Uncertainty of Point Clouds
Part 1/2

Advanced Techniques for Mobile Sensing and Robotics (Geodesy Track)

Dr.-Ing. Christoph Holst
SS 2020
6 Uncertainty of point clouds

6.1 Basics
6.2 Error sources at laser scanning
6.3 Strategies for minimizing the uncertainty
6.4 Stochastic model of single static laser scans
6.5 Stochastic model of geo-referenced point clouds
6.6 Determining the uncertainty of existing point clouds
• Which error sources make a point cloud uncertain?
• How do these errors contribute to the uncertainty?
• How can they be minimized?
6 Uncertainty of point clouds

6.1 Basics
6.1.1 Visual inspection of uncertainty
6.1.2 Error types and their treatment
6.2 Error sources at laser scanning
6.3 Strategies for minimizing the uncertainty
High-end terrestrial laser scanners

www.leica-geosystems.com

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Smoothness (Leica P50)
Smoothness (Z+F Imager 5016)
High-end mobile mapping system

- GNSS antenna
- Inertial navigation system
- 2D laser scanner
- Profiler 9012A

scanning plane

30°
Smoothness (Z+F Profiler 9012A)

Courtesy: Lasse Klingbeil & Erik Heinz
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Uncertainty?
2-Face-Scan

FACE 1

FACE 2
Separation of two scans
Face 1 vs. Face 2

- Quantity: millimeters, even at short distances
- Visible in smooth point cloud

3mm @ 10m
• Errors of up to +/- 1.5 mm at 10m distance
• Dependend on vertical angle and horizontal angle
• Consequence: offset between face 1 and face 2
• Other probable consequences: innere deformation of scanned objects, tilt or bending of object

=> Errors exist in any case, maybe just visually hidden!
6 Uncertainty of point clouds

6.1 Basics

6.1.1 Visual inspection of uncertainty

6.1.2 Error types and their treatment

6.2 Error sources at laser scanning

6.3 Strategies for minimizing the uncertainty
Separation into error types

- Precision
- Outliers
- Resolution
- Bias

Courtesy: Bertold Witte
## Characterization of error types

<table>
<thead>
<tr>
<th></th>
<th>Outliers</th>
<th>Systematic errors</th>
<th>Random errors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Appearance</strong></td>
<td>Extremely erroneous individual measurements</td>
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<td><strong>Quantitative measure</strong></td>
<td>–</td>
<td>Bias / absolute accuracy</td>
<td>Precision / relative accuracy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Uncertainty</td>
<td></td>
</tr>
</tbody>
</table>
Error sources at laser scanning

Atmosphere
Temperature, Air pressure, ...

Geo-referencing
Targets, point cloud, add. hardware

Measurement Geometry
Angle of incidence, distance

Object
Reflectance, roughness

Instrument
Misalignments, Eccentricities, Offset, …
6 Uncertainty of point clouds

6.1 Basics

6.2 Error sources at laser scanning
   6.2.1 Instrument
   6.2.2 Atmosphere
   6.2.3 Geometry / Object
   6.2.4 Geo-referencing

6.3 Strategies for minimizing the uncertainty
Construction of 3D laser scanner

- Laser source
- Collimation axis
- Vertical profile
- Rotational mirror
- Vertical axis
- Horizontal rotation of the instrument

Courtesy: Tomislav Medic
Mirror tilt

Error in Face 1:
\[ \Delta \varphi_j^i = + \frac{2x_6}{\sin(\theta_j^i)} \]

Error in Face 2:
\[ \Delta \varphi_j^i = - \frac{2x_6}{\sin(\theta_j^i)} \]

... with \( \theta \) = vertical angle

=> Equals collimation axis error at total stations

Courtesy: Tomislav Medic
Error in Face 1:
\[ \Delta \varphi^i_j = + \frac{x_7}{\tan(\theta^i_j)} \]

Error in Face 2:
\[ \Delta \varphi^i_j = - \frac{x_7}{\tan(\theta^i_j)} \]

