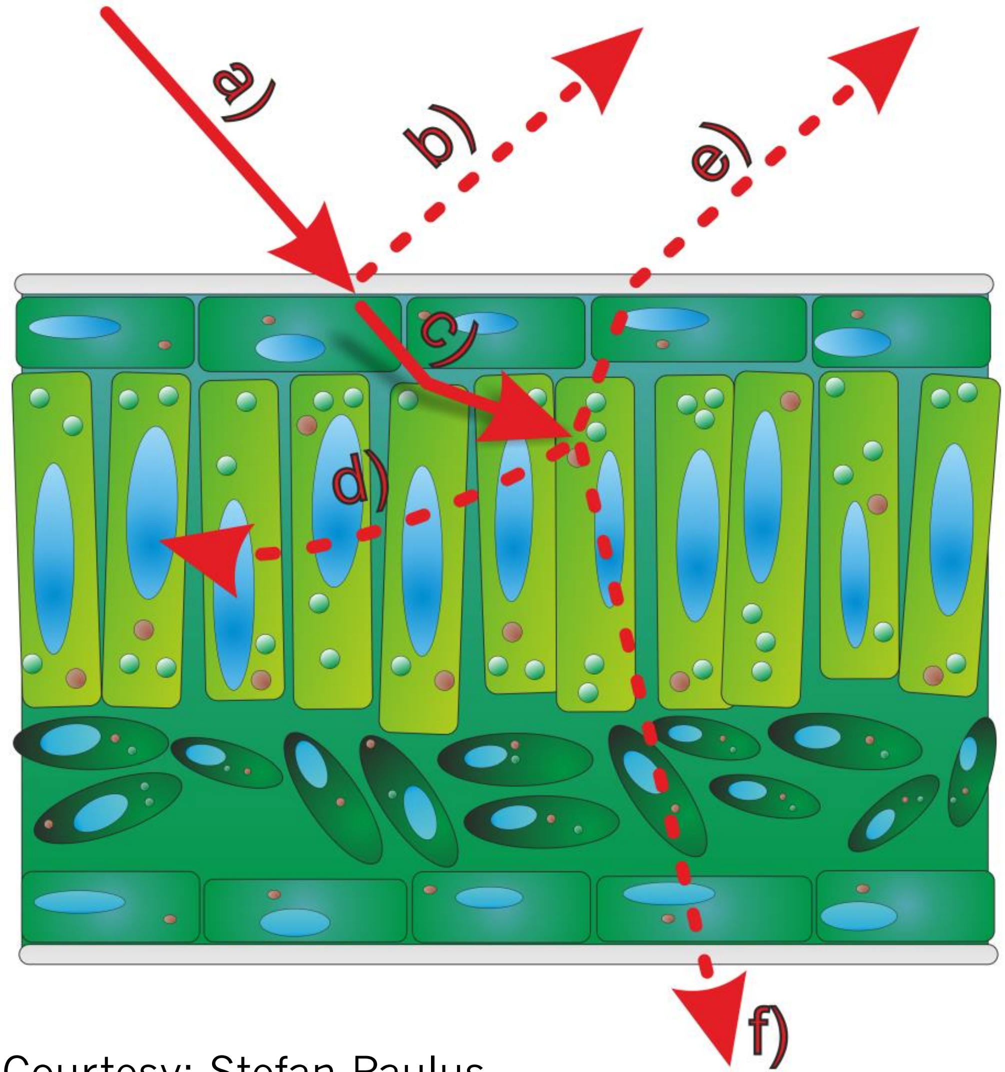
Angle of incidence: empirical values



Courtesy: Carsten Goy

- Bias in distance measurement up to -1.6mm (Z+F Imager 5016)
- Dependent on distance => laser spot size
- Dependent on angle of incidence => laser spot deformation

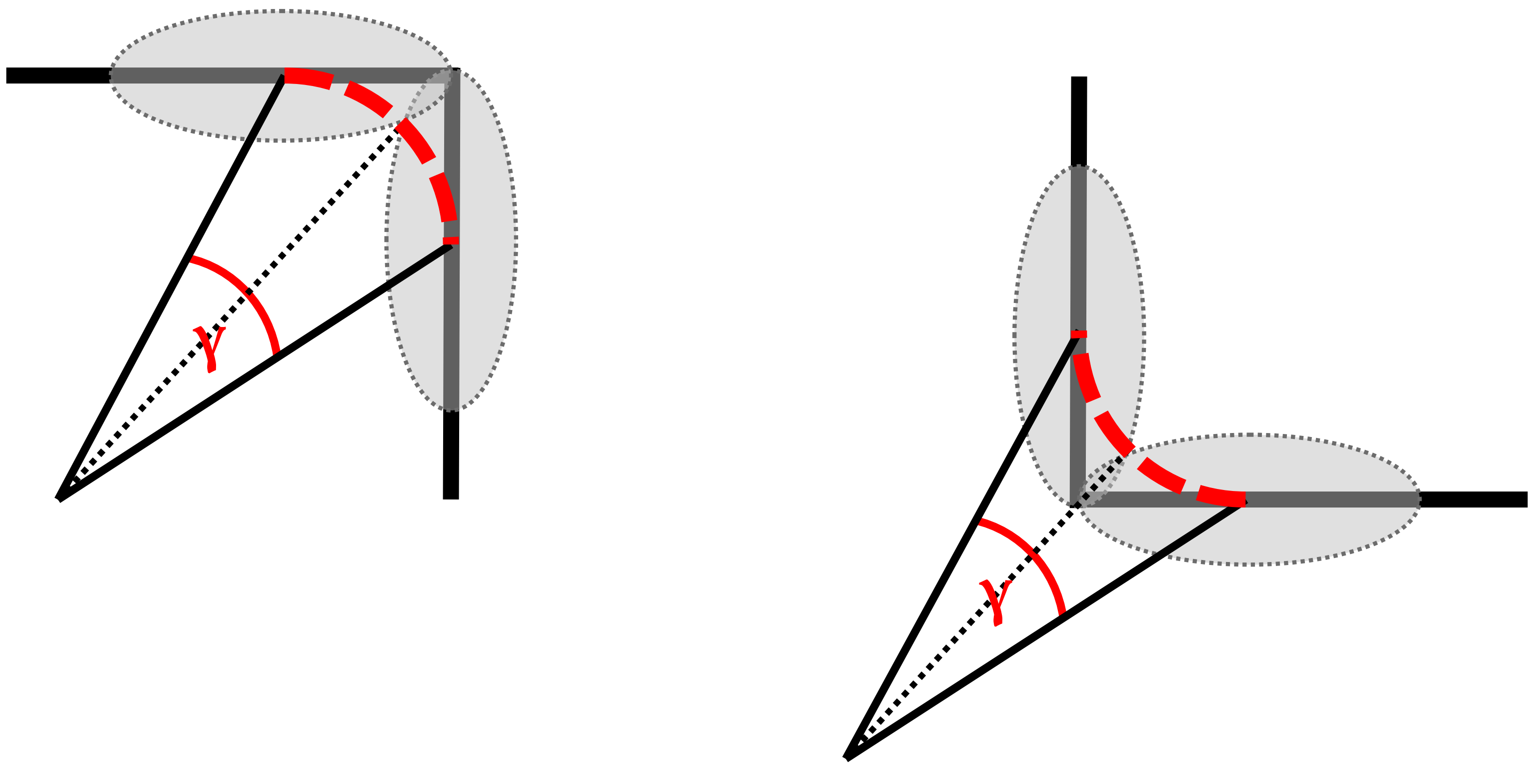
Penetration of the surface



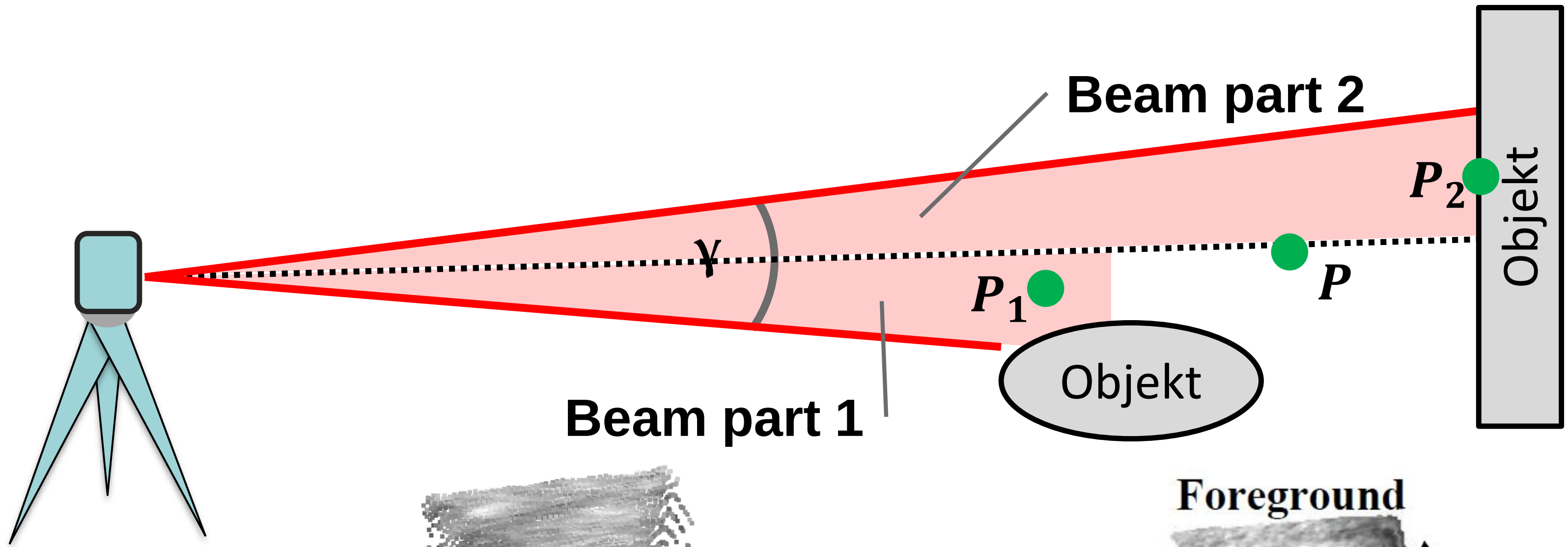
Courtesy: Stefan Paulus

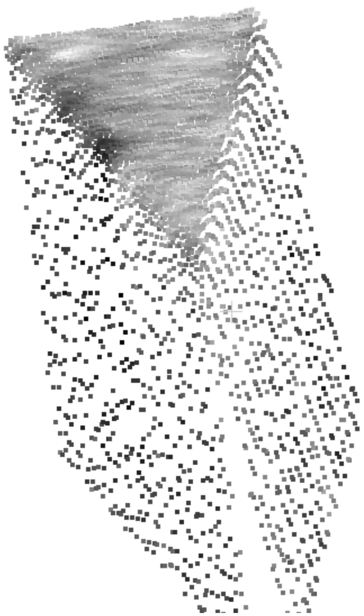
a) Laser beam
b) Reflection at cuticula
c) Transmission through epidermis
d) Absorption by chlorophyll
e) Reflection at cell wall
f) Complete transmission

