Uncertainty of Point Clouds
Part 2/2

Advanced Techniques for Mobile Sensing and Robotics (Geodesy Track)

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SS 2020

6 Uncertainty of point clouds

6.1 Basics

6.2 Error sources at laser scanning

6.3 Strategies for minimizing the uncertainty

6.4 Stochastic model of single static laser scan

6.5 Stochastic model of geo-referenced point clouds

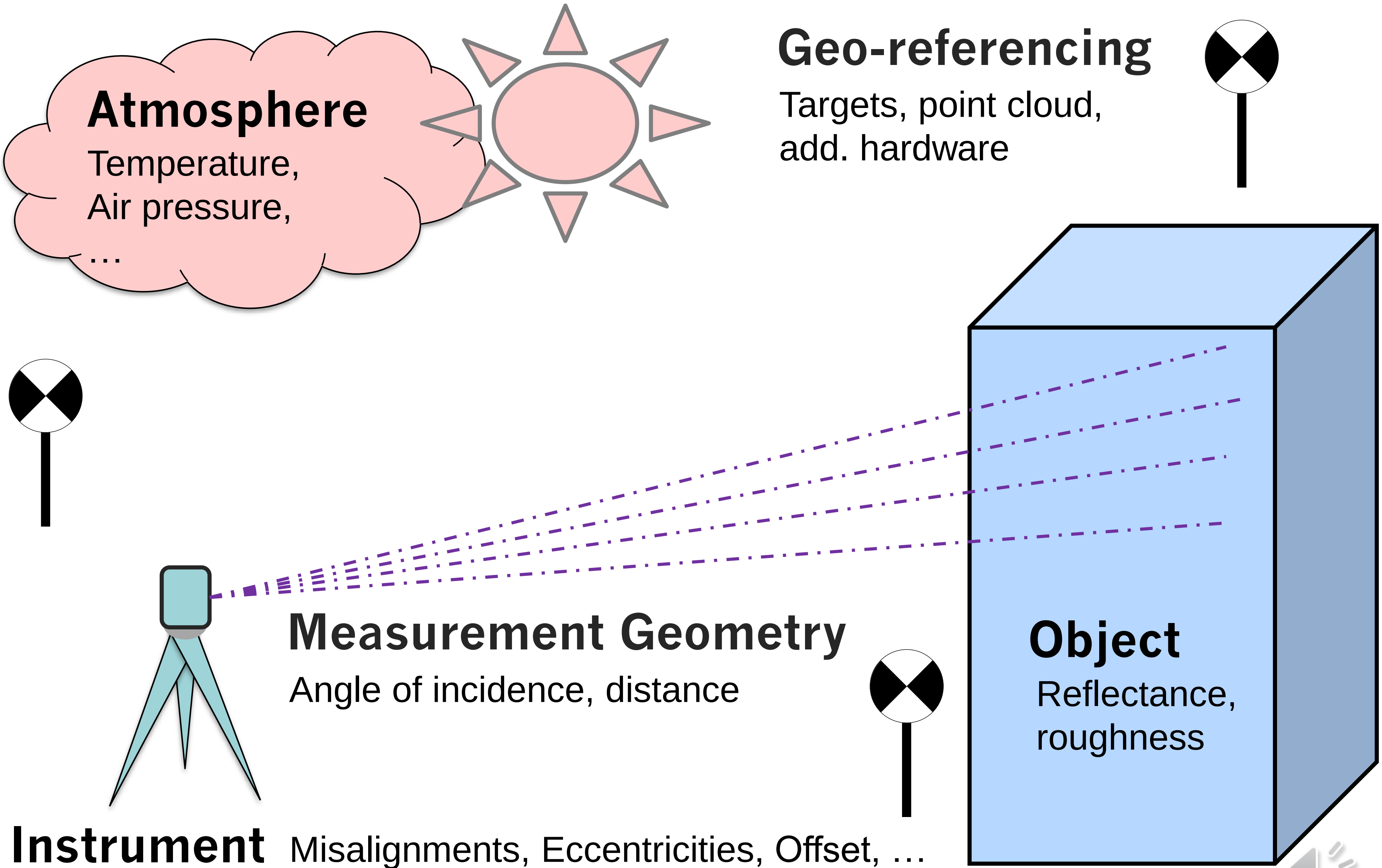
6.6 Determining the uncertainty of existing point clouds

Learning objectives

- How to describe the uncertainty of a point cloud within a variance-covariance matrix?
- What makes the variance-covariance matrix complex?
- How to determine the uncertainty of existing point clouds?

6 Uncertainty of point clouds

Error sources at laser scanning



Characterization of error types

	Outliers	Systematic errors	Random errors
Appear-ance	Extremely erroneous individual measurements	One-sided effects	Gaußian distributed (positive and negative, more often small than large)
Source	Mistakes of operator, Inappropriate used of instrument	Imperfection of measurement procedure, insufficient calibration of instrument	Fluctuations not trackable by calibration, uncontrolled changes of measured object and environment
Minimi-zing	Avoidance by care and control	**Calibration, math. compensation, measurem. strategies**	**Averaging by multiple measurements**
Quanti-tative measure	–	Bias / absolute accuracy	Precision / relative accuracy
	Uncertainty		

Minimizing systematic errors

Category	Element	Random	Systematic
Geometry	Distance	x	
	Angle of impact	x	x
Object	Smoothness	x	x
	Reflectance	x	x
Atmosphere	Propagation delay		x
	Refraction		x
Instrument	Construction	x	x / x
Geo-referencing	*various*	x	x / x / x

- Calibration
- Mathematical compensation
- Measurement strategies

In the end:

Randomized errors v

6 Uncertainty of point clouds

VCM = variance-covariance matrix

$$\boldsymbol{l} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ \vdots \\ l_{n-2} \\ l_{n-1} \\ l_n \end{bmatrix} = \begin{bmatrix} r_1 \\ \varphi_1 \\ \theta_1 \\ \vdots \\ r_m \\ \varphi_m \\ \theta_m \end{bmatrix}$$

$n = \#$ Observations
$m = \#$ Points

$$\boldsymbol{\Sigma}_{ll} = \begin{bmatrix} \overbrace{\begin{bmatrix} \sigma_{r_1}^2 & \sigma_{\varphi_1 r_1} & \sigma_{\theta_1 r_1} \\ \sigma_{r_1 \varphi_1} & \sigma_{\varphi_1}^2 & \sigma_{\theta_1 \varphi_1} \\ \sigma_{r_1 \theta_1} & \sigma_{\varphi_1 \theta_1} & \sigma_{\theta_1}^2 \end{bmatrix}}^{\boldsymbol{\Sigma}_{1,1}} & \cdots & \overbrace{\begin{bmatrix} \sigma_{r_m r_1} & \sigma_{\varphi_m r_1} & \sigma_{\theta_m r_1} \\ \sigma_{r_m \varphi_1} & \sigma_{\varphi_m \varphi_1} & \sigma_{\theta_m \varphi_1} \\ \sigma_{r_m \theta_1} & \sigma_{\varphi_m \theta_1} & \sigma_{\theta_m \theta_1} \end{bmatrix}}^{\boldsymbol{\Sigma}_{m,1}} \\ \vdots & \ddots & \vdots \\ \underbrace{\begin{bmatrix} \sigma_{r_1 r_m} & \sigma_{\varphi_1 r_m} & \sigma_{\theta_1 r_m} \\ \sigma_{r_1 \varphi_m} & \sigma_{\varphi_1 \varphi_m} & \sigma_{\theta_1 \varphi_m} \\ \sigma_{r_1 \theta_m} & \sigma_{\varphi_1 \theta_m} & \sigma_{\theta_1 \theta_m} \end{bmatrix}}_{\boldsymbol{\Sigma}_{1,m}} & \cdots & \underbrace{\begin{bmatrix} \sigma_{r_m}^2 & \sigma_{\varphi_m r_m} & \sigma_{\theta_m r_m} \\ \sigma_{r_m \varphi_m} & \sigma_{\varphi_m}^2 & \sigma_{\theta_m \varphi_m} \\ \sigma_{r_m \theta_m} & \sigma_{\varphi_m \theta_m} & \sigma_{\theta_m}^2 \end{bmatrix}}_{\boldsymbol{\Sigma}_{m,m}} \end{bmatrix}$$

Elementary error model

$$\boldsymbol{v} = \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\boldsymbol{\delta}_k, \boldsymbol{\xi}, \boldsymbol{\gamma}_h)$$

Randomized errors consist of a summation of countless individual and small errors

$\boldsymbol{\delta_k}$... non correlating errors (# p)