... with \( \theta \) = vertical angle

=> Equals horizontal axis errors at total stations

Horizontal axis

Vertical axis

Collimation axis

True horizon

x_7

Courtesy: Tomislav Medic

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Further misalignments

Eccentricity of laser

Alignment of laser

Eccentricity of mirror

Eccentricity of axes

Courtesy: Tomislav Medic
## Calibration parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>TS</th>
<th>2-faces</th>
<th>high-end</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal beam offset</td>
<td>(x_{1n})</td>
<td>similar</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Vertical beam offset</td>
<td>(x_{1z})</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Horizontal axis offset</td>
<td>(x_2)</td>
<td>similar</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Mirror offset</td>
<td>(x_3)</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Vertical index offset</td>
<td>(x_4)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Horizontal beam tilt</td>
<td>(x_{5n})</td>
<td>similar</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Vertical beam tilt</td>
<td>(x_{5z})</td>
<td>similar</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Mirror tilt</td>
<td>(x_6)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Horizontal axis error (tilt)</td>
<td>(x_7)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Horizontal angle encoder eccentricity</td>
<td>(x_{8x})</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Horizontal angle encoder eccentricity</td>
<td>(x_{8y})</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Vertical angle encoder eccentricity</td>
<td>(x_{9n})</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Vertical angle encoder eccentricity</td>
<td>(x_{9z})</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Second order scale error in the horizontal angle encoder</td>
<td>(x_{11a})</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Second order scale error in the horizontal angle encoder</td>
<td>(x_{11b})</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Second order scale error in the vertical angle encoder</td>
<td>(x_{12a})</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Second order scale error in the vertical angle encoder</td>
<td>(x_{12b})</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Rangefinder offset</td>
<td>(x_{10})</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Rangefinder scale error</td>
<td>/</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Rangefinder cyclic error</td>
<td>/</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

* existing only in TLSs using phase-shift distance measuring principle
Functional model

\[ \Delta r_j^i = x_2 \sin(\theta_j^i) + x_{10} + v_{r_j^i} \]

\[ \Delta \varphi_j^i = \frac{x_{1z}}{r_j^i \tan(\theta_j^i)} + \frac{x_3}{r_j^i \sin(\theta_j^i)} + \frac{x_{5z-7}}{\tan(\theta_j^i)} + \frac{2x_6}{\sin(\theta_j^i)} + \frac{x_{1n}}{r_j^i} + v_{\varphi_j^i} \]

\[ \Delta \theta_j^i = \frac{x_{1n+2} \cos(\theta_j^i)}{r_j^i} + x_4 + x_{5n} \cos(\theta_j^i) - \frac{x_{1z} \sin(\theta_j^i)}{r_j^i} - x_{5z} \sin(\theta_j^i) + v_{\theta_j^i} \]

**Horizontal axis error**

**Vertical beam tilt**
Interim summary

• Systematic and random errors of angular and range measurements
• Sources similar to total station, but more complex
• Impacts on point cloud large (larger than smoothness / random errors)

• Here: 3D panoramic scanner
• 2D profile scanner analogous, but with less error sources
6 Uncertainty of point clouds

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6.2.2 Atmosphere
6.2.3 Geometry / Object
6.2.4 Geo-referencing
6.3 Strategies for minimizing the uncertainty
**Time of flight principle**

\[
r = \frac{1}{2} \cdot c \cdot t
\]

- \(c\): speed of light in atmosphere
- \(t\): time of flight

\[
c = \frac{c_0}{n}
\]

- \(c_0\): speed of light in vacuum
  (299 792 458 m/s)
- \(n\): refraction index in atmosphere

\[
r = \frac{1}{2} \cdot \frac{c_0}{n} \cdot t
\]

**Phase shift principle**

\[
r = \frac{1}{2} \cdot \left( N \cdot \lambda + \frac{\Delta \varphi}{2\pi} \cdot \lambda \right)
\]

- \(N\): number of waves
- \(\lambda\): wavelength
- \(\Delta \varphi\): phase shift
- \(f\): frequency

\[
\lambda = \frac{c}{f}
\]

\[
r = \frac{1}{2} \cdot \left( N + \frac{\Delta \varphi}{2\pi} \right) \cdot \frac{c_0}{n \cdot f}
\]
Propagation delay of signal

\[ \Delta n \cdot 10^{-6} = -0.99 \cdot \Delta t + 0.28 \cdot \Delta p - 0.04 \cdot \Delta e \]

- \( \Delta n \): Change of refraction index...
- \( \Delta t \): Change of temperature...
- \( \Delta p \): Change of air pressure...
- \( \Delta e \): Change of air humidity...