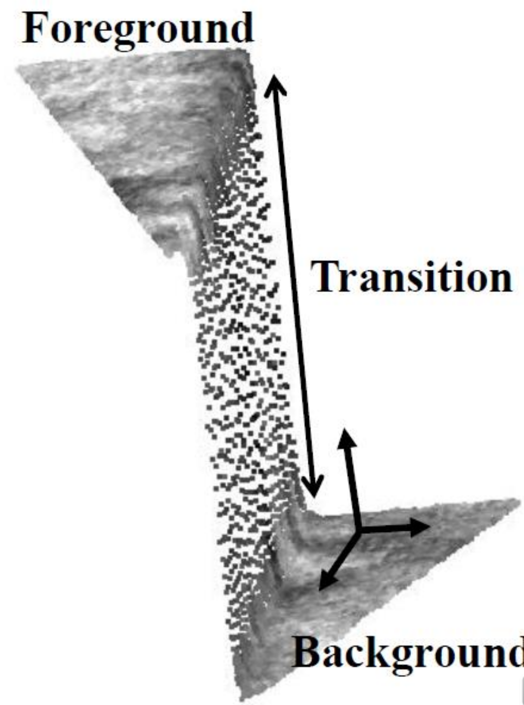
Edge effects



Mixed pixels

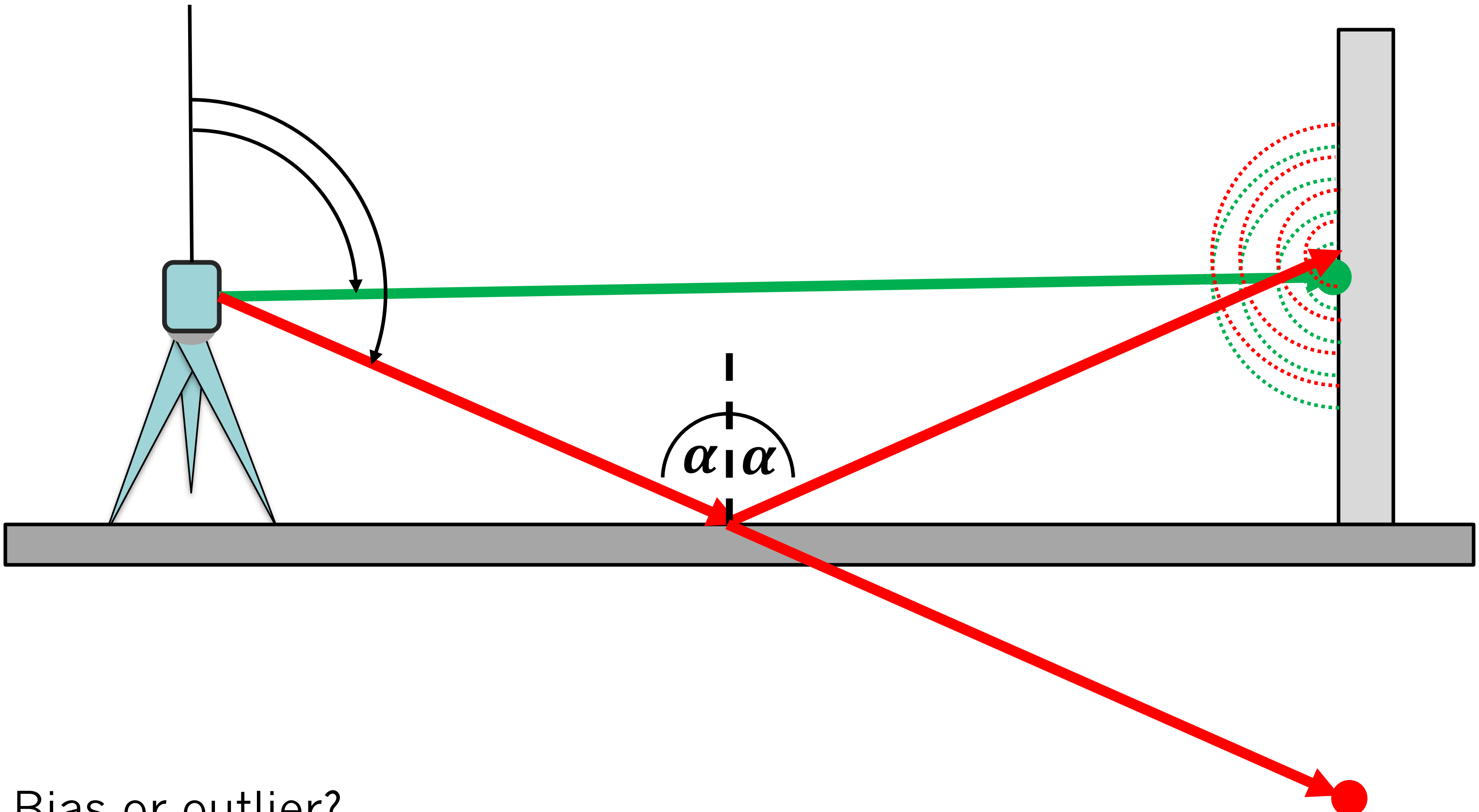






- Bias or outlier?

Total reflection

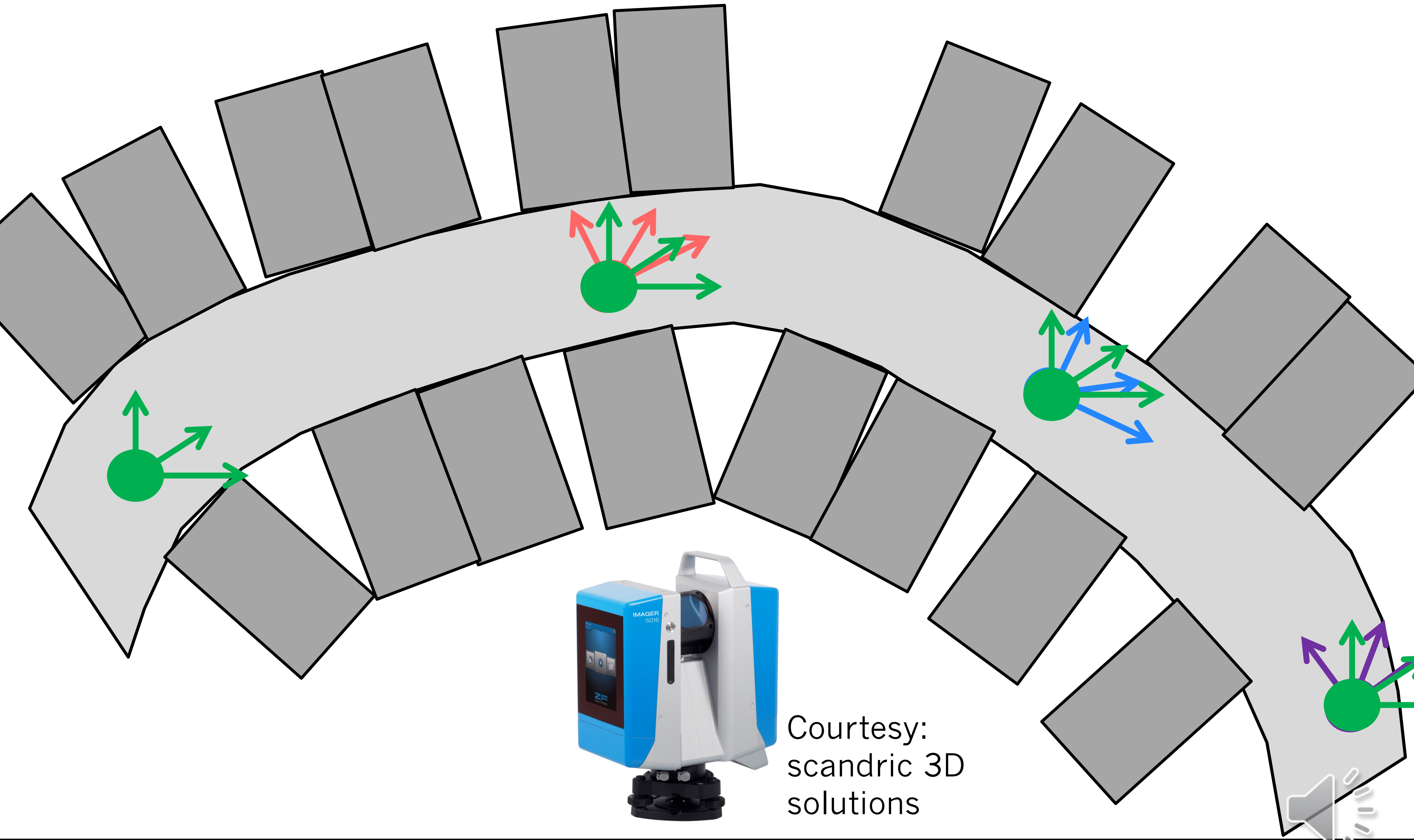


- Bias or outlier?

Agenda

6 Uncertainty of point clouds

Static laser scanning

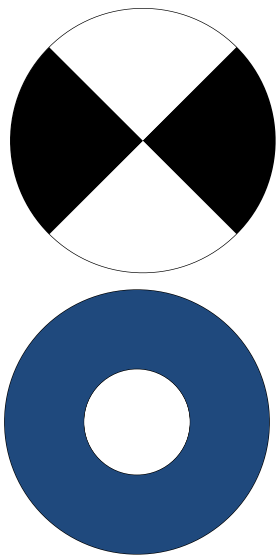


Courtesy:
scandric 3D
solutions

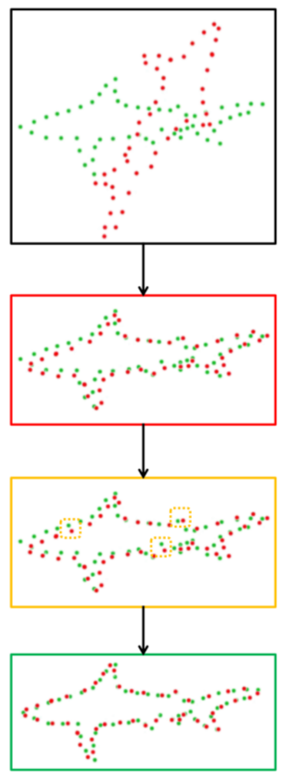
Geo-referencing 3D point clouds

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^g = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \boldsymbol{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot \begin{bmatrix} r \cdot \sin\theta \cdot \sin\varphi \\ r \cdot \sin\theta \cdot \cos\varphi \\ r \cdot \cos\theta \end{bmatrix}^s$$