$\boldsymbol{\xi}$... functional correlating errors

$\boldsymbol{\gamma_h}$... stochastic correlating errors (# q)

Each individual error contributes to the overall error budget

$$\boldsymbol{v} = \sum_{k=1}^{p} \boldsymbol{D}_k \cdot \boldsymbol{\delta}_k + \boldsymbol{F} \cdot \boldsymbol{\xi} + \sum_{h=1}^{q} \boldsymbol{G}_h \cdot \boldsymbol{\gamma}_h$$

$$\boldsymbol{\Sigma_{ll}} = \sum_{k=1}^{p} \boldsymbol{D_k} \cdot \boldsymbol{\Sigma_{\delta\delta,k}} \cdot \boldsymbol{D_k^T} + \boldsymbol{F} \cdot \boldsymbol{\Sigma_{\xi\xi}} \cdot \boldsymbol{F^T} + \sum_{h=1}^{q} \boldsymbol{G_h} \cdot \boldsymbol{\Sigma_{\gamma\gamma,h}} \cdot \boldsymbol{G_h^T}$$

Non-correlating errors δ_k

$$\boldsymbol{D_k} = \begin{bmatrix} \frac{\partial l_1}{\partial \delta_{1k}} & 0 & \cdots & 0 \\ 0 & \frac{\partial l_2}{\partial \delta_{2k}} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \frac{\partial l_n}{\partial \delta_{nk}} \end{bmatrix}$$

- **Random noise**
- Only influence individual measurement elements
- No functional or stochastic dependencies

$$\boldsymbol{\Sigma_{\delta\delta,k}} = \begin{bmatrix} \sigma_{1k}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2k}^2 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_{nk}^2 \end{bmatrix}$$

Example: random errors

$$\boldsymbol{\Sigma}_{\boldsymbol{\delta\delta},\boldsymbol{k}} = \sigma_{r_i}^2 = \left(a \cdot intensity_i^b\right)^2, \text{for } k = 1,4,7, \dots$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\delta\delta},\boldsymbol{k}} = \sigma_{\varphi}^2, \text{for } k = 2,5,8, \dots$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\delta\delta},\boldsymbol{k}} = \sigma_{\theta}^2, \text{for } k = 3,6,9, \dots$$

$$\boldsymbol{D}_{\boldsymbol{k}} = \begin{bmatrix} \frac{\partial l_1}{\partial \delta_{1k}} & 0 & \cdots & 0 \\ 0 & \frac{\partial l_2}{\partial \delta_{2k}} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \frac{\partial l_n}{\partial \delta_{nk}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}; \qquad \sum_{\boldsymbol{k}=\boldsymbol{1}}^{\boldsymbol{p}} \boldsymbol{D}_{\boldsymbol{k}} = \boldsymbol{I}$$

$$\sum_{\boldsymbol{k}=\boldsymbol{1}}^{\boldsymbol{p}} \boldsymbol{\Sigma}_{\boldsymbol{\delta\delta},\boldsymbol{k}} = \boldsymbol{\Sigma}_{\boldsymbol{\delta\delta}} = \begin{bmatrix} \left(a \cdot intensity_i^b\right)^2 & & \\ & \ddots & \\ & & \sigma_{\theta_m}^2 \end{bmatrix}$$

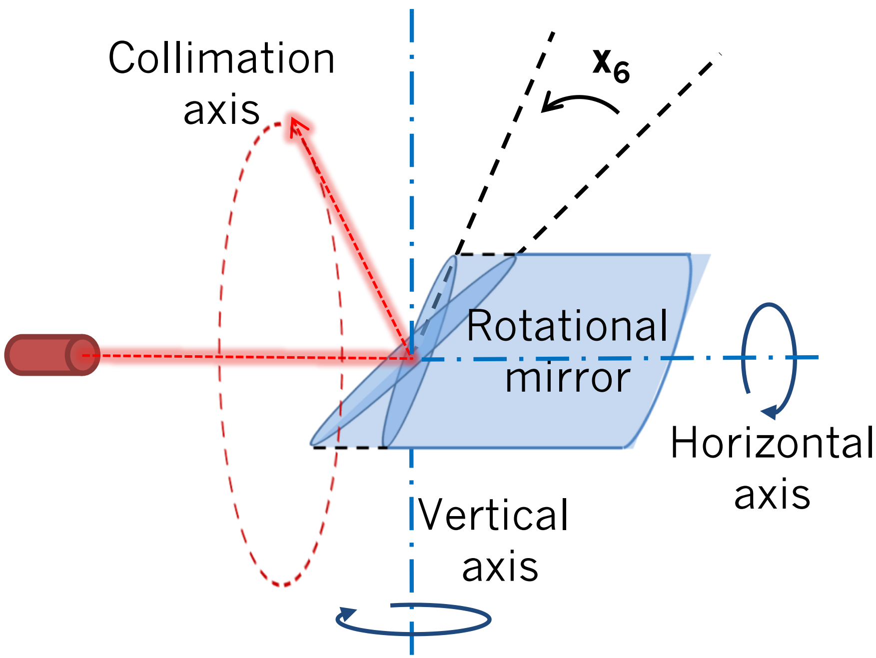
Functional correlating errors ξ

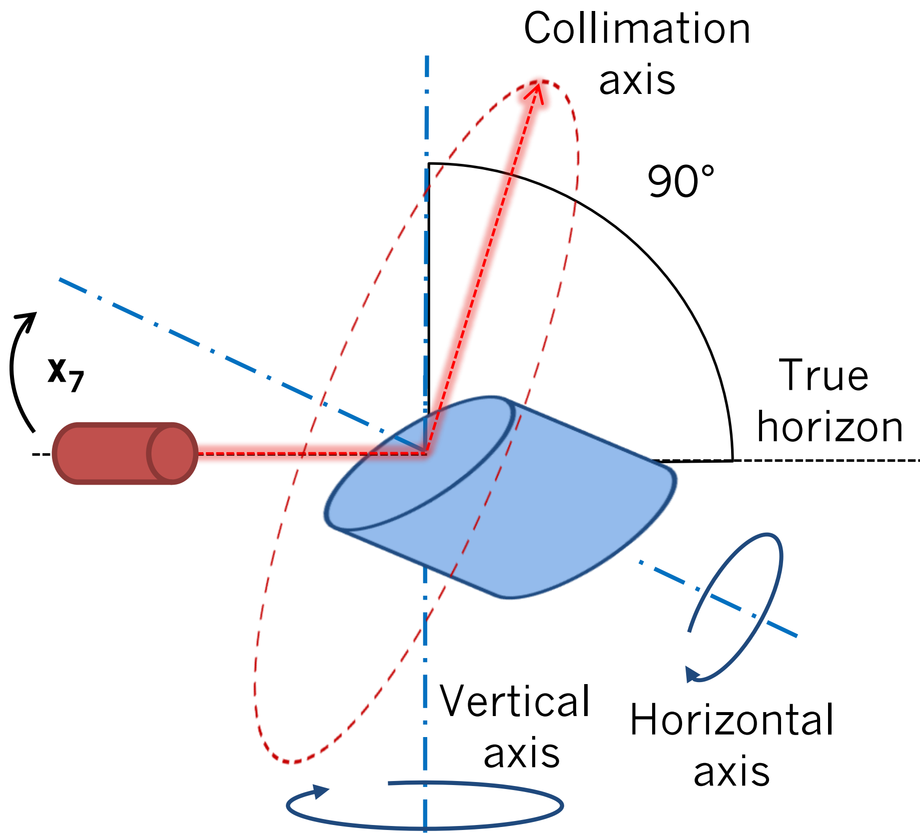
$$\boldsymbol{F} = \begin{bmatrix} \frac{\partial l_1}{\partial \xi_1} & \frac{\partial l_1}{\partial \xi_2} & \cdots & \frac{\partial l_1}{\partial \xi_m} \\ \frac{\partial l_2}{\partial \xi_1} & \frac{\partial l_2}{\partial \xi_2} & \cdots & \frac{\partial l_2}{\partial \xi_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial l_n}{\partial \xi_1} & \frac{\partial l_n}{\partial \xi_2} & \cdots & \frac{\partial l_n}{\partial \xi_m} \end{bmatrix}$$

$$\boldsymbol{\Sigma}_{\xi\xi} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_m^2 \end{bmatrix}$$

- **Mathematical correlations** between observations due to variance propagation
- Potentially impact all observations
- All errors sources that have been minimized not completely by calibration, mathem. compensation or measurement strategies

Example: Instrumental (rest) errors





- Error in Face 1: $\Delta\varphi_j^i = +\frac{2x_6}{\sin(\theta_j^i)}$
- Error in Face 2: $\Delta\varphi_j^i = -\frac{2x_6}{\sin(\theta_j^i)}$