... in relation to reference atmosphere

Temperature: 1°C = -1 ppm
Air pressure: 3 hPa = +1 ppm
Air humidity: 25 hPa = -1 ppm (neglectable)

- Neglectable in close range applications
- At larger distances (>100m) presumably important
• Atmospheric correction in ppm
• Reference atmosphere: 12°C, 1013.25 hPa, 60%
• Here: carrier wavelength of 700 nm

Courtesy: Thomas Schäfer
Atmosphere composes of horizontal layers with varying density.

- Vertical gradient of refraction index leads to curved beam.
- Measured distance is assigned to biased angle (refraction angle $\beta$).
- Hard to compensate due to unknown layering of atmosphere.
- Avoiding by measuring during low atmospheric turbulence.

\[
\beta = \int_{0}^{r} \frac{\partial n}{\partial q} \frac{r - s}{r} \ ds
\]
Refraction: empirical values

- Distances of 2km
- Variation of 10 mgon during day (see fig.), 2 mgon during night

Courtesy: Ephraim Friedli & Andreas Wieser
Interim summary

• Systematic errors due to atmospheric effects
• Important for longer distances and at changing atmospheric conditions
• In any case: mathematical correction of propagation delay
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Laser Radar Equation

\[ P_r = \frac{P_t D_r^2 \eta_{sys}}{4} \eta_{atm} \cos \alpha \frac{\rho}{r^2} \]

- **Transmitted Power**
- **Receiver Aperture Diameter**
- **System Transmission Factor**
- **Atmospheric Transmission Factor**
- **Angle of Incidence**
- **Reflectance of Surface**
- **Measured Range**

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Interpretation

Angle of incidence

Reflectance

Distance

object

laser
Reflectance (color) and distance

STD of ranges [mm]

Intensity [Inc]

Courtesy: Daniel Wujanz
Reflectance (material)

Courtesy: Daniel Wujanz
Angle of incidence and distance

STD of ranges [mm]

Intensity [Inc]

- 127 kHz
- 15 m
- 30 m

Courtesy: Daniel Wujanz
Rotational speed of laser scanner

\[ \sigma = a \cdot \text{intensity}^b \]

- \(a, b\) to be estimated per scanner model and measurement frequency
- Raw intensities needed, scaled intensities are only limitedly valid

Courtesy: Daniel Wujanz
Estimating the parameters

1D-mode (Wujanz et al. 2017)
- Repetitive measurements without mirror rotation
- Varying distances, materials, angles of incidence
- Std. dev. + averaged intensity

2D-mode (Heinz et al. 2018)
- Scanning printed pages with varying grey scale
- Fitting of plane through 3D scans + averaged intensity
- Scaled intensity sometimes also usable

3D-mode (Schmitz et al. 2019)
- Scanning printed pages with varying grey scale
- Fitting of plane through 3D scans + averaged intensity
- Scaled intensity sometimes also usable

$$\alpha = 80^\circ$$
$$\alpha = 0^\circ$$
Quantifying random errors

Leica ScanStation P20, 1.6mm @ 10m, Quality level 1

- $a = 60.34 \frac{\text{mm}}{\text{inc}}, \sigma_a = 3.67 \frac{\text{mm}}{\text{inc}}$
- $b = -0.61, \sigma_b = 0.01$

Courtesy: Berit Schmitz
Quantifying random errors

Intensity: 123.000

\( \sigma_p \)
1.98 mm

STD of ranges [mm]

Intensity [Inc]

Courtesy: Daniel Wujanz
Systematic errors

- Laser Radar Equation only explains random errors
- Further systematic errors exist due to physical and geometric effects

- Errors due to varying reflectance of the surface
- Errors due to large angles of incidence
- Errors due to transmission / penetration of the surface
- Errors due to measurements on edges
- Errors due to mixed pixels
- Errors due to total reflection
• Up to now no physical parameterization
• Error due to distance and reflectance
• 70% compensatable