Target based

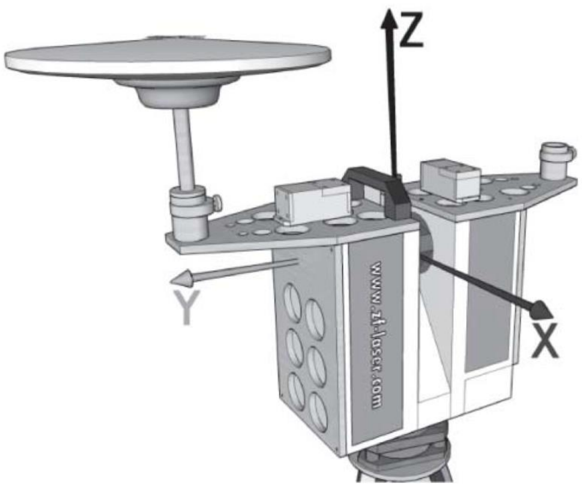


Point cloud based



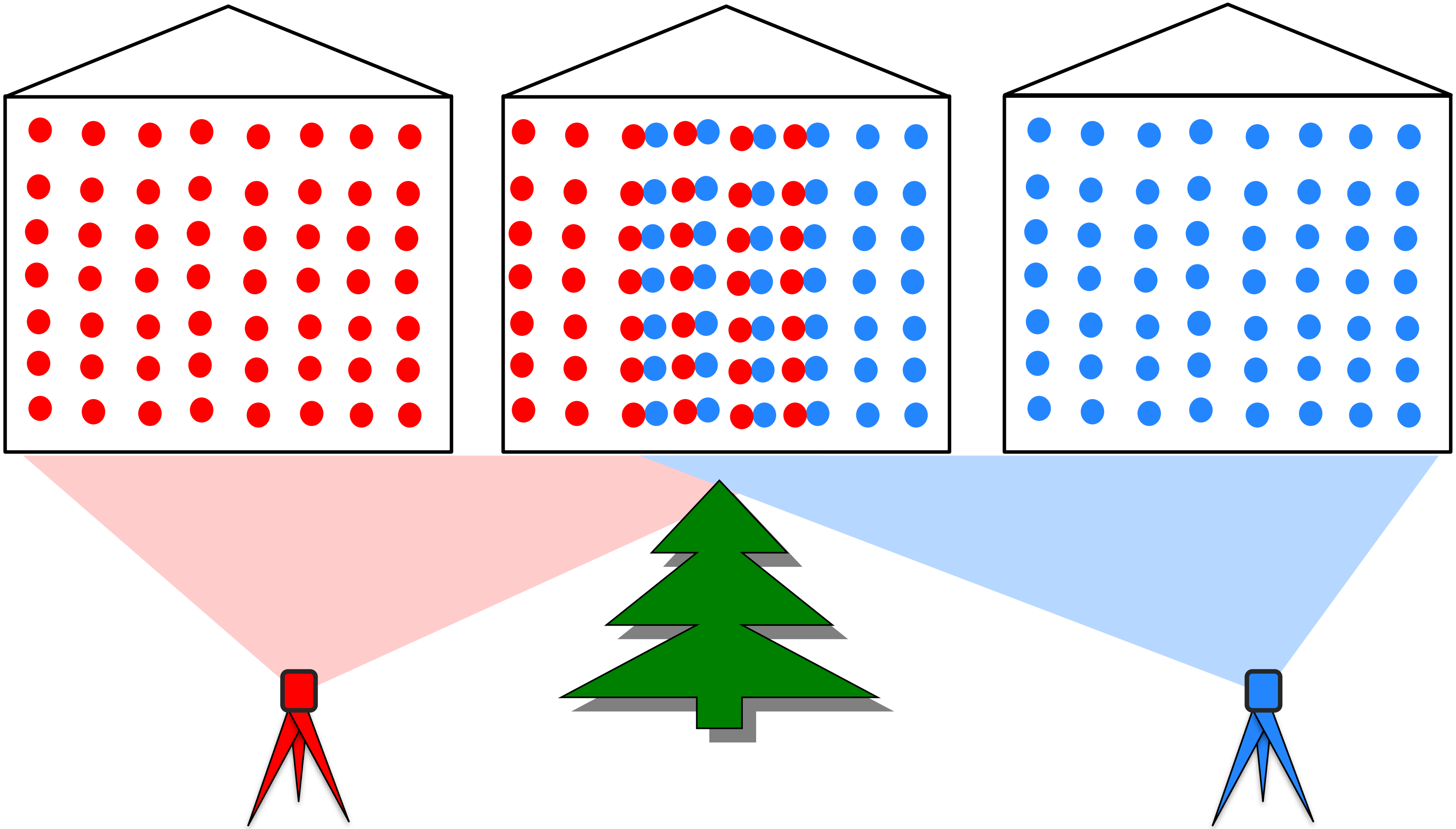
Courtesy: Daniel Wujanz

Hardware based



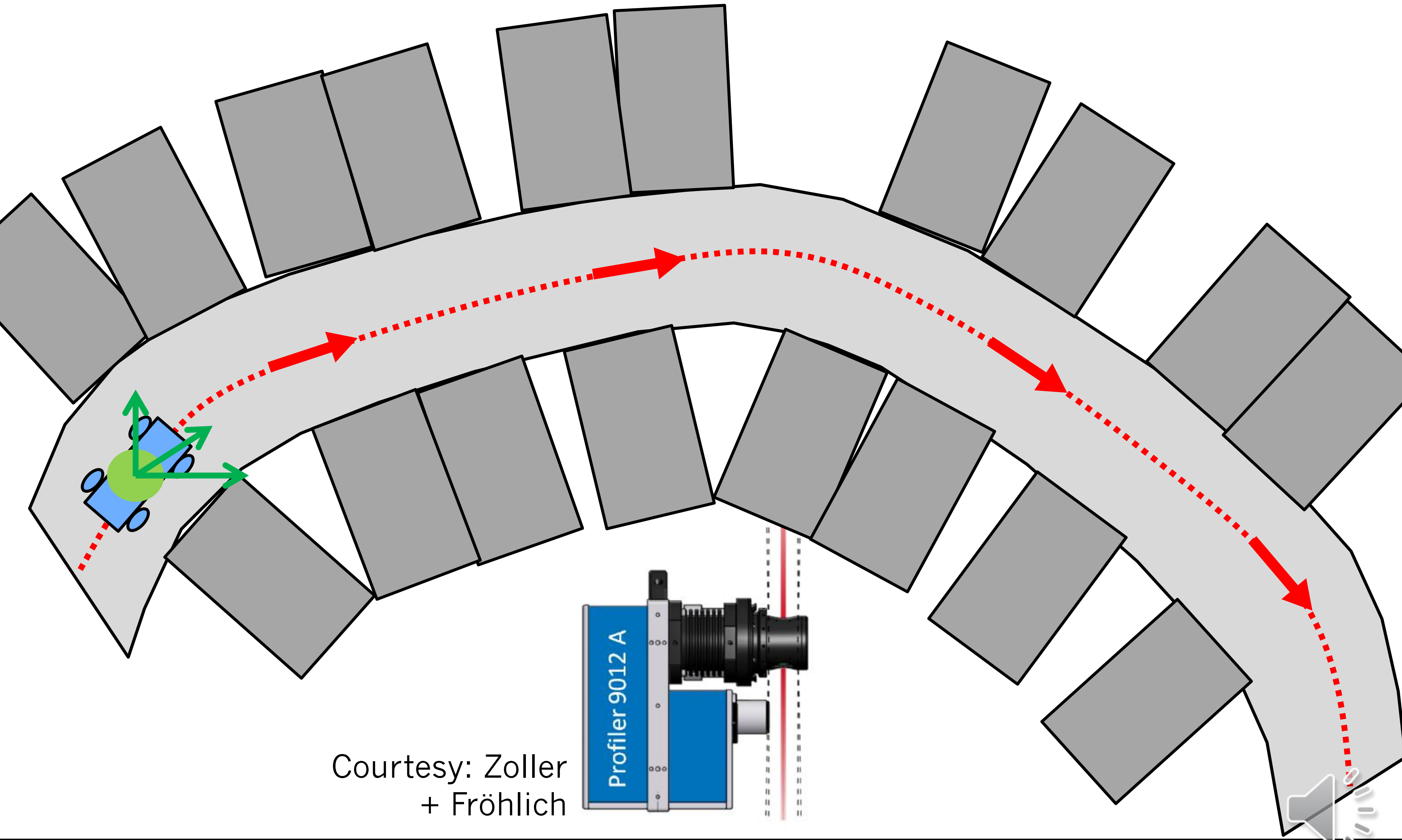
Courtesy: Jens-André Paffenholz

Impact of uncertainty



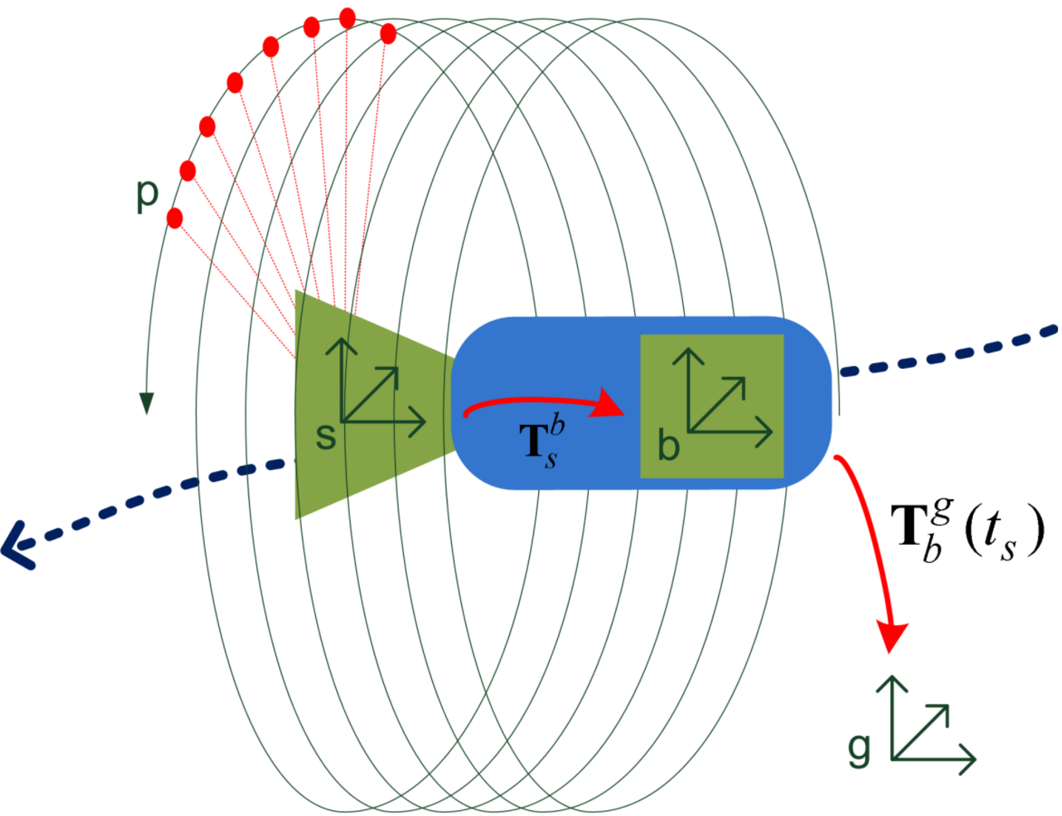
- **Bias / absolute accuracy: complete scans**
- **Precision / relative accuracy: only at overlaps**

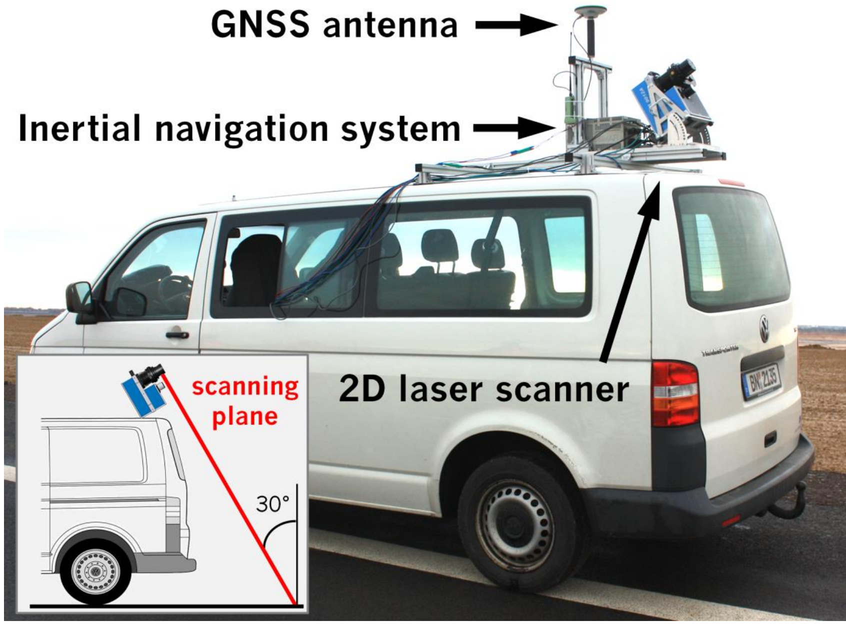
Mobile laser scanning



Courtesy: Zoller
+ Fröhlich

Mobile laser scanning = profile scan





Courtesy: Lasse Klingbeil

Geo-referencing: static vs. mobile

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^g = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \boldsymbol{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot \begin{bmatrix} r \cdot \sin\theta \cdot \sin\varphi \\ r \cdot \sin\theta \cdot \cos\varphi \\ r \cdot \cos\theta \end{bmatrix}^s$$

A) Transformation from scanner to geo-referenced system

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^g = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \boldsymbol{R}_n^e(L, B) \cdot \boldsymbol{R}_b^n(\phi, \theta, \psi) \cdot \left(\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \boldsymbol{R}_s^b(\alpha, \beta, \gamma) \cdot \begin{bmatrix} 0 \\ d \cdot \sin b \\ d \cdot \cos b \end{bmatrix}^s \right)$$

A) Transformation from scanner to body system

B) Transformation from body to geo-referenced system

Geo-referencing: static vs. mobile

Trafo s to g

Geo-referenced 3D point

Local 3D point

$$\boldsymbol{p}^{\boldsymbol{g}} = \mathbf{T}_{\boldsymbol{s}}^{\boldsymbol{g}} \cdot \boldsymbol{p}^{\boldsymbol{s}}$$

Local 2D point

$$\boldsymbol{p}^{\boldsymbol{g}} = \boldsymbol{T}_{\boldsymbol{b}}^{\boldsymbol{g}}(t_S) \cdot \boldsymbol{T}_{\boldsymbol{s}}^{\boldsymbol{b}} \cdot \boldsymbol{p}^{\boldsymbol{s}}$$

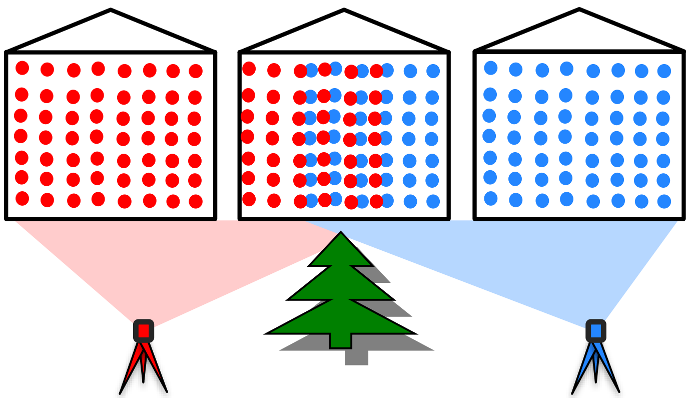
Trafo b to g (for each time step t_s individually)

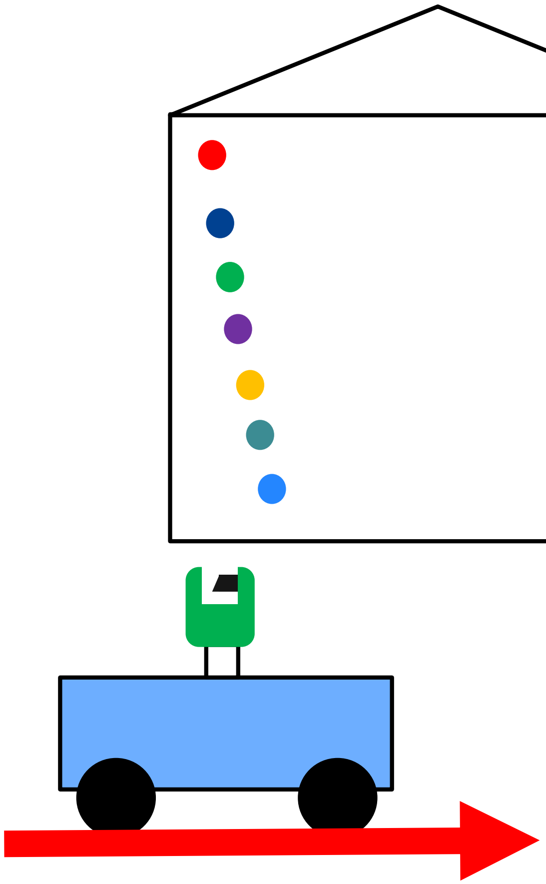
Trafo from s to b (temporally stable)