- Error in Face 1: $\Delta\varphi_j^i = +\frac{x_7}{\tan(\theta_j^i)}$
- Error in Face 2: $\Delta\varphi_j^i = -\frac{x_7}{\tan(\theta_j^i)}$

Example: Instrumental (rest) errors

$$\xi_1 = x_6$$
$$\xi_2 = x_7$$

$$\varphi_j^i = \bar{\varphi}_j^i + \frac{x_7}{\tan(\theta_j^i)} + \frac{2x_6}{sin(\theta_j^i)} + v_{\varphi_j^i}$$

$$\boldsymbol{F} = \begin{bmatrix} \frac{\partial l_1}{\partial \xi_1} & \frac{\partial l_1}{\partial \xi_2} & \cdots & \frac{\partial l_1}{\partial \xi_m} \\ \frac{\partial l_2}{\partial \xi_1} & \frac{\partial l_2}{\partial \xi_2} & \cdots & \frac{\partial l_2}{\partial \xi_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial l_n}{\partial \xi_1} & \frac{\partial l_n}{\partial \xi_2} & \cdots & \frac{\partial l_n}{\partial \xi_m} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{2}{sin(\theta_j^i)} & \frac{1}{\tan(\theta_j^i)} \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_{\xi\xi} = \begin{bmatrix} \sigma_{x_6}^2 & 0 \\ 0 & \sigma_{x_7}^2 \end{bmatrix}$$

Stochastic correlating errors γ_h

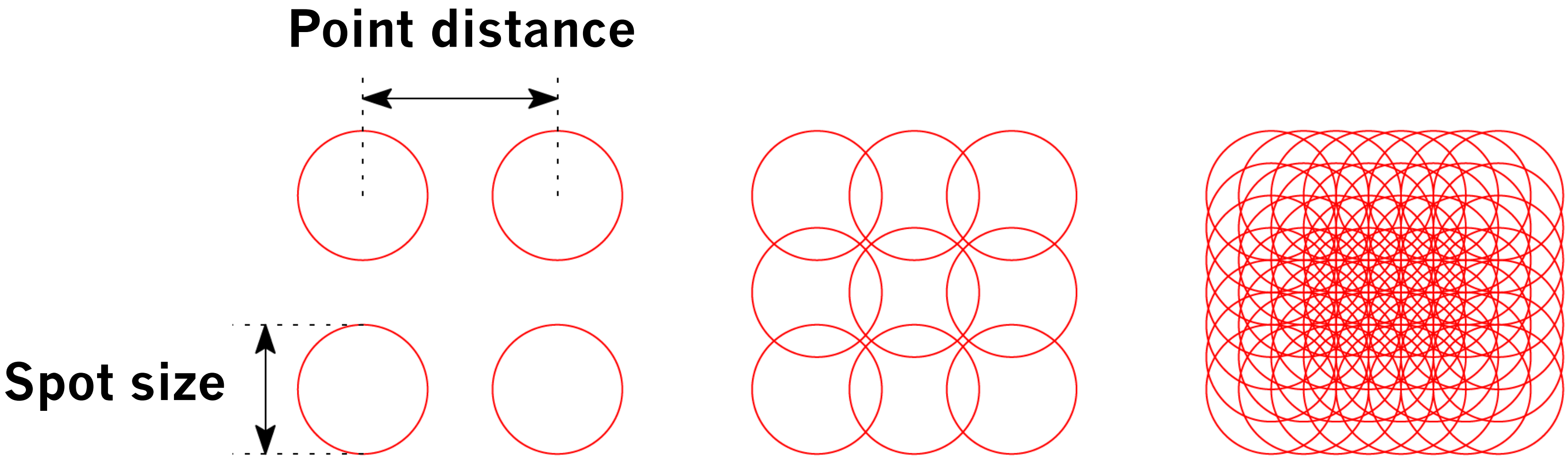
$$\boldsymbol{G_h} = \begin{bmatrix} \frac{\partial l_1}{\partial \gamma_{1h}} & 0 & \cdots & 0 \\ 0 & \frac{\partial l_2}{\partial \gamma_{2h}} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \frac{\partial l_n}{\partial \gamma_{nh}} \end{bmatrix}$$

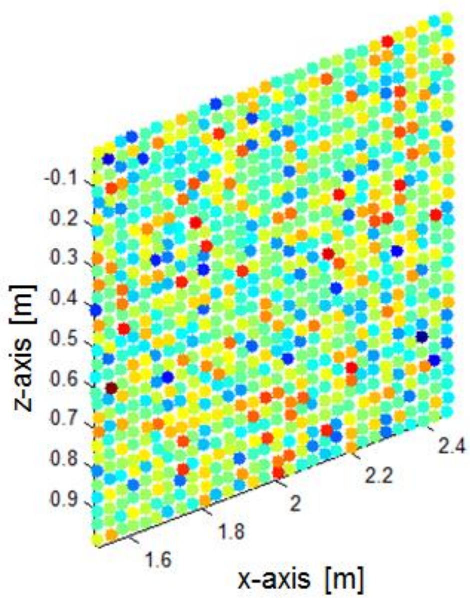
$$\boldsymbol{\Sigma_{\gamma\gamma,h}} = \begin{bmatrix} \sigma_{1h}^2 & \sigma_{12h} & \cdots & \sigma_{1nh} \\ \sigma_{12h} & \sigma_{2h}^2 & \cdots & \sigma_{2nh} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1nh} & \cdots & \cdots & \sigma_{nh}^2 \end{bmatrix}$$

- **Physical correlations** due to measurement process
- Also basin for errors that are not easily to model functionally

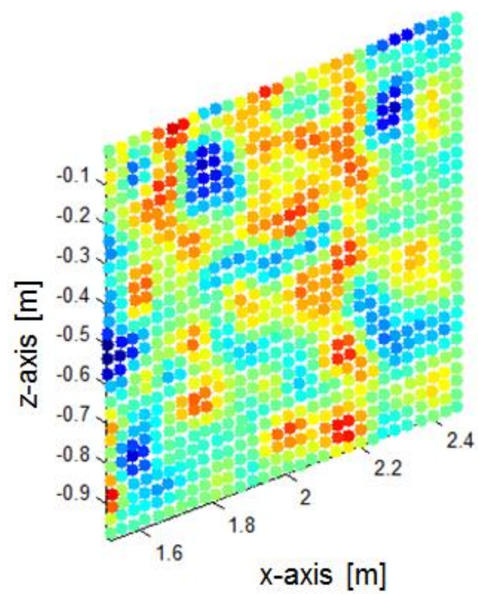
- Overlapping laser spots
- Fluctuations in atmosphere
- Temporally interpolated angular observations
- ...

Example: Overlapping laser spots





Increase of correlations



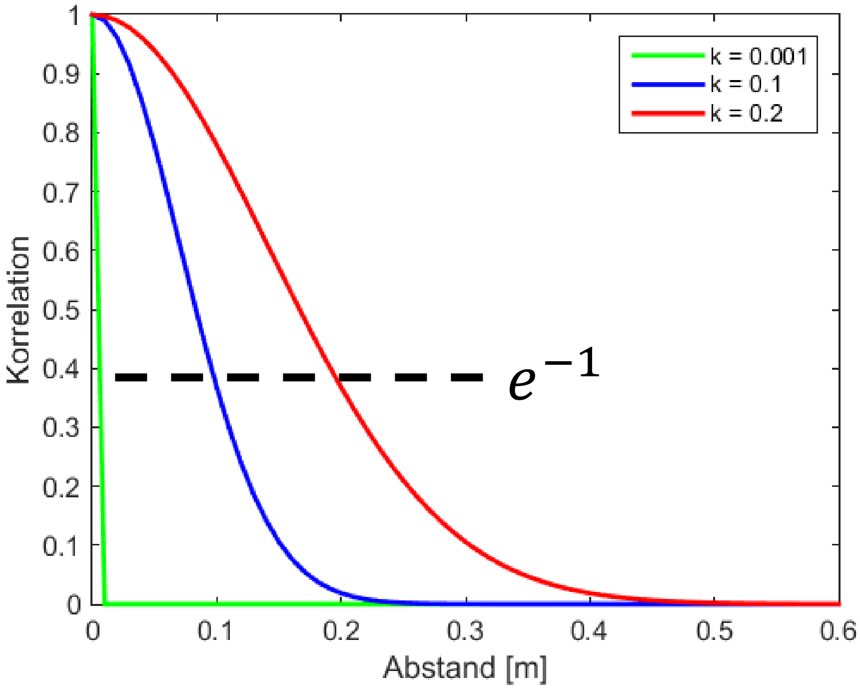
Estimating correlation coefficient

$$R_{ij} = \exp\left(-\left(\frac{d_{ij}}{k}\right)^2\right)$$

R_{ij} … Correlation

d_{ij} … Distance [m]

k … Correlation length [m]