<table>
<thead>
<tr>
<th>ID</th>
<th>Reflectance</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>S99</td>
<td>99 %</td>
<td>Spectralon</td>
</tr>
<tr>
<td>S80</td>
<td>87 %</td>
<td>Spectralon</td>
</tr>
<tr>
<td>S40</td>
<td>47 %</td>
<td>Spectralon</td>
</tr>
<tr>
<td>S05</td>
<td>7 %</td>
<td>Spectralon</td>
</tr>
<tr>
<td>KaH</td>
<td>102 %</td>
<td>white cardboard</td>
</tr>
<tr>
<td>KaG</td>
<td>81 %</td>
<td>grey cardboard</td>
</tr>
<tr>
<td>KaS</td>
<td>10 %</td>
<td>black cardboard</td>
</tr>
<tr>
<td>MeW</td>
<td>650 %</td>
<td>aluminium</td>
</tr>
</tbody>
</table>
Reflectance of the surface

Surphaser 25HSX-IR  Leica P20  Leica P50

12mm  0mm  5mm
Angle of incidence: geometry
Angle of incidence: geometry

- **Mitte Strahl**

<table>
<thead>
<tr>
<th>$\alpha$ / $r$</th>
<th>3,5 m</th>
<th>10 m</th>
<th>20 m</th>
<th>30 m</th>
<th>50 m</th>
<th>70 m</th>
<th>100 m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>15°</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>30°</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>45°</strong></td>
<td></td>
<td></td>
<td></td>
<td>+10 µm</td>
<td>+10 µm</td>
<td>+10 µm</td>
<td></td>
</tr>
<tr>
<td><strong>60°</strong></td>
<td>+10 µm</td>
<td>+10 µm</td>
<td>+20 µm</td>
<td>+20 µm</td>
<td>+30 µm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
- Deformation of energy wave front if reflected with non-zero angle of incidence
- Intensities are pulled together at $\Delta r_1$ and pulled apart at $\Delta r_2$
- Intensity distribution at photo detector shifted towards $\Delta r_1$ by time shift $\Delta t_1$
- Consequence is shortened distance measurement
Bias in distance measurement up to -1.6mm (Z+F Imager 5016)

- Dependent on distance => laser spot size
- Dependent on angle of incidence => laser spot deformation

Courtesy: Carsten Goy
Penetration of the surface

- a) Laser beam
- b) Reflection at cuticula
- c) Transmission through epidermis
- d) Absorption by chlorophyll
- e) Reflection at cell wall
- f) Complete transmission

Courtesy: Stefan Paulus
Edge effects
Mixed pixels

- Bias or outlier?

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Folie 54
• Bias or outlier?

Total reflection
6 Uncertainty of point clouds

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6.2.4 Geo-referencing
6.3 Strategies for minimizing the uncertainty
Static laser scanning

Courtesy: scandric 3D solutions
Geo-referencing 3D point clouds

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}^g = \begin{bmatrix}
  t_x \\
  t_y \\
  t_z
\end{bmatrix} + R(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot \begin{bmatrix}
  r \cdot \sin \theta \cdot \sin \varphi \\
  r \cdot \sin \theta \cdot \cos \varphi \\
  r \cdot \cos \theta
\end{bmatrix}^s
\]

Target based

Point cloud based

Hardware based

Courtesy: Daniel Wujanz

Courtesy: Jens-André Paffenholz

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Impact of uncertainty

- **Bias / absolute accuracy**: complete scans
- **Precision / relative accuracy**: only at overlaps
Mobile laser scanning

Courtesy: Zoller + Fröhlich
Mobile laser scanning = profile scan

Courtesy: Lasse Klingbeil
Geo-referencing: static vs. mobile

\[
\begin{bmatrix}
\chi \\
y \\
z
\end{bmatrix}^g =
\begin{bmatrix}
t_x \\
t_y \\
t_z
\end{bmatrix} +
\mathbf{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot
\begin{bmatrix}
r \cdot \sin \theta \cdot \sin \varphi \\
r \cdot \sin \theta \cdot \cos \varphi \\
r \cdot \cos \theta
\end{bmatrix}^s
\]