=> Trajectory estimation

=> System calibration

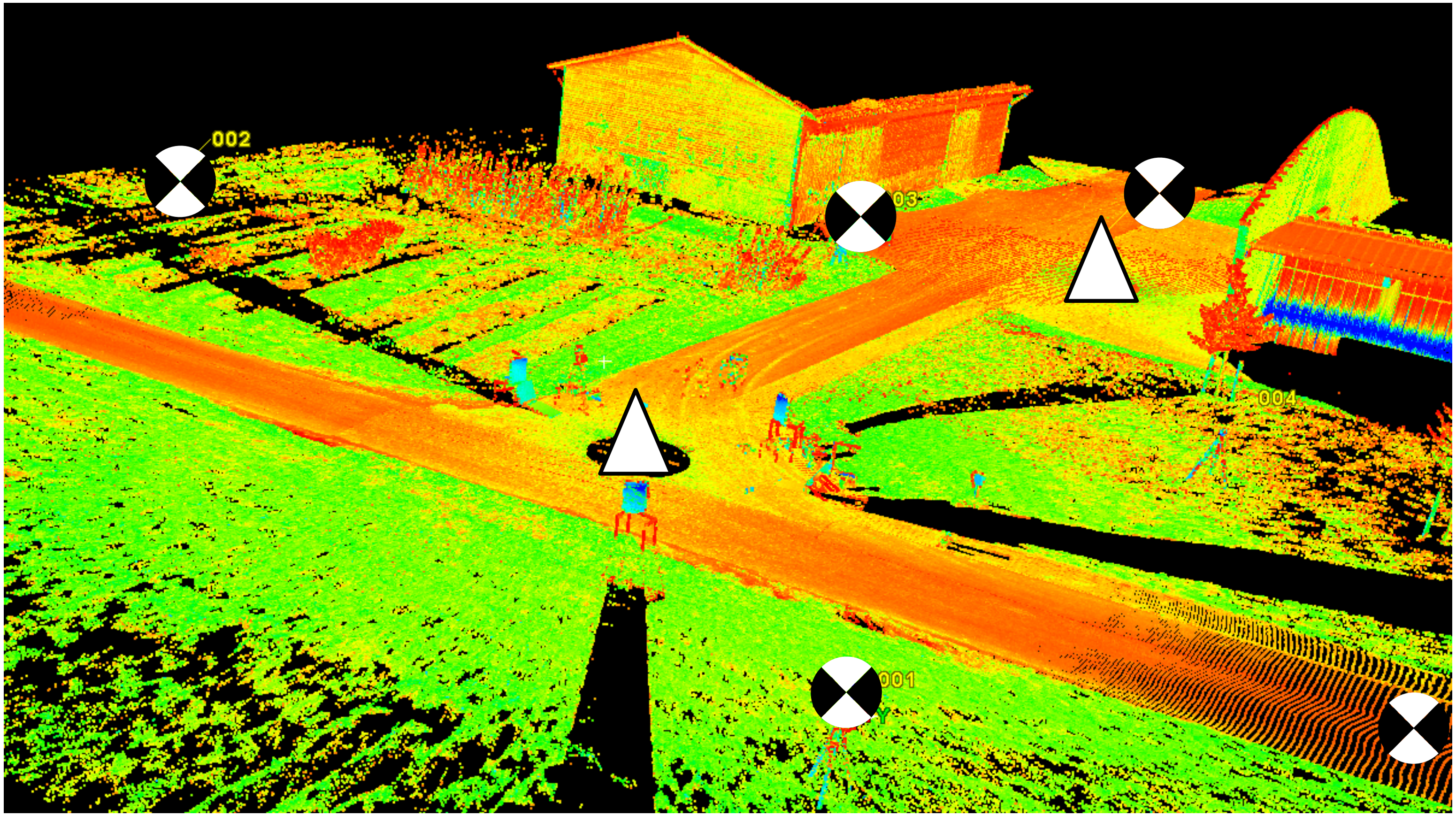
Impact of uncertainty

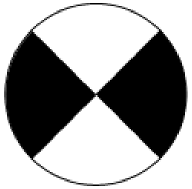




- **Bias / absolute accuracy: complete scans**
- **Precision / relative accuracy: complete scans**

Numerical example: static

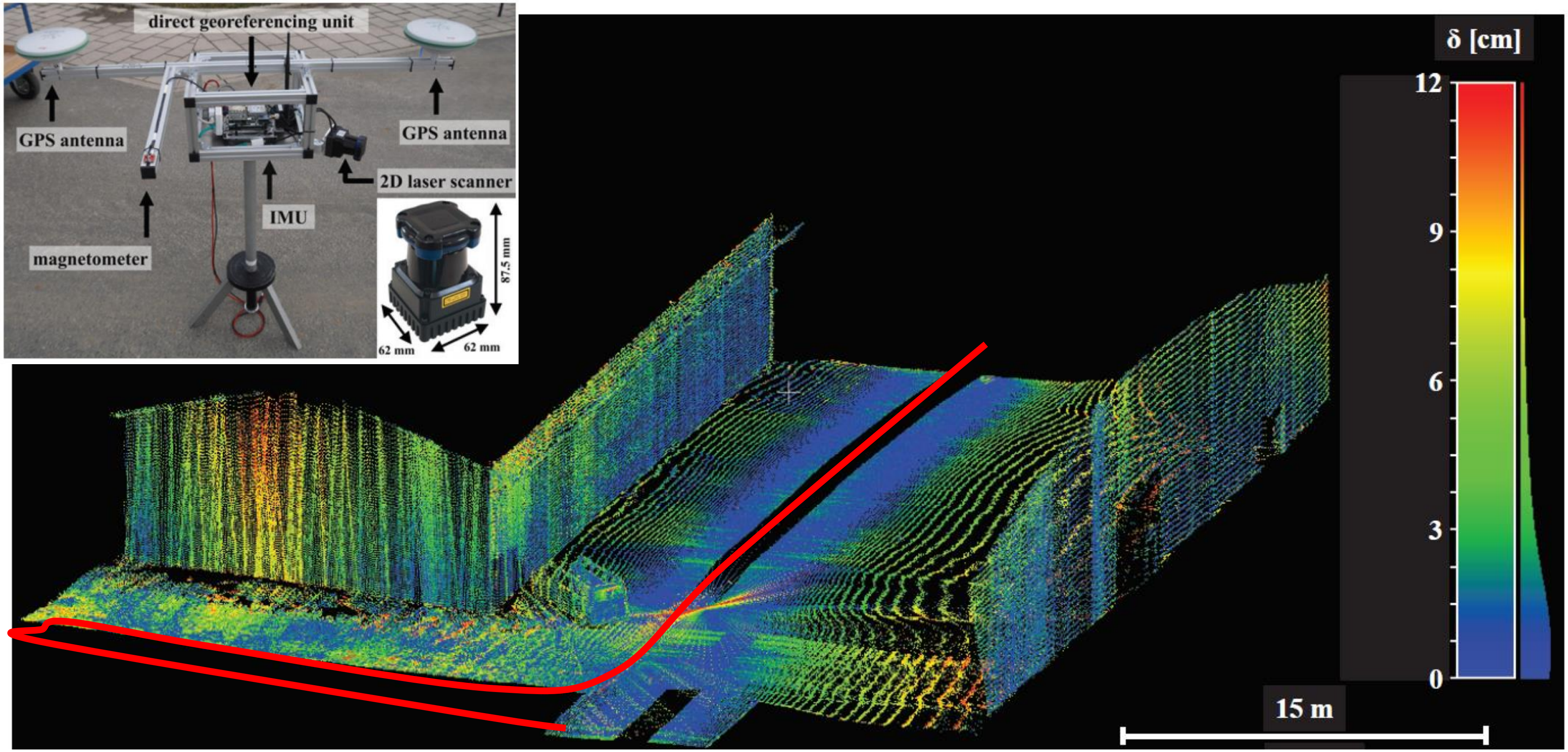




- Geo-referencing using targets
- Uncertainty: several mm

Courtesy: Erik Heinz

Numerical example: mobile



- Cloud2Cloud comparison: errors < 12 cm
- Uncertainty due to geo-referencig of mobile scans: systematic GPS-errors

Courtesy: Erik Heinz

Interim summary

- Uncertainty of geo-referencing occurs when transforming local scan points into another coordiante system
- Static laser scanning
  - Geo-referencing of complete 3D point clouds
  - Uncertainty potentially lower
  - Affecting bias all over the point cloud
  - Affecting precision only at overlap of laser scans
- Mobile laser scanning
  - Geo-referencing of individual 2D points
  - Uncertainty potentially higher (trajectory estimation, system calibration)
  - Affecting bias and precision all over the point cloud

6 Uncertainty of point clouds

Characterization of error types

	Outliers	Systematic errors	Random errors
Appear-ance	Extremely erroneous individual measurements	One-sided effects	Gaußian distributed (positive and negative, more often small than large)
Source	Mistakes of operator, Inappropriate used of instrument	Imperfection of measurement procedure, insufficient calibration of instrument	Fluctuations not trackable by calibration, uncontrolled changes of measured object and environment
Minimi-zing	Avoidance by care and control	**Calibration, math. compensation, measurem. strategies**	**Averaging by multiple measurements**
Quanti-tative measure	–	Bias / absolute accuracy	Precision / relative accuracy
	Uncertainty		

Minimizing systematic errors

Category	Element	Random	Systematic
Geometry	Distance	x	
	Angle of impact	x	x
Object	Smoothness	x	x
	Reflectance	x	x
Atmosphere	Propagation delay		x
	Refraction		x
Instrument	Construction	x	x / x
Geo-referencing	*various*	x	x / x / x

- **Calibration**
- **Mathematical compensation**
- **Measurement strategies**

In the end:

Randomized errors v

6 Uncertainty of point clouds

6.1 Basics

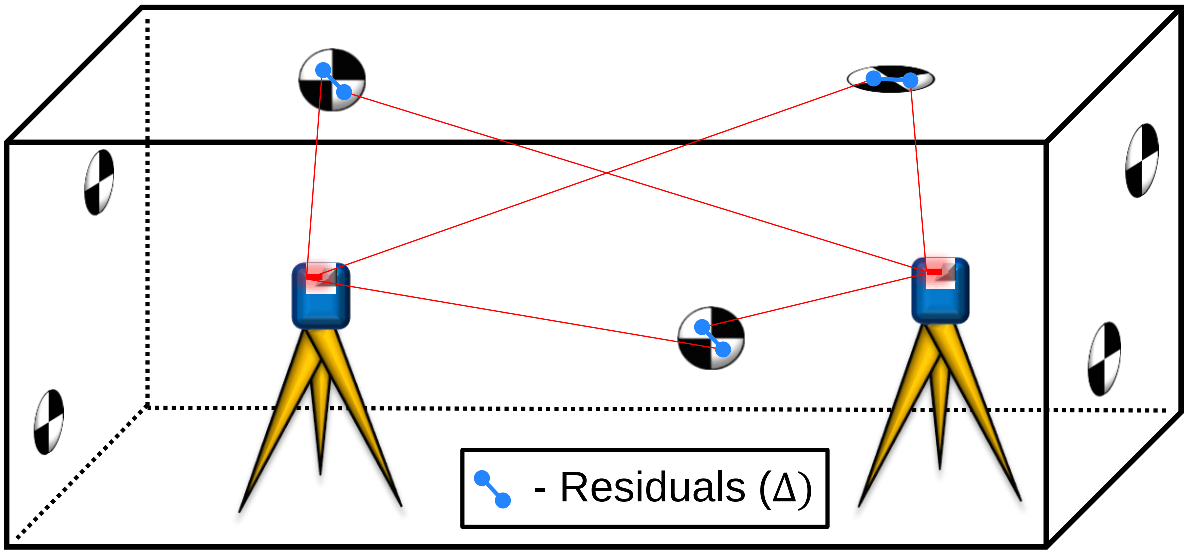
6.2 Error sources at laser scanning

6.3 Strategies for minimizing the uncertainty

6.3.1 Instrumental errors

6.3.2 Geo-referencing errors

A) Calibrating the laser scanner



$$\Delta r_j^i = x_2 \sin(\theta_j^i) + x_{10} + v_{r_j^i}$$

$$\Delta \varphi_j^i = \frac{x_{1z}}{r_j^i \tan(\theta_j^i)} + \frac{x_3}{r_j^i \sin(\theta_j^i)} + \frac{x_{5z-7}}{\tan(\theta_j^i)} + \frac{2x_6}{\sin(\theta_j^i)} + \frac{x_{1n}}{r_j^i} + v_{\varphi_j^i}$$

$$\Delta \theta_j^i = \frac{x_{1n+2} \cos(\theta_j^i)}{r_j^i} + x_4 + x_{5n} \cos(\theta_j^i) - \frac{x_{1z} \sin(\theta_j^i)}{r_j^i} - x_{5z} \sin(\theta_j^i) + v_{\theta_j^i}$$