- **Correlation length and funtion unknown!**
- Dependent on intensity (as also random noise)
- But: Most probably not as „easy" to model as intensity based noise for distance measurement

Consequence:

- $\boldsymbol{G_h}$ most probably unknown
- $\boldsymbol{\Sigma_{\gamma\gamma,h}}$ might be easier to assess

Resulting VCM for examples

$$\boldsymbol{\Sigma_{ll}} = \sum_{k=1}^{p} \boldsymbol{D_k} \cdot \boldsymbol{\Sigma_{\delta\delta,k}} \cdot \boldsymbol{D_k^T} + \boldsymbol{F} \cdot \boldsymbol{\Sigma_{\xi\xi}} \cdot \boldsymbol{F^T} + \sum_{h=1}^{q} \boldsymbol{G_h} \cdot \boldsymbol{\Sigma_{\gamma\gamma,h}} \cdot \boldsymbol{G_h^T}$$

$$\boldsymbol{\Sigma_{ll}} = \begin{bmatrix} \left(a \cdot intensity_i^b\right)^2 & & \\ & \ddots & \\ & & \sigma_{\theta_m}^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2}{sin(\theta_j^i)} & \frac{1}{\tan(\theta_j^i)} \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_{x_6}^2 & 0 \\ 0 & \sigma_{x_7}^2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ \frac{2}{\sin(\theta_j^i)} & \frac{1}{\tan(\theta_j^i)} \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}^{\boldsymbol{T}} + ?$$

Here: although quite complex, still extremely simplified, only considering six error sources (noise distance, noise horizontal angle, noise vertical angle, mirror tilt, horizontal axis error, overlapping laser spots)

Synthetic VCM

Purely random, non-correlated errors

(mostly known)

Physical correlations due to measurement process

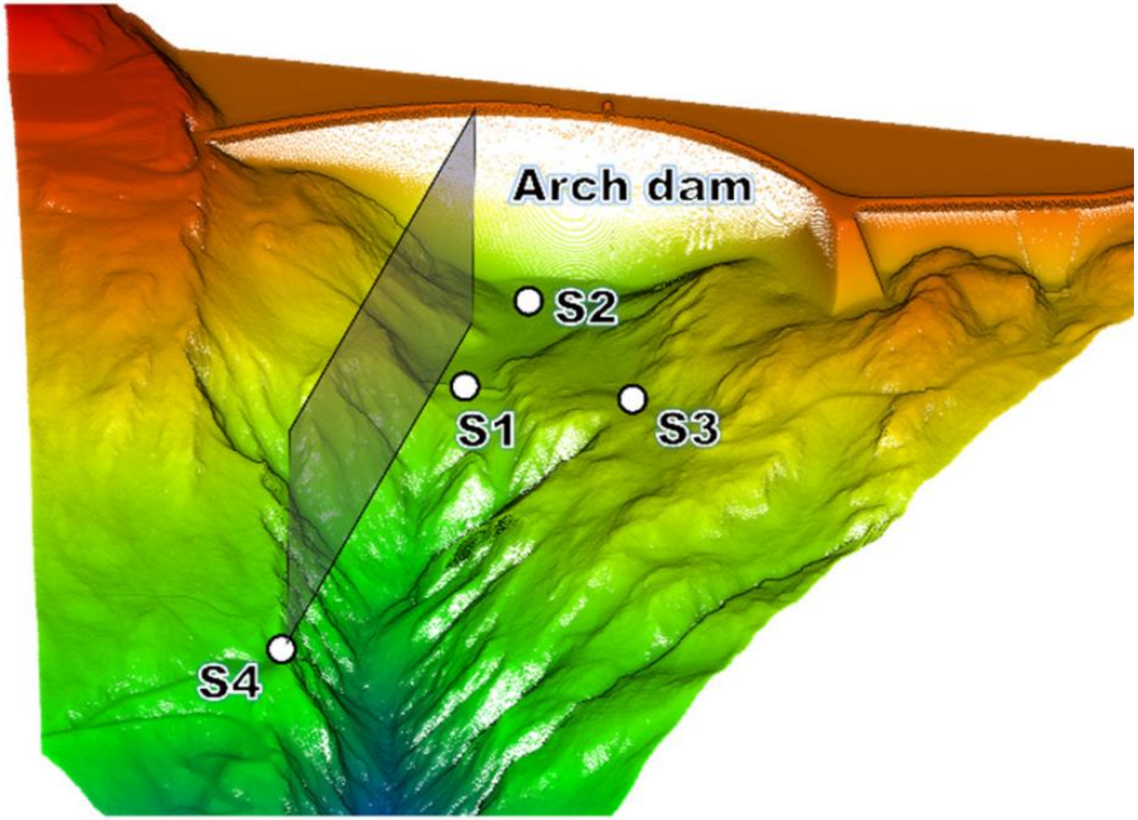
(mostly unknown)

$$\boldsymbol{\Sigma}_{ll} = \sum_{k=1}^{p} \boldsymbol{D}_k \cdot \boldsymbol{\Sigma}_{\delta\delta,k} \cdot \boldsymbol{D}_k^T + \boldsymbol{F} \cdot \boldsymbol{\Sigma}_{\xi\xi} \cdot \boldsymbol{F}^T + \sum_{h=1}^{q} \boldsymbol{G}_h \cdot \boldsymbol{\Sigma}_{\gamma\gamma,h} \cdot \boldsymbol{G}_h^T$$

Mathematical correlations due to variance propagation of systematic errors that have not been eliminated completely

(complex, but known to a certain degree)

Simulation of elementary errors

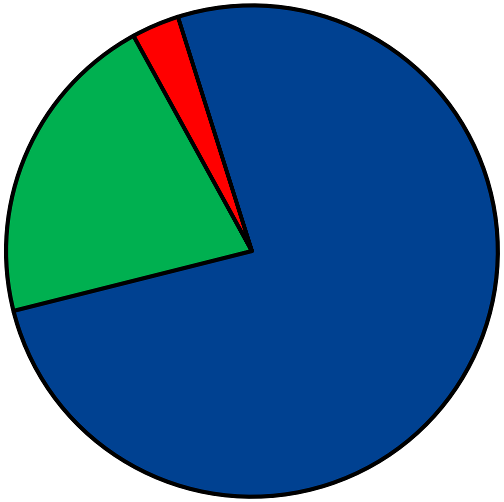


Courtesy: Gabriel Kerekes
& Volker Schwieger

Ratio at total uncertainty

- Non-correlating: noise
- Functional correlating: Instrumental calibration
- Stochastic correlating: atmosphere

(others not modelled)



6 Uncertainty of point clouds

6.4 Stochastic model of single static laser scans

6.4.1 Elementary error model of static laser scans

6.4.2 Resulting stochastic model of a laser scan

6.4.3 Resulting stochastic model of a point cloud

VCM of all measurements

$$\boldsymbol{\Sigma}_{ll} = \begin{bmatrix} \overbrace{\begin{bmatrix} \sigma_{r_1}^2 & \sigma_{\varphi_1 r_1} & \sigma_{\theta_1 r_1} \\ \sigma_{r_1 \varphi_1} & \sigma_{\varphi_1}^2 & \sigma_{\theta_1 \varphi_1} \\ \sigma_{r_1 \theta_1} & \sigma_{\varphi_1 \theta_1} & \sigma_{\theta_1}^2 \end{bmatrix}}^{\boldsymbol{\Sigma}_{1,1}} & \cdots & \overbrace{\begin{bmatrix} \sigma_{r_m r_1} & \sigma_{\varphi_m r_1} & \sigma_{\theta_m r_1} \\ \sigma_{r_m \varphi_1} & \sigma_{\varphi_m \varphi_1} & \sigma_{\theta_m \varphi_1} \\ \sigma_{r_m \theta_1} & \sigma_{\varphi_m \theta_1} & \sigma_{\theta_m \theta_1} \end{bmatrix}}^{\boldsymbol{\Sigma}_{m,1}} \\ \vdots & \ddots & \vdots \\ \underbrace{\begin{bmatrix} \sigma_{r_1 r_m} & \sigma_{\varphi_1 r_m} & \sigma_{\theta_1 r_m} \\ \sigma_{r_1 \varphi_m} & \sigma_{\varphi_1 \varphi_m} & \sigma_{\theta_1 \varphi_m} \\ \sigma_{r_1 \theta_m} & \sigma_{\varphi_1 \theta_m} & \sigma_{\theta_1 \theta_m} \end{bmatrix}}_{\boldsymbol{\Sigma}_{1,m}} & \cdots & \underbrace{\begin{bmatrix} \sigma_{r_m}^2 & \sigma_{\varphi_m r_m} & \sigma_{\theta_m r_m} \\ \sigma_{r_m \varphi_m} & \sigma_{\varphi_m}^2 & \sigma_{\theta_m \varphi_m} \\ \sigma_{r_m \theta_m} & \sigma_{\varphi_m \theta_m} & \sigma_{\theta_m}^2 \end{bmatrix}}_{\boldsymbol{\Sigma}_{m,m}} \end{bmatrix}$$