A) **Transformation from scanner to geo-referenced system**

\[
\begin{bmatrix}
\chi \\
y \\
z
\end{bmatrix}^g =
\begin{bmatrix}
t_x \\
t_y \\
t_z
\end{bmatrix} +
\mathbf{R}_n^e(L, B) \cdot \mathbf{R}_n^m(\phi, \theta, \psi) \cdot
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} +
\mathbf{R}_s^b(\alpha, \beta, \gamma) \cdot
\begin{bmatrix}
0 \\
d \cdot \sin b \\
d \cdot \cos b
\end{bmatrix}^s
\]

A) **Transformation from scanner to body system**

B) **Transformation from body to geo-referenced system**
Geo-referencing: static vs. mobile

Geo-referenced 3D point

\[ p^g = T_s^g \cdot p^s \]

Local 3D point

Local 2D point

\[ p^g = T_b^g(t_s) \cdot T_s^b \cdot p^s \]

Trafo \( s \) to \( g \)

Trafo \( b \) to \( g \) (for each time step \( t_s \) individually)

Trafo from \( s \) to \( b \) (temporally stable)

\[ \Rightarrow \text{Trajectory estimation} \]

\[ \Rightarrow \text{System calibration} \]
Impact of uncertainty

- Bias / absolute accuracy: complete scans
- Precision / relative accuracy: complete scans
Numerical example: static

- Geo-referencing using targets
- Uncertainty: several mm

Courtesy: Erik Heinz
• Cloud2Cloud comparison: errors < 12 cm
• Uncertainty due to geo-referencing of mobile scans: systematic GPS-errors

Courtesy: Erik Heinz
• Uncertainty of geo-referencing occurs when transforming local scan points into another coordinate system

• Static laser scanning
  – Geo-referencing of complete 3D point clouds
  – Uncertainty potentially lower
  – Affecting bias all over the point cloud
  – Affecting precision only at overlap of laser scans

• Mobile laser scanning
  – Geo-referencing of individual 2D points
  – Uncertainty potentially higher (trajectory estimation, system calibration)
  – Affecting bias and precision all over the point cloud
6 Uncertainty of point clouds

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6.3 Strategies for minimizing the uncertainty
6.3.1 Instrumental errors
6.3.2 Geo-referencing errors
## Characterization of error types

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## Minimizing systematic errors

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<th>Random</th>
<th>Systematic</th>
</tr>
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<tbody>
<tr>
<td>Geometry</td>
<td>Distance</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angle of impact</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Object</td>
<td>Smoothness</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Reflectance</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Atmosphere</td>
<td>Propagation delay</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Refraction</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Instrument</td>
<td>Construction</td>
<td>x</td>
<td>x / x</td>
</tr>
<tr>
<td>Geo-referencing</td>
<td>various</td>
<td>x</td>
<td>x / x / x</td>
</tr>
</tbody>
</table>

- **Calibration**
- **Mathematical compensation**
- **Measurement strategies**

In the end:

Randomized errors $\nu$
6 Uncertainty of point clouds

6.1 Basics

6.2 Error sources at laser scanning

6.3 Strategies for minimizing the uncertainty

6.3.1 Instrumental errors

6.3.2 Geo-referencing errors
A) Calibrating the laser scanner

\[ \Delta r_j^i = x_2 \sin(\theta_j^i) + x_{10} + v_{r_j^i} \]

\[ \Delta \varphi_j^i = \frac{x_{1z}}{r_j^i \tan(\theta_j^i)} + \frac{x_3}{r_j^i \sin(\theta_j^i)} + \frac{x_{5z-7}}{\tan(\theta_j^i)} + \frac{2x_6}{\sin(\theta_j^i)} + \frac{x_{1n}}{r_j^i} + v_{\varphi_j^i} \]

\[ \Delta \theta_j^i = \frac{x_{1n+2} \cos(\theta_j^i)}{r_j^i} + x_4 + x_{5n} \cos(\theta_j^i) - \frac{x_{1z} \sin(\theta_j^i)}{r_j^i} - x_{5z} \sin(\theta_j^i) + v_{\theta_j^i} \]
Estimation of parameters

\[ f_i(l, x) = R(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_i + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_j = 0 \]

\( i = 1, \ldots, \# \text{Targets} \)

\( j, j + k \ldots \text{Stations} \)

\[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \cdot \sin \theta \cdot \sin \varphi \\ r \cdot \sin \theta \cdot \cos \varphi \\ r \cdot \cos \theta \end{bmatrix} \]