Estimation of parameters

$$f_i(\boldsymbol{l},\boldsymbol{x}) = \boldsymbol{R}(\varepsilon_x,\varepsilon_y,\varepsilon_z)\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}_j^i + \begin{bmatrix}t_x\\t_y\\t_z\end{bmatrix} - \begin{bmatrix}x\\y\\z\end{bmatrix}_{j+k}^i = 0$$

$i = 1, \dots, \#\,\text{Targets}$

$j, j+k \dots \text{Stations}$

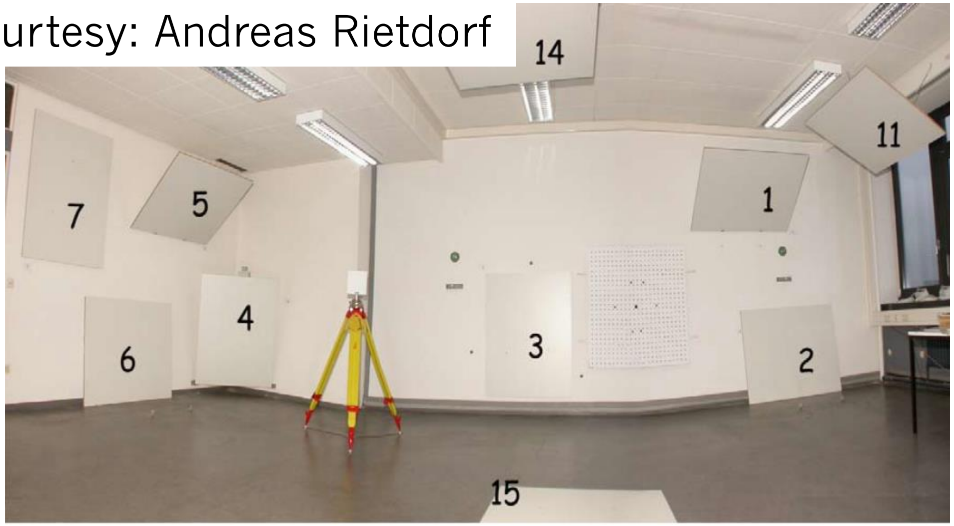
$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}r\cdot\sin\theta\cdot\sin\varphi\\ r\cdot\sin\theta\cdot\cos\varphi\\ r\cdot\cos\theta\end{bmatrix}$$

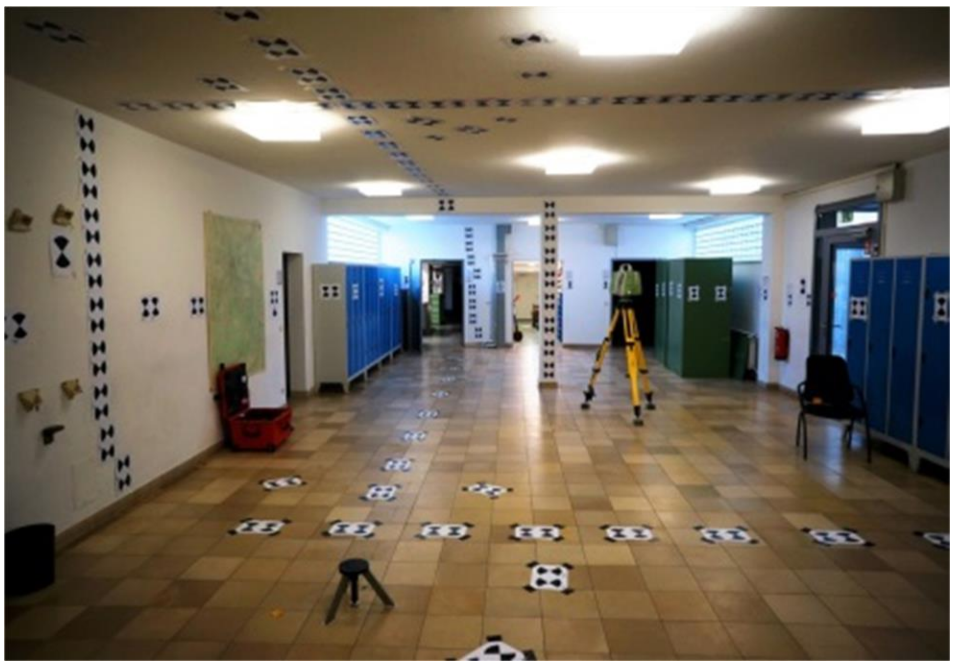
$$r = \bar{r} + \Delta r$$
$$\varphi = \bar{\varphi} + \Delta\varphi$$
$$\theta = \bar{\theta} + \Delta\theta$$

(Registration – for $j + k = 2$ – or Bundle Block Adjustment)