VCM of a single point

$$\boldsymbol{\Sigma}_{\boldsymbol{i},\boldsymbol{i}} = \begin{bmatrix} \sigma_{r_i}^2 & \sigma_{\varphi_i r_i} & \sigma_{\theta_i r_i} \\ \sigma_{r_i \varphi_i} & \sigma_{\varphi_i}^2 & \sigma_{\theta_i \varphi_i} \\ \sigma_{r_i \theta_i} & \sigma_{\varphi_i \theta_i} & \sigma_{\theta_i}^2 \end{bmatrix}$$

- $\sigma_{r_i}^2$: Laser Radar Equation / Intensity based or empirical
- $\sigma_{\varphi_i}^2, \sigma_{\theta_i}^2$: manufacturer‘s specifications
- $\sigma_{r_i\varphi_i}, \sigma_{r_i\theta_i}, \sigma_{\varphi_i\theta_i} = 0$

Covariances between points

$$\boldsymbol{\Sigma_{i,j}} = \begin{bmatrix} \sigma_{r_i r_j} & \sigma_{\varphi_i r_j} & \sigma_{\theta_i r_j} \\ \sigma_{r_i \varphi_j} & \sigma_{\varphi_i \varphi_j} & \sigma_{\theta_i \varphi_j} \\ \sigma_{r_i \theta_j} & \sigma_{\varphi_i \theta_j} & \sigma_{\theta_i \theta_j} \end{bmatrix}$$

- $\sigma_{r_i r_j}$: Overlapping laser spots *(current research)*
- $\sigma_{\varphi_i \varphi_j}$: Temporal interpolation, slow rotation *(current research)*
- $\sigma_{\theta_i \theta_j}$: Temporal interpolation, fast rotation *(current research)*
- $\sigma_{r_i \varphi_j}, \sigma_{r_i \theta_j}, \sigma_{\varphi_i \theta_j} = 0$

Mostly used VCM

$$\boldsymbol{\Sigma}_{ll} = \begin{bmatrix} \overbrace{\begin{bmatrix} \sigma_{r_1}^2 & & \\ & \sigma_{\varphi_1}^2 & \\ & & \sigma_{\theta_1}^2 \end{bmatrix}}^{\textbf{first point}} & \cdots & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} & \cdots & \underbrace{\begin{bmatrix} \sigma_{r_m}^2 & & \\ & \sigma_{\varphi_m}^2 & \\ & & \sigma_{\theta_m}^2 \end{bmatrix}}_{\textbf{last point}} \end{bmatrix}$$

- $\sigma_{r_i}^2$: Laser Radar Equation / Intensity based or empirical
- $\sigma_{\varphi_i}^2, \sigma_{\theta_i}^2$: manufacturer‘s specifications

6 Uncertainty of point clouds

6.4 Stochastic model of single static laser scans

6.4.1 Elementary error model of static laser scans

6.4.2 Resulting stochastic model of a laser scan

6.4.3 Resulting stochastic model of a point cloud

VCM of 3D point

Polar observations to 3D cartesian coordinates

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} r \cdot \sin\theta \cdot \sin\varphi \\ r \cdot \sin\theta \cdot \cos\varphi \\ r \cdot \cos\theta \end{bmatrix}}_{f(\boldsymbol{l}) = f(r,\varphi,\theta)}$$

Variance propagation

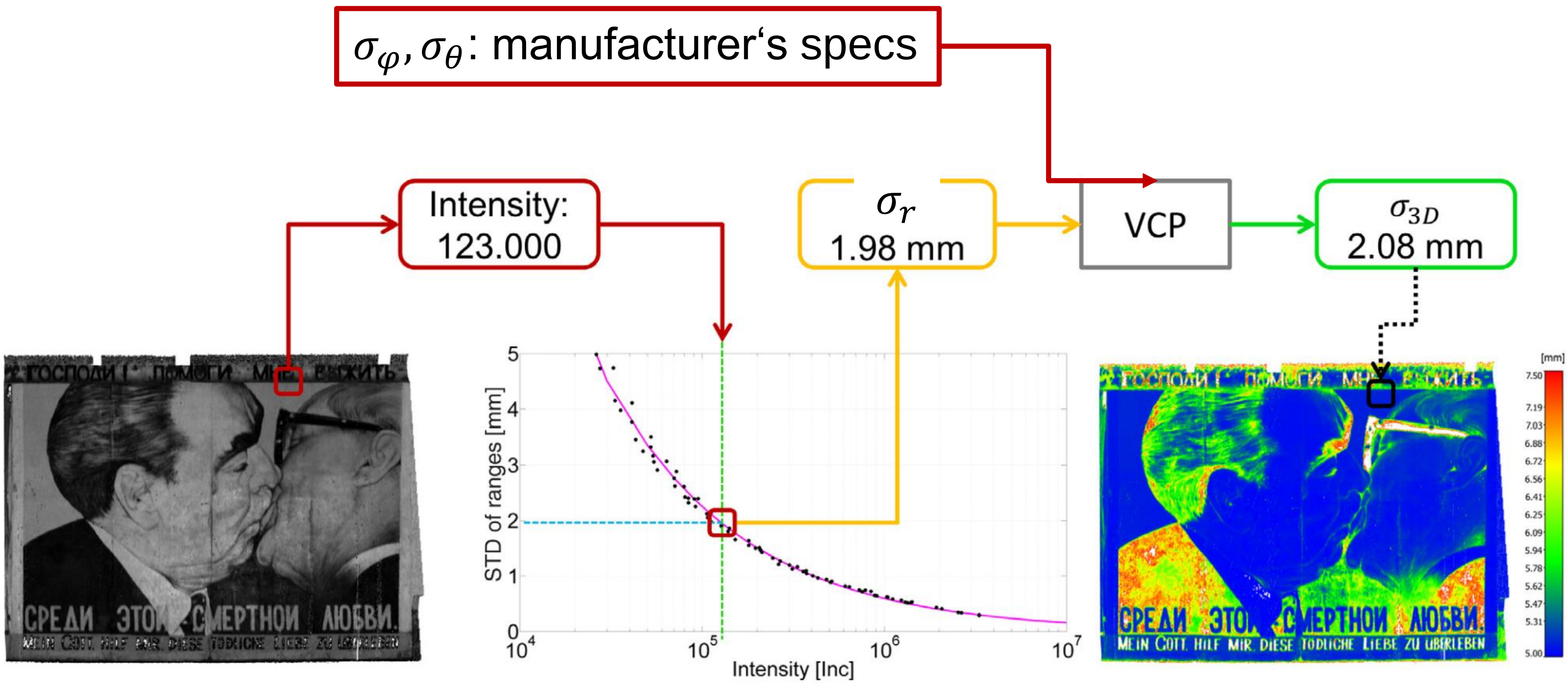
$$\boldsymbol{\Sigma}_{\boldsymbol{xyz}} = \left[\frac{\partial f}{\partial \boldsymbol{l}}\right] \cdot \boldsymbol{\Sigma}_{\boldsymbol{ll}} \cdot \left[\frac{\partial f}{\partial \boldsymbol{l}}\right]^{\boldsymbol{T}}$$

VCM of complete point cloud

$$\boldsymbol{\Sigma}_{xyz} = \begin{bmatrix} \overbrace{\begin{bmatrix} \sigma_{x_1}^2 & \sigma_{y_1x_1} & \sigma_{z_1x_1} \\ \sigma_{x_1y_1} & \sigma_{y_1}^2 & \sigma_{z_1y_1} \\ \sigma_{x_1z_1} & \sigma_{y_1z_1} & \sigma_{z_1}^2 \end{bmatrix}}^{\boldsymbol{\Sigma}_{xyz,1,1}} & \cdots & \boldsymbol{\Sigma}_{xyz,m,1} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_{xyz,1,m} & \cdots & \underbrace{\begin{bmatrix} \sigma_{x_m}^2 & \sigma_{y_mx_m} & \sigma_{z_mx_m} \\ \sigma_{x_my_m} & \sigma_{y_m}^2 & \sigma_{z_my_m} \\ \sigma_{x_mz_m} & \sigma_{y_mz_m} & \sigma_{z_m}^2 \end{bmatrix}}_{\boldsymbol{\Sigma}_{xyz,m,m}} \end{bmatrix}$$