\( r = \bar{r} + \Delta r \)

\( \varphi = \bar{\varphi} + \Delta \varphi \)

\( \theta = \bar{\theta} + \Delta \theta \)

\( (\text{Registration – for } j + k = 2 – \text{or Bundle Block Adjustment}) \)
Measurement configuration

Courtesy: Andreas Rietdorf

Courtesy: Derek D. Lichti
Calibration field of University of Bonn

25 m

21 m

7.5 m

Courtesy: Tomislav Medic
Estimated calibration parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Leica P50</th>
<th>Imager 5016</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{10}$ [mm] (Nullpunktabw.)</td>
<td>-0.03</td>
<td>0.37</td>
</tr>
<tr>
<td>$x_{2}$ [mm]</td>
<td>-0.14</td>
<td>0.03</td>
</tr>
<tr>
<td>$x_{1z}$ [mm]</td>
<td>0.14</td>
<td>0.59</td>
</tr>
<tr>
<td>$x_{3}$ [mm]</td>
<td>-0.03</td>
<td>-0.15</td>
</tr>
<tr>
<td>$x_{7}$ [''] (Kippachsabw.)</td>
<td>-33.50</td>
<td>26.49</td>
</tr>
<tr>
<td>$x_{6}$ [''] (Zielachsabw.)</td>
<td>3.41</td>
<td>0.40</td>
</tr>
<tr>
<td>$x_{1n}$ [mm]</td>
<td>0.01</td>
<td>-0.45</td>
</tr>
<tr>
<td>$x_{4}$ [''] (Höhenindexabw.)</td>
<td>4.51</td>
<td>-6.44</td>
</tr>
<tr>
<td>$x_{5n}$ ['']</td>
<td>5.38</td>
<td>12.74</td>
</tr>
<tr>
<td>$x_{5z}$ ['']</td>
<td>-16.78</td>
<td>2.66</td>
</tr>
</tbody>
</table>

$\hat{\sigma} \approx 0.5'' / \hat{\sigma} \approx 0.5\text{mm}$ (high correlations!)
Leica P50: Face 1 minus Face 2

Courtesy: Tomislav Medic
Z+F Imager 5016: Face 1 minus Face 2

Courtesy: Tomislav Medic

[Image of point cloud data]
B) Compensation by two-face scans

lage 1

lage 2
Two-face scans
Face 1 minus Face 2

3mm @ 10m
Two-face scan reveals systematic errors in the point cloud (anyway contained)

Cause are instrumental errors whose sign switches between faces

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical beam offset</td>
</tr>
<tr>
<td>Horizontal axis offset</td>
</tr>
<tr>
<td>Mirror offset</td>
</tr>
<tr>
<td>Vertical index offset</td>
</tr>
<tr>
<td>Horizontal beam tilt</td>
</tr>
<tr>
<td>Mirror tilt</td>
</tr>
<tr>
<td>Horizontal axis error (tilt)</td>
</tr>
</tbody>
</table>

=> Minimizing by modelling

Areal modeling per face + averaging (of parameters)
Areal modelling combinedly
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A) Calibrating the mobile mapping system

• Calibrating single sensors: GNSS, IMU, laser scanner, ...
• Calibrating geometrical relationships => system calibration \( (T^b_s) \)
• Calibrating temporal relationships => sensor synchronization \( (t_s) \)

Uncertainty of point clouds, Part 1/2, Christoph Holst

Courtesy: Erik Heinz

Courtesy: Lasse Klingbeil
B) Compensation by smoothing

- Smoothing for mobile scanning ≈ network adjustment for static laser scanning
C) Repetitive geo-referencing

- Static laser scanning: repetitive scanning of identical targets from several stations

- Mobile laser scanning: doubled navigation to identical objects
Uncertainty of point cloud might be minimized by
- Calibration
- Mathematical compensation
- Measurement strategies

Choice depends on error source

Remaining uncertainty might be considered as only containing randomized (rest) errors

Quantification in variance-covariance matrix $\Sigma_u$
References

• Which error sources make a point cloud uncertain?
• How do these errors contribute to the uncertainty?
• How can they be minimized?