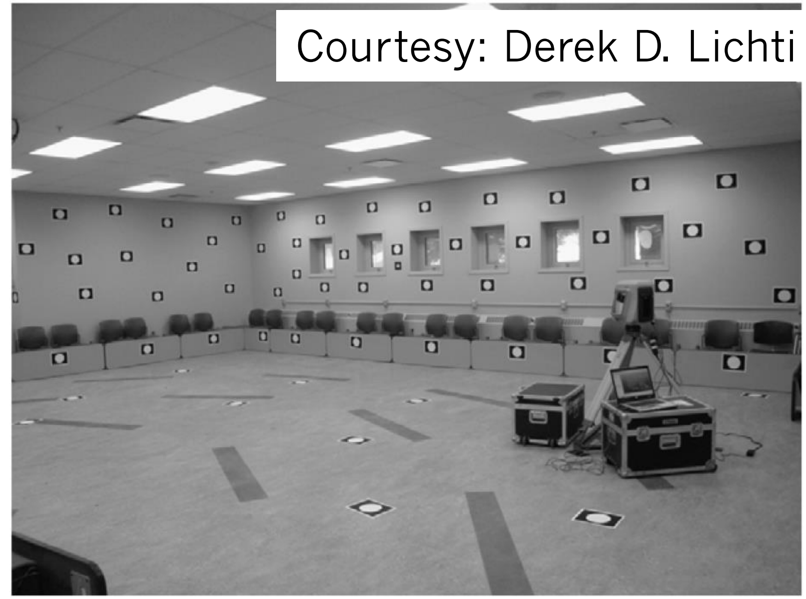
Measurement configuration

Courtesy: Andreas Rietdorf

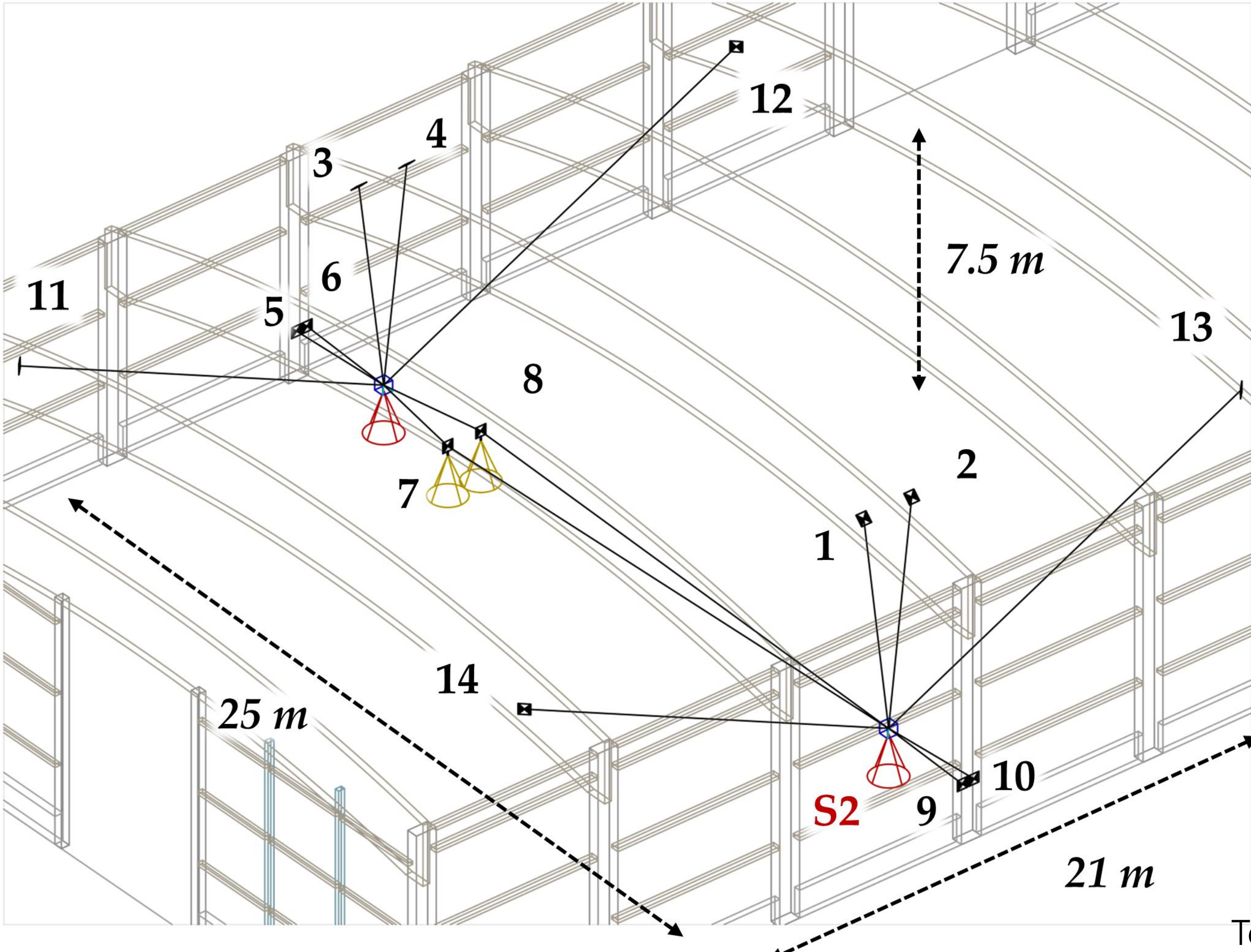




Courtesy: Derek D. Lichti



Calibration field of University of Bonn



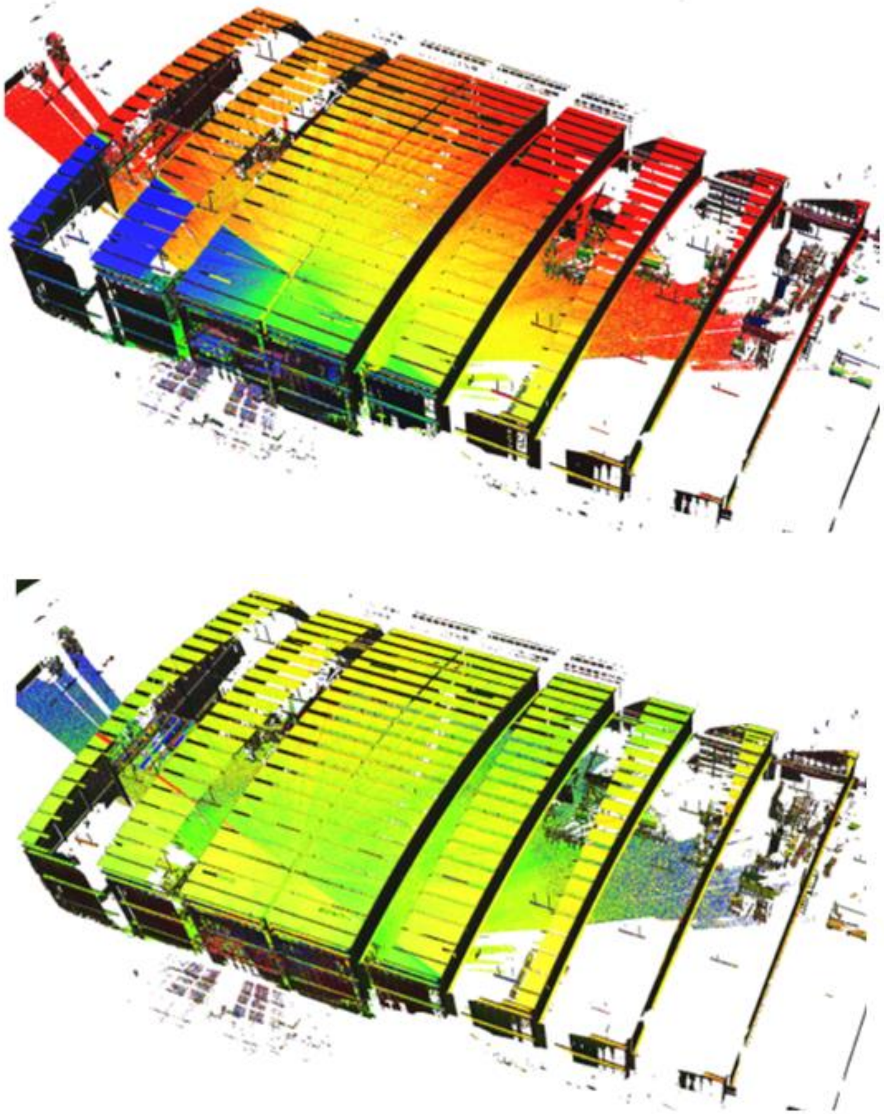
Courtesy:
Tomislav Medic

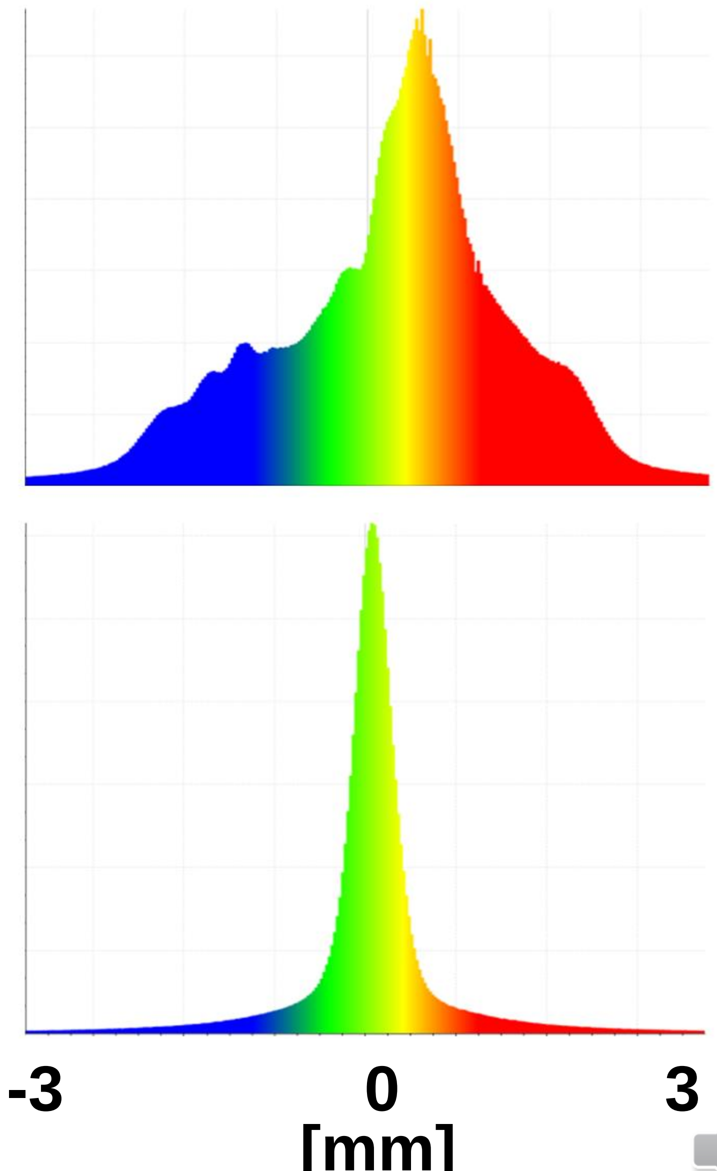
Estimated calibration parameters

Parameter	Leica P50	Imager 5016
x_{10} [mm] (Nullpunktabw.)	-0.03	0.37
x_2 [mm]	-0.14	0.03
x_{1z} [mm]	0.14	0.59
x_3 [mm]	-0.03	-0.15
x_7 [“] (Kippachsabw.)	-33.50	26.49
x_6 [“] (Zielachsabw.)	3.41	0.40
x_{1n} [mm]	0.01	-0.45
x_4 [“] (Höhenindexabw.)	4.51	-6.44
x_{5n} [“]	5.38	12.74
x_{5z} [“]	-16.78	2.66

$\hat{\sigma} \approx 0.5''$ / $\hat{\sigma} \approx 0.5mm$ **(high correlations!)**