- $\Sigma_{xyz,1,1}$: fully-populated based on variance propagation
- $\Sigma_{xyz,1,m}$: dependent on Σ_{ll}

Example for mostly used VCM



Courtesy: Daniel Wujanz
(fig. changed)

Interim summary

- Stochastic model expressed by VCM assuming that systematic errors have been minimized sufficiently
- Synthetic VCM includes non-correlating, functional correlating and stochastic correlating errors
- Not all entries are known / easy to include => mostly used VCM is too simplistic, esp. neglecting physical correlations
- If too simplistic, significance of VCM is questionable

6 Uncertainty of point clouds

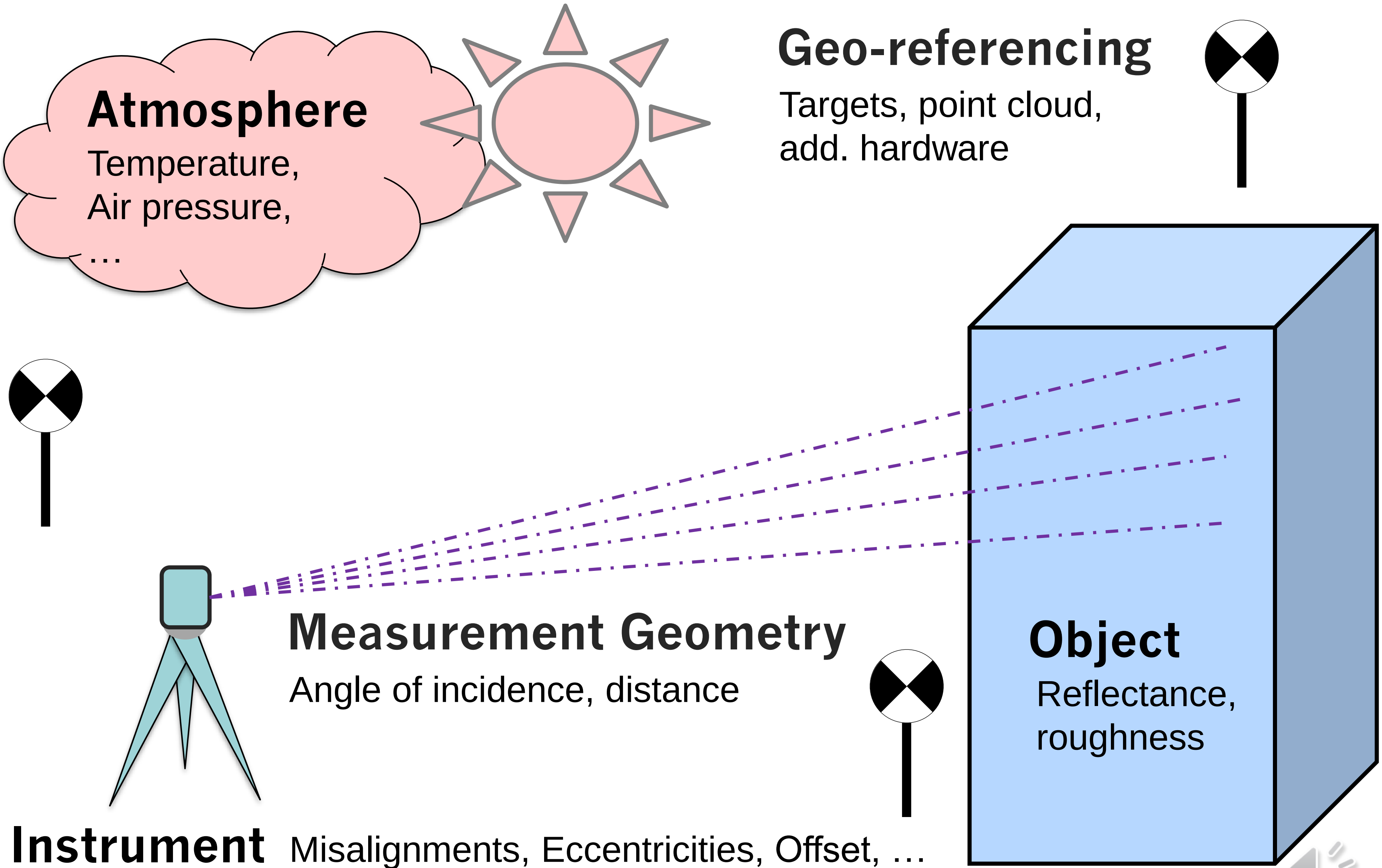
6.5 Stochastic model of geo-referenced point clouds

6.5.1 Static laser scanning

6.5.2 Mobile laser scanning

6.5.3 Camera based mobile mapping

Error sources at laser scanning



Static laser scanning

$$f: \begin{bmatrix} x \\ y \\ z \end{bmatrix}^g = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \boldsymbol{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot \underbrace{\begin{bmatrix} r \cdot \sin\theta \cdot \sin\varphi \\ r \cdot \sin\theta \cdot \cos\varphi \\ r \cdot \cos\theta \end{bmatrix}^s}_{\begin{bmatrix} x \\ y \\ z \end{bmatrix}^s}$$

- Until now:

$$\boldsymbol{\Sigma}_{ll} = \boldsymbol{\Sigma}_{ll} \text{ or } \boldsymbol{\Sigma}_{xyz} = \boldsymbol{\Sigma}_{xyz}^s$$

- New:

$$\boldsymbol{\Sigma}_{xyz}^g = \left[\frac{\partial f}{\partial(\boldsymbol{p},\boldsymbol{l})}\right] \cdot \begin{bmatrix} \boldsymbol{\Sigma}_{pp} & \boldsymbol{\Sigma}_{pl} \\ \boldsymbol{\Sigma}_{lp} & \boldsymbol{\Sigma}_{ll} \end{bmatrix} \cdot \left[\frac{\partial f}{\partial(\boldsymbol{p},\boldsymbol{l})}\right]^T$$

$$\boldsymbol{p} = [t_x, t_y, t_z, \varepsilon_x, \varepsilon_y, \varepsilon_z]^T$$

VCM of geo-referencing

$$\begin{bmatrix} \Sigma_{pp} & \Sigma_{pl} \\ \Sigma_{lp} & \Sigma_{ll} \end{bmatrix} \Rightarrow \Sigma_{pp}$$