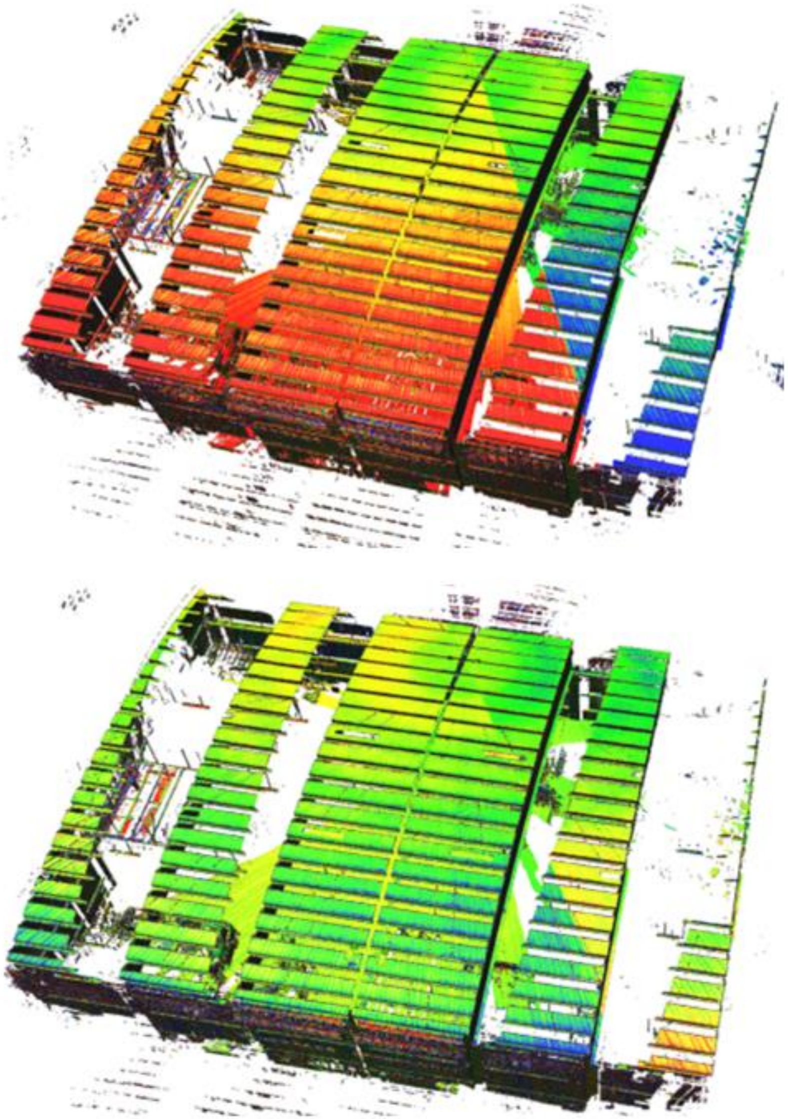
Leica P50: Face 1 minus Face 2

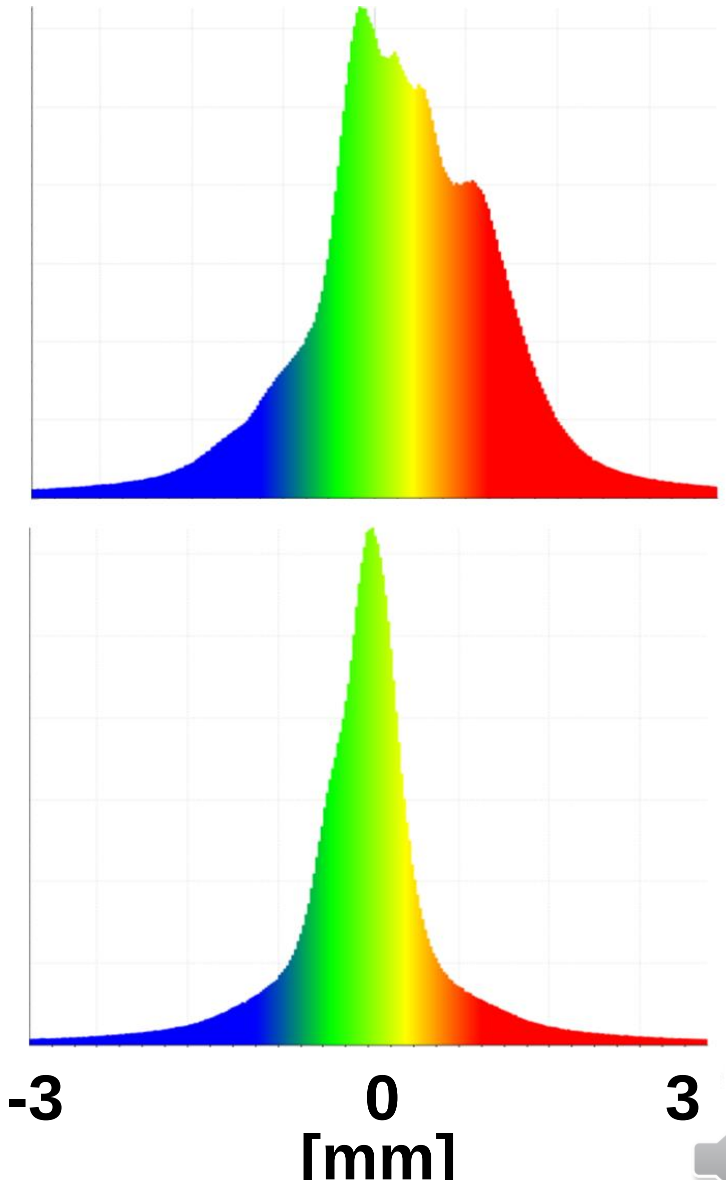




Courtesy: Tomislav Medic

Z+F Imager 5016: Face 1 minus Face 2

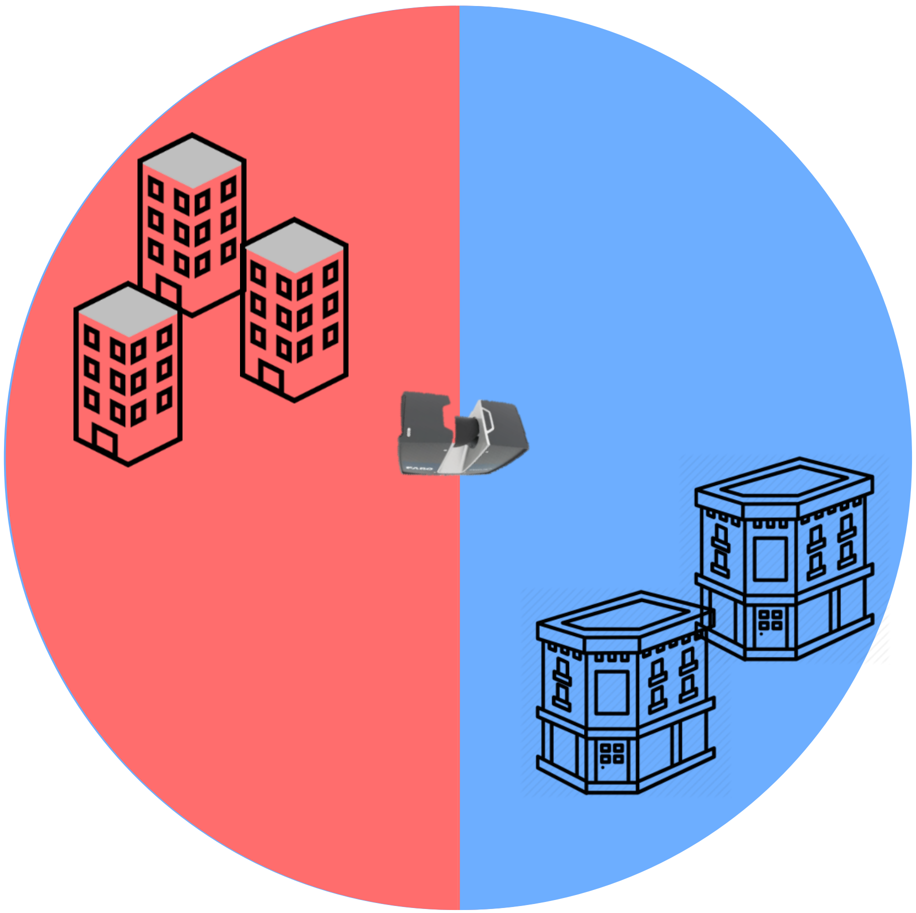




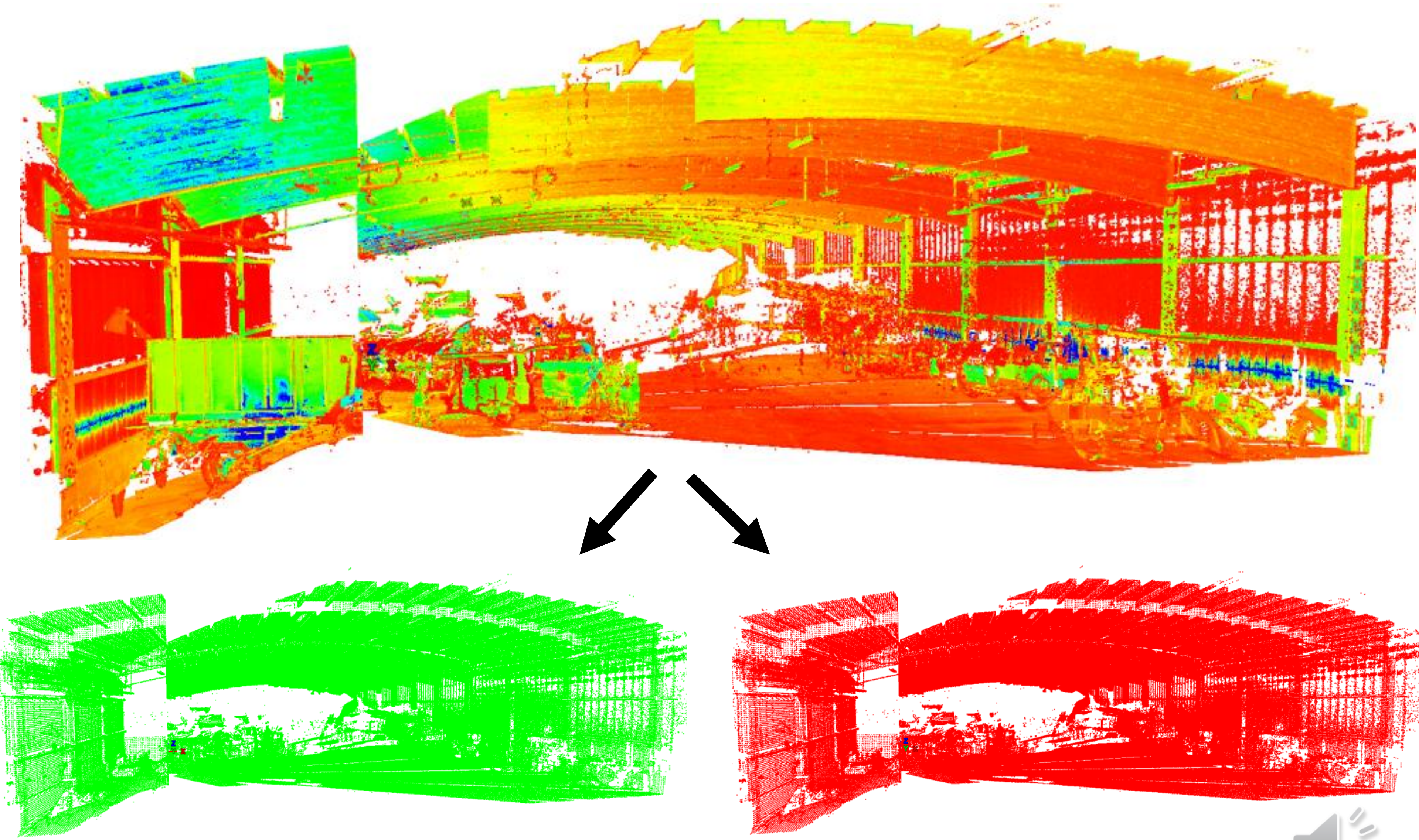
Courtesy: Tomislav Medic

B) Compensation by two-face scans

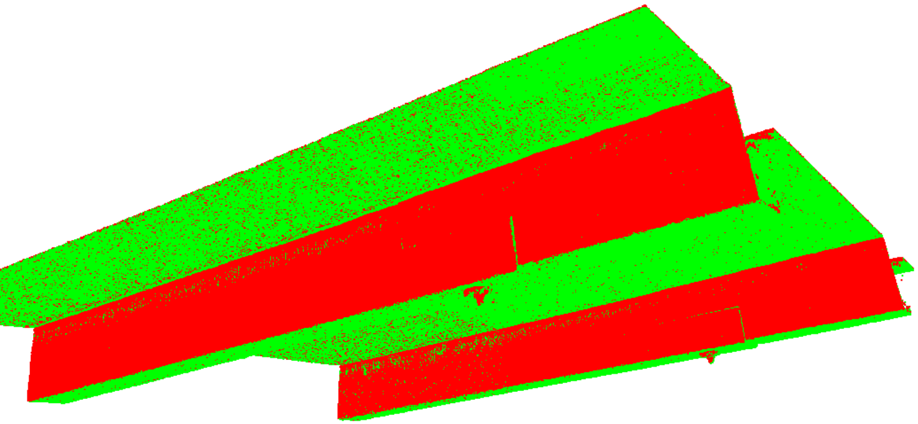
LAGE 1 **LAGE 2**

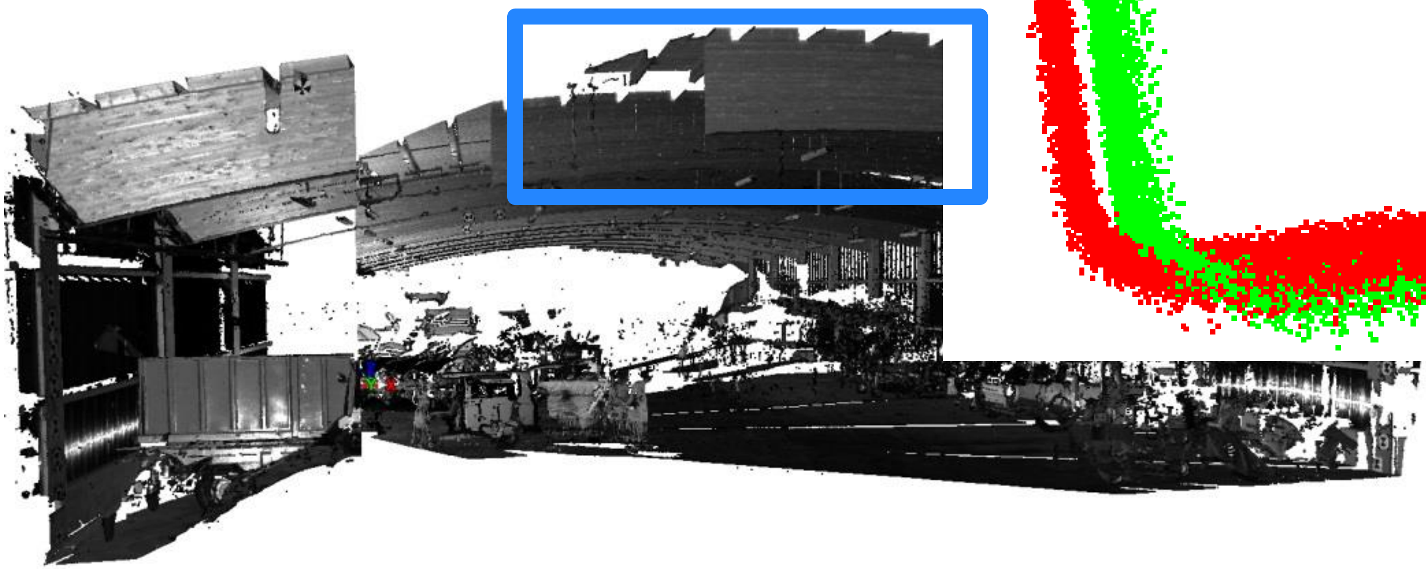


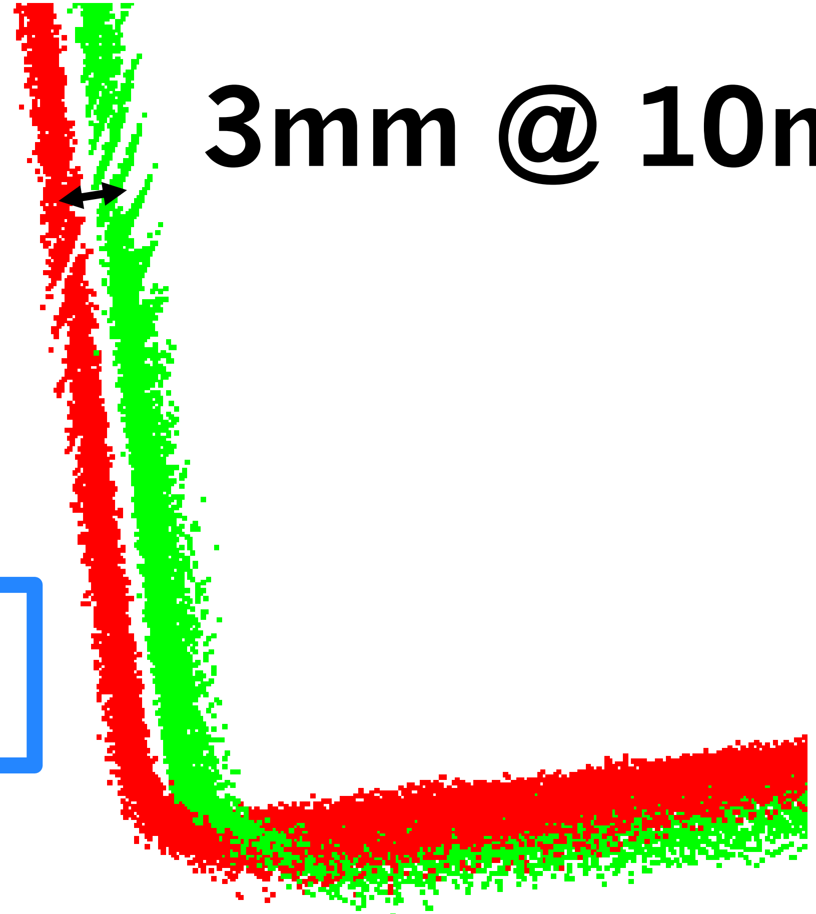
Two-face scans



Face 1 minus Face 2







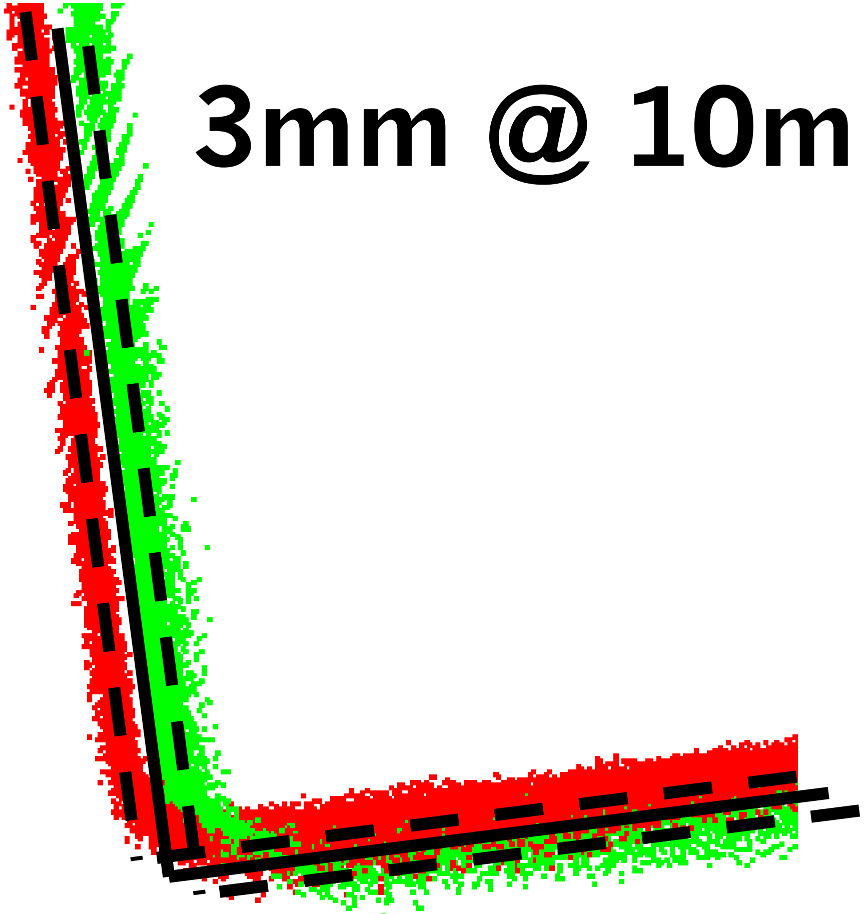
3mm @ 10m

Why reasonable?

- Two-face scan reveals systematic errors in the point cloud (anyway contained)
- Cause are instrumental errors whose sign switches between faces

Description
Vertical beam offset
Horizontal axis offset
Mirror offset
Vertical index offset
Horizontal beam tilt
Mirror tilt
Horizontal axis error (tilt)

=> Minimizing by modelling



- Areal modeling per face + averaging (of parameters)
- Areal modelling combinedly

Agenda

6 Uncertainty of point clouds

6.1 Basics

6.2 Error sources at laser scanning

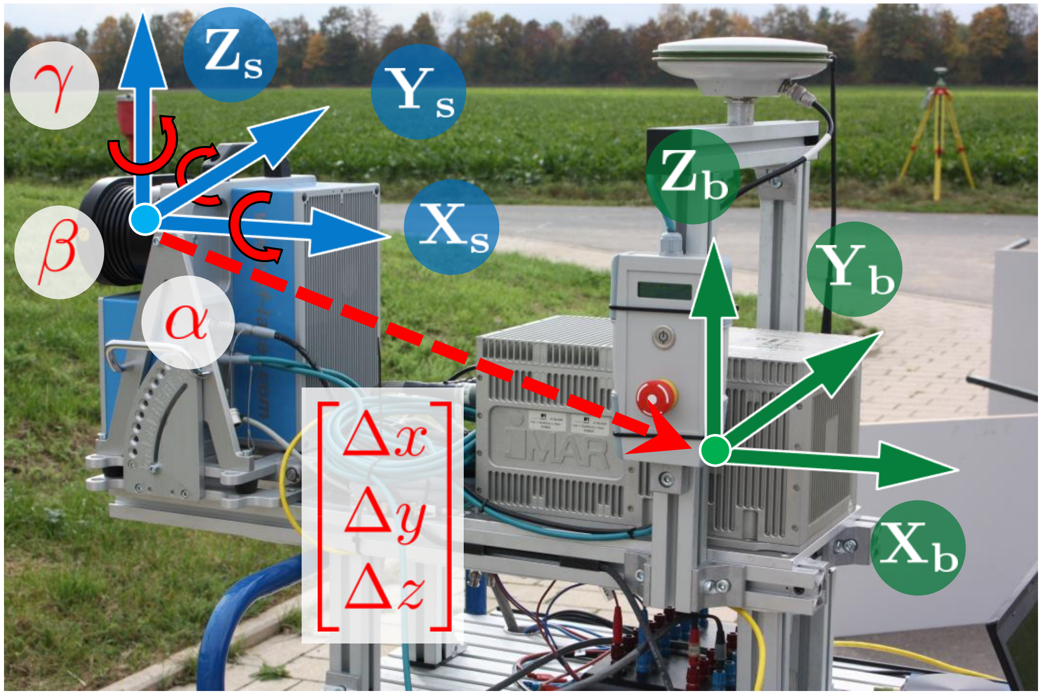
6.3 Strategies for minimizing the uncertainty

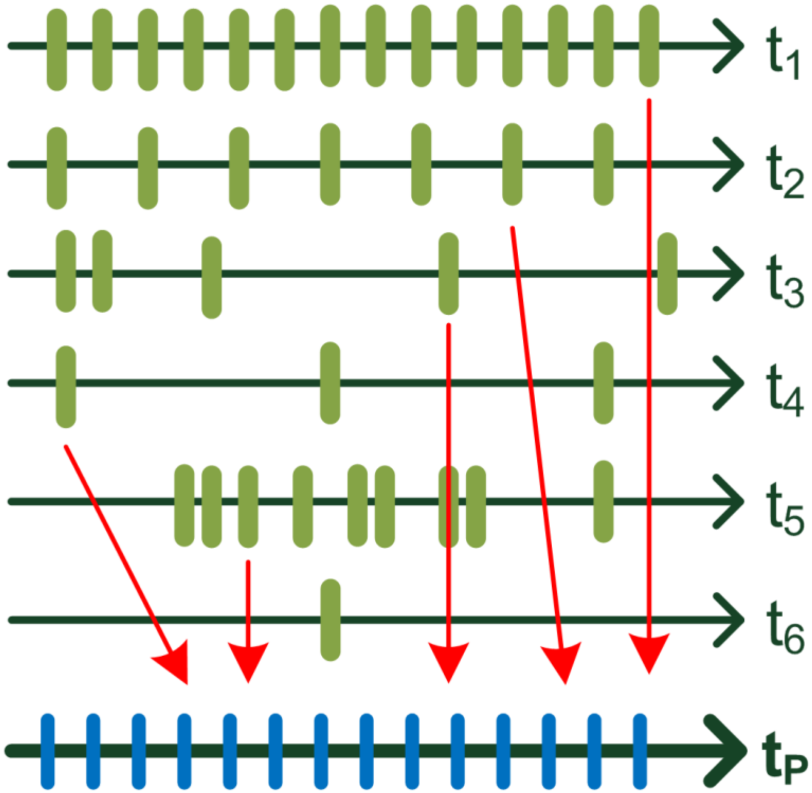
6.3.1 Instrumental errors

6.3.2 Geo-referencing errors

A) Calibrating the mobile mapping system

- Calibrating single sensors: GNSS, IMU, laser scanner, ...
- Calibrating geometrical relationships => **system calibration** (T_s^b)
- Calibrating temporal relationships => **sensor synchronization** (t_S)

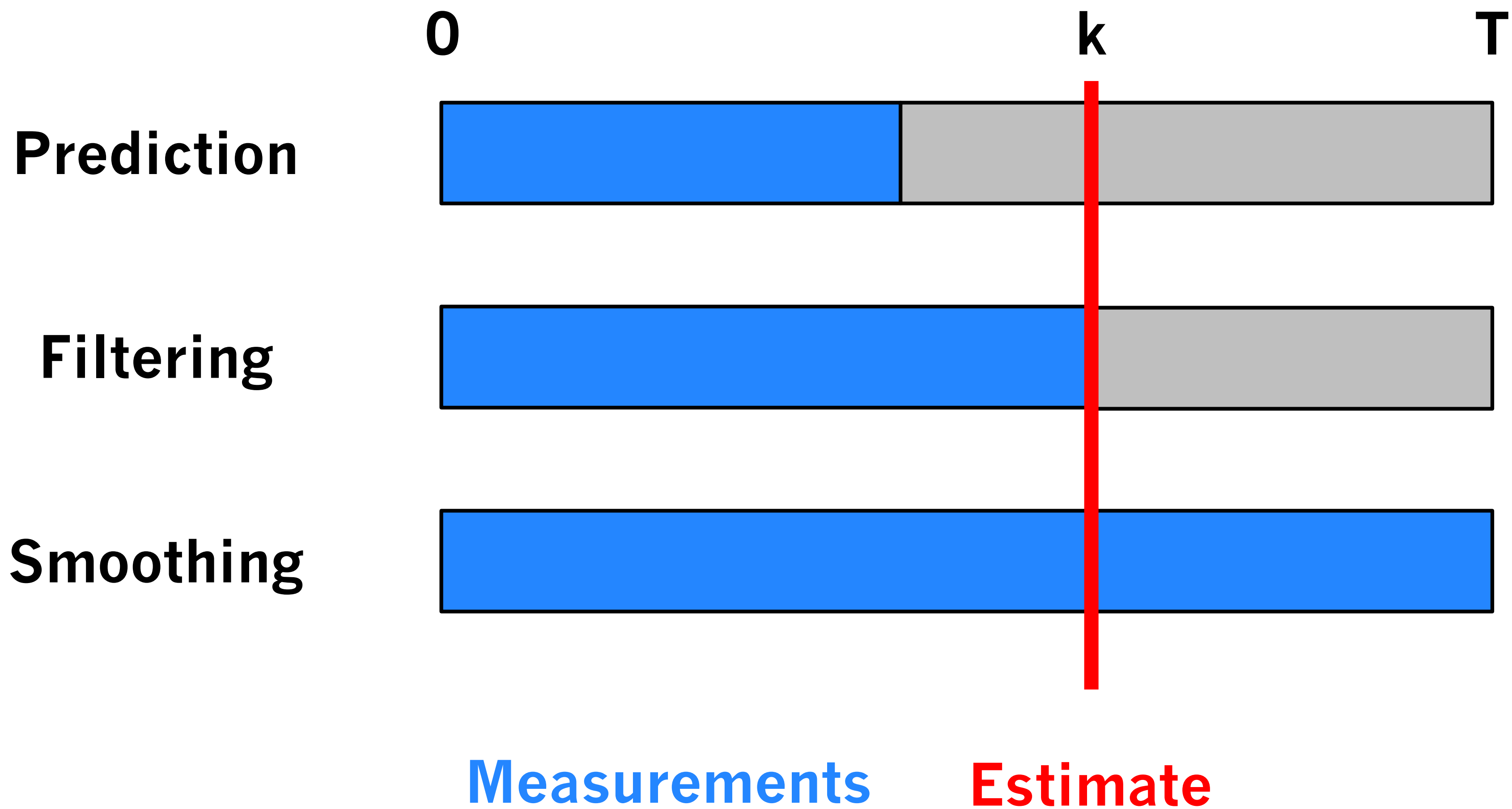




Courtesy: Erik Heinz

Courtesy: Lasse Klingbeil

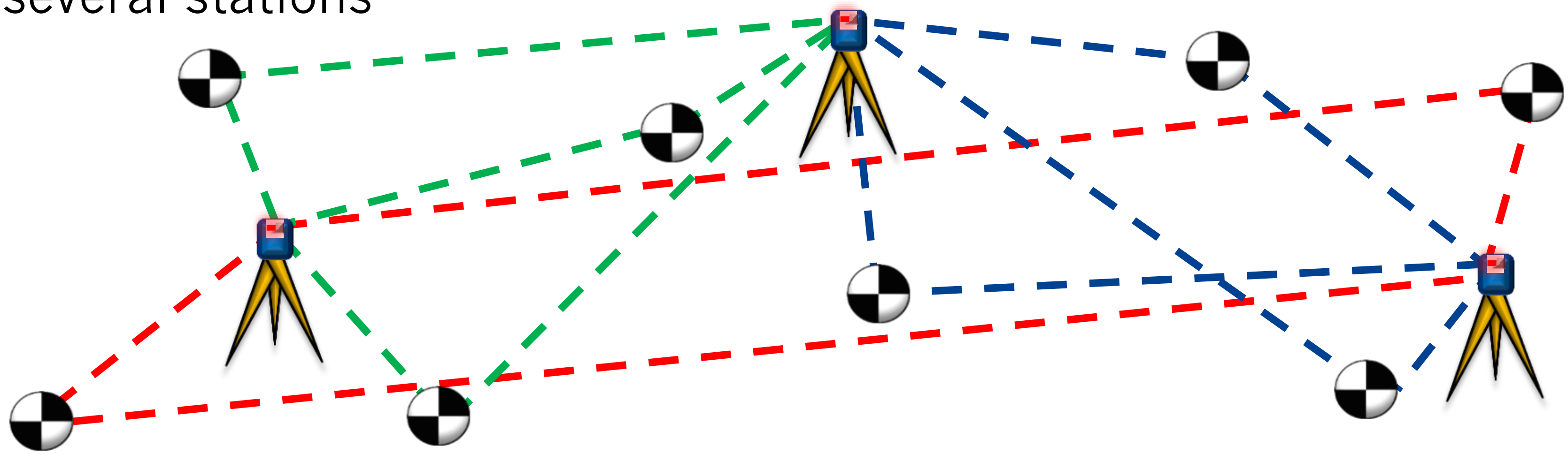
B) Compensation by smoothing



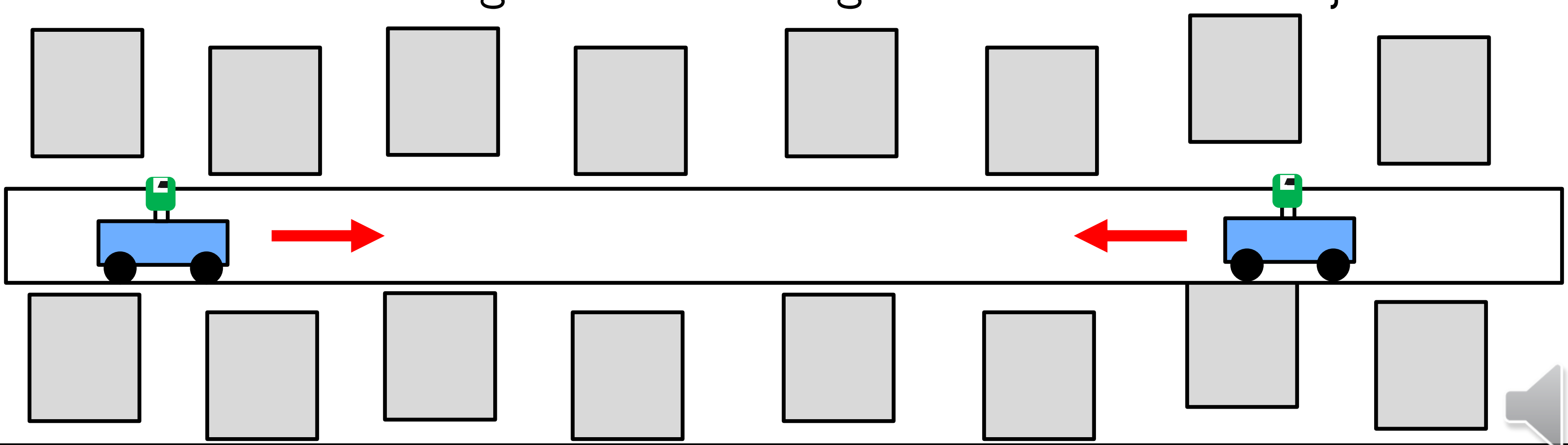
- Smoothing for mobile scanning ≈ network adjustment for static laser scanning

C) Repetitive geo-referencing

- Static laser scanning: repetative scanning of identical targets from several stations



- Mobile laser scanning: doubled navigation to identical objects



Interim summary

- Uncertainty of point cloud might be minimized by
  - Calibration
  - Mathematical compensation
  - Measurement strategies
- Choice depends on error source

- **Remaining uncertainty might be considered as only containing randomized (rest) errors**
- Quantification in variance-covariance matrix $\boldsymbol{\Sigma}_{ll}$

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Learning objectives

- Which error sources make a point cloud uncertain?
- How do these errors contribute to the uncertainty?
- How can they be minimized?