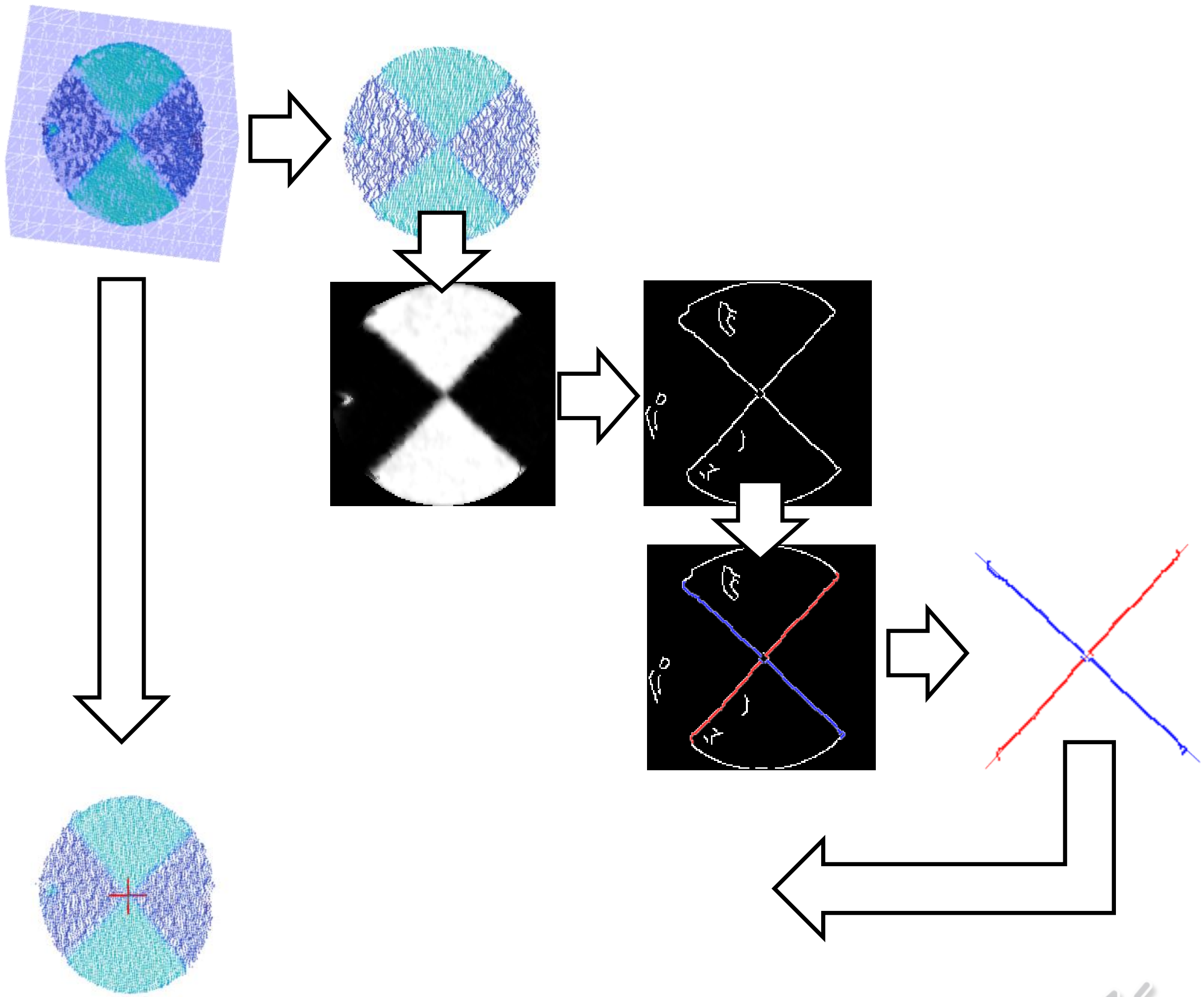
- Example: GNSS based geo-referencing with targets as identical points

1. **Estimating local target coordinates using laser scans => Σ_{target}**
   – *See next slide*
2. **Estimating geo-referenced coordinates using GNSS => Σ_{GNSS}**
   – Multipath, satellite orbits, clock errors, antenna errors, ...
3. **Estimating transformation parameters => Σ_{trafo}**
   – Number and geometric distribution of targets

$$\Rightarrow \Sigma_{pp} = \Sigma_{pp}\left(\Sigma_{target}, \Sigma_{GNSS}, \Sigma_{trafo}\right)$$

Estimating target coordinates

1. Plane estimation
2. Image gradients
3. Line intersection
4. 3D coordinate of target



Summarising uncertainty

GNSS uncertainty + Target estimation uncertainty (+ further) => Transformation parameter uncertainty

$$\boldsymbol{\Sigma}_{xyz}^{g} = \left[\frac{\partial f}{\partial(\boldsymbol{p},\boldsymbol{l})}\right] \cdot \begin{bmatrix} \boldsymbol{\Sigma}_{pp} & \boldsymbol{\Sigma}_{pl} \\ \boldsymbol{\Sigma}_{lp} & \boldsymbol{\Sigma}_{ll} \end{bmatrix} \cdot \left[\frac{\partial f}{\partial(\boldsymbol{p},\boldsymbol{l})}\right]^{T}$$

Uncertainty of local laser scan

=> Much more complex to model than for local scan

6 Uncertainty of point clouds

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^g = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \boldsymbol{R}_n^e(L,B) \cdot \boldsymbol{R}_b^n(\phi,\theta,\psi) \cdot \left(\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \boldsymbol{R}_s^b(\alpha,\beta,\gamma) \cdot \begin{bmatrix} 0 \\ d \cdot \sin b \\ d \cdot \cos b \end{bmatrix}^s \right)$$

- Until now:

$$\boldsymbol{\Sigma}_{ll} = \boldsymbol{\Sigma}_{ll}$$

- New:

$$\boldsymbol{\Sigma}_{xyz}^{g} = \left[\frac{\partial f}{\partial(\boldsymbol{p},\boldsymbol{l})}\right] \cdot \begin{bmatrix} \boldsymbol{\Sigma}_{pp} & \boldsymbol{\Sigma}_{pl} \\ \boldsymbol{\Sigma}_{lp} & \boldsymbol{\Sigma}_{ll} \end{bmatrix} \cdot \left[\frac{\partial f}{\partial(\boldsymbol{p},\boldsymbol{l})}\right]^T$$

$$\boldsymbol{p} = \left[t_x, t_y, t_z, L, B, \phi, \theta, \psi, \Delta x, \Delta y, \Delta z, \alpha, \beta, \gamma\right]^T$$

VCM of geo-referencing

$$\begin{bmatrix} \Sigma_{pp} & \Sigma_{pl} \\ \Sigma_{lp} & \Sigma_{ll} \end{bmatrix} \Rightarrow \Sigma_{pp}$$

- **Participating sensors: GNSS, IMU, …**
  - Changing environment for kinematic GNSS?
  - Drifts => IMU?
- **Geometrical relationships => system calibration**
  - Stable and uncertainty can be assessed with acceptable effort
- **Temporal relationships => sensor synchronization**
  - Process rather deterministic
- **Combination in Kalman filter**
  - Complex, also due to process noise

=> Again, much more complex and many quantities unknown

6 Uncertainty of point clouds

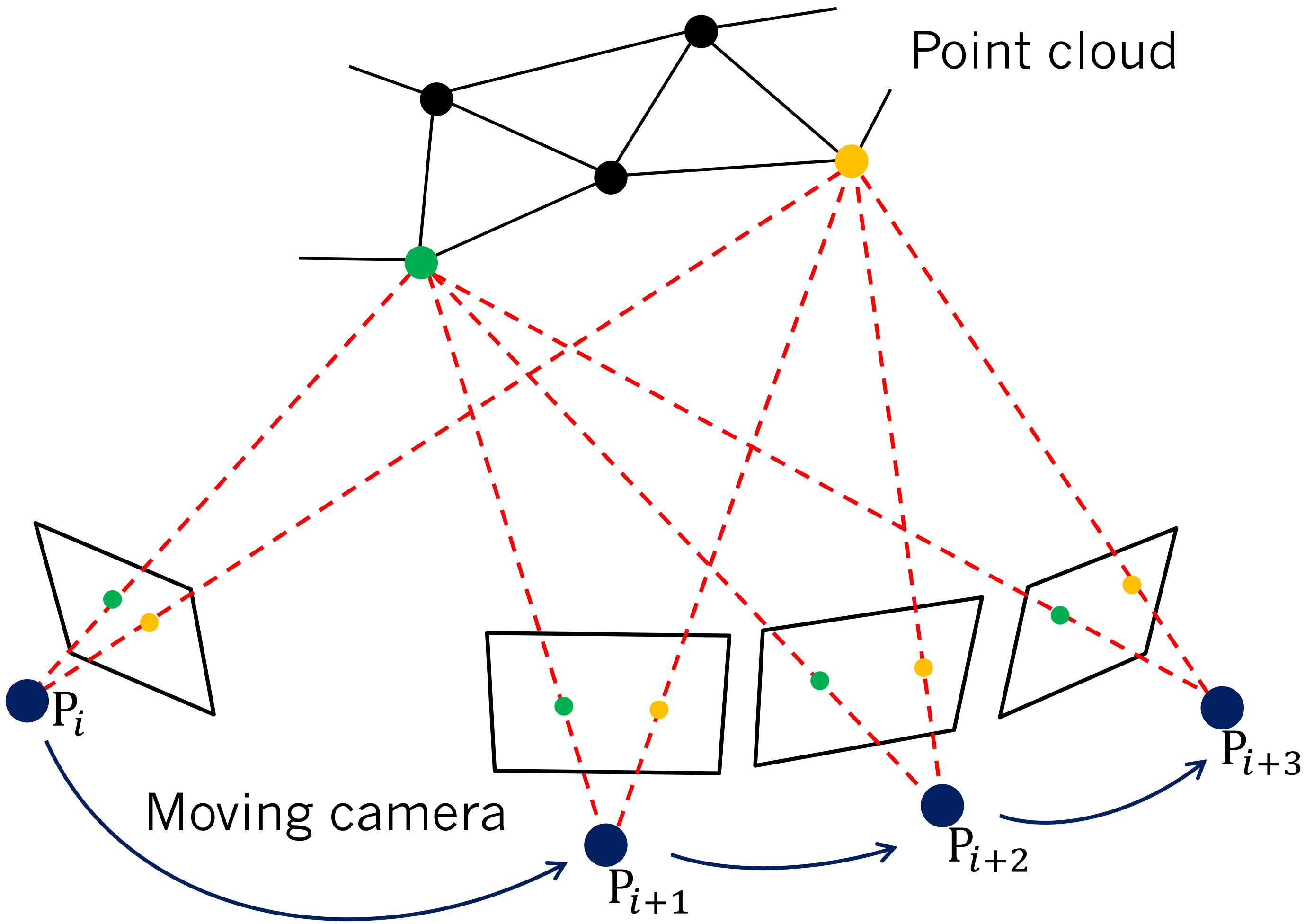
6.5 Stochastic model of geo-referenced point clouds

6.5.1 Static laser scanning

6.5.2 Mobile laser scanning

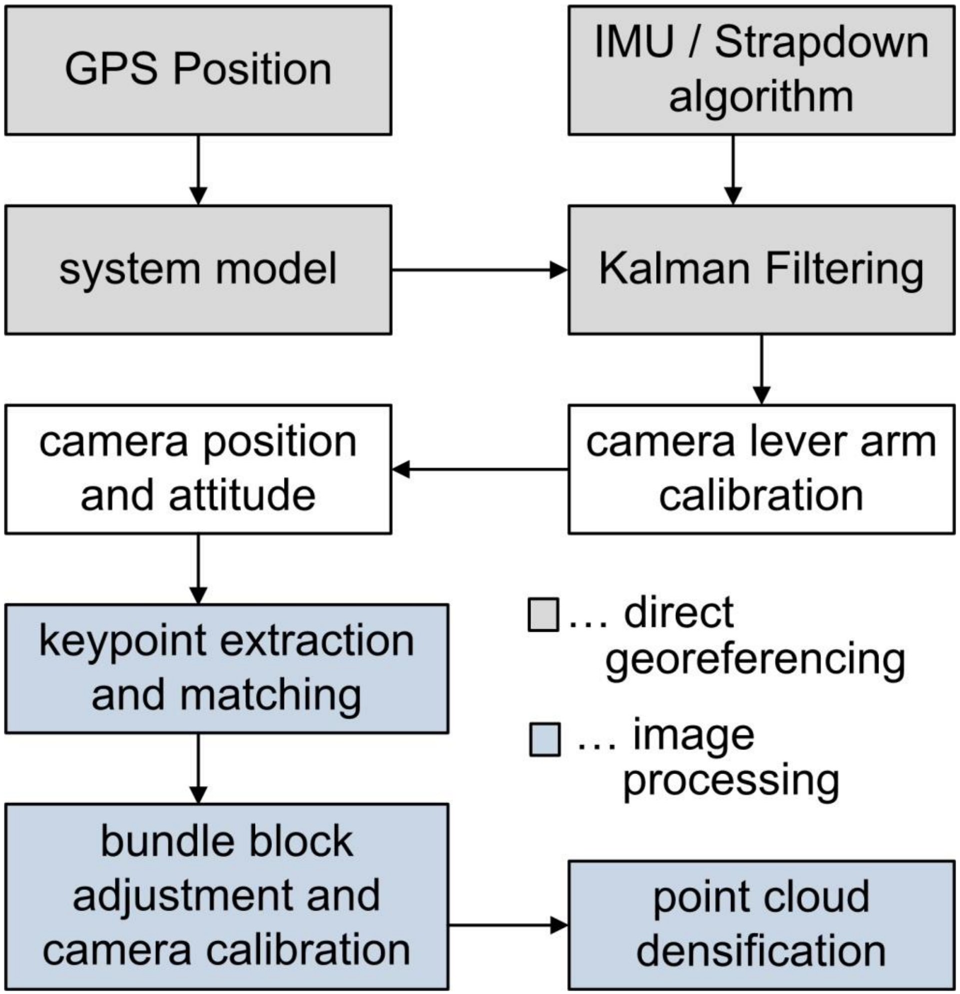
6.5.3 Camera based mobile mapping

From image to point cloud



● and ● : corresponding feature points

VCM of point cloud



- Direct georeferencing
  - GPS influences (baseline length, multi path, shadowing, satellite geometry, ...)
  - Duration of measurement / GPS outages => INS
  - Movement of UAV
- Calibration parameters
- Image processing
  - Distance to object
  - Number and geometric distribution of key points
  - ....

Courtesy: Christian Eling & Lasse Klingbeil

$\Rightarrow \boldsymbol{\Sigma}_{xyz}^{g} = ?$

Interim summary

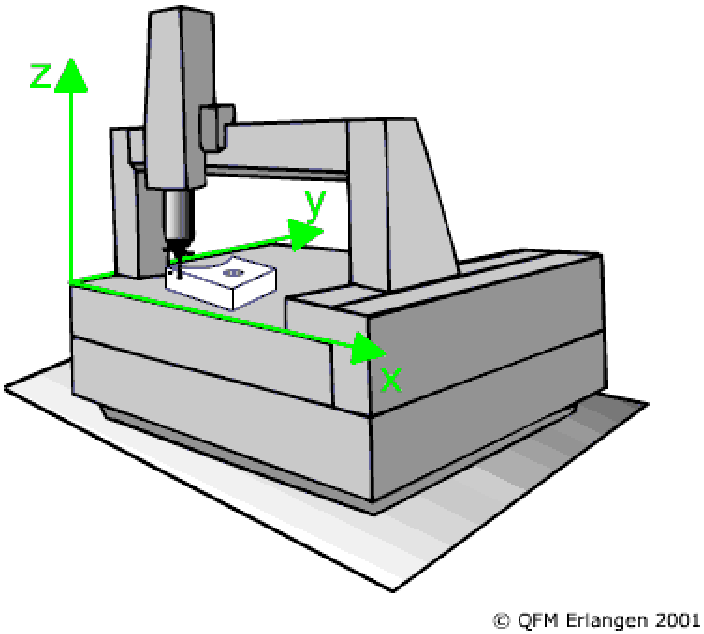
- Complexity of stochastic model increases noticeably when geo-referencing point clouds
- For mobile laser scanning, the VCM might still be modelled by variance propagation to some amount, but individual uncertainties are partially not accessible (e.g., current GNSS uncertainty)
- For camera based mapping systems, the variance propagation cannot be performed due to a too complex processing chain with unknown distribution functions of participating methods
- Instead, the uncertainty is often rather determined empirically based on the given point clouds

6 Uncertainty of point clouds

6.6 Determining the uncertainty of existing point clouds

Relation to metrology in general

- **VDI/VDE 2617, Part 2.3:** Accuracy of coordinate measuring machines (CMM) of large dimensions



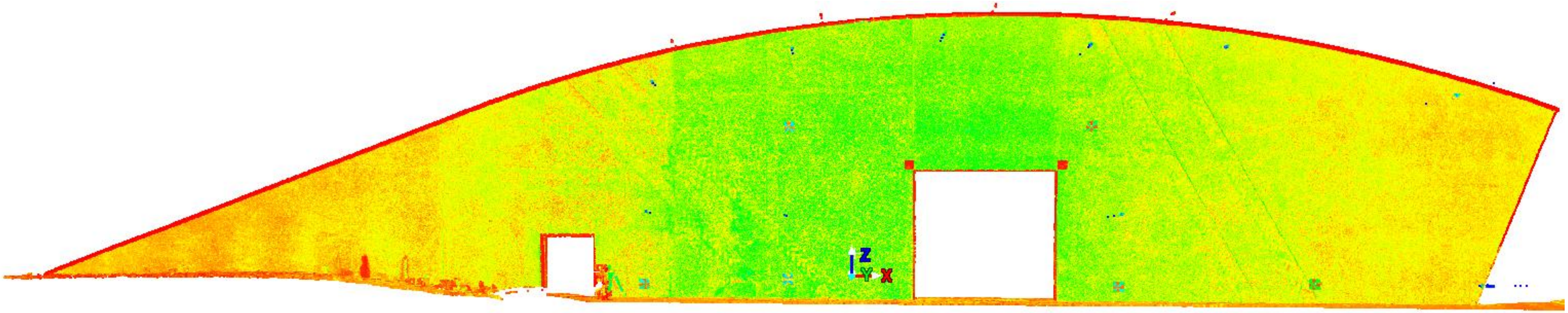
a) **Probing error:** Error of indication within which the range of radii of a spherical material standard of size can be determined by a CMM.

b) **Error of indication for size measurement:** Error of indication when determining the size of a material standard of size, using a coordinate measuring machine.

c) **Error of indication of a multi-arm CMM for size Measurement:** Error of indication when determining the size of a material standard of size, using two probing systems within the same coordinate system.

Courtesy: A. Weckenmann;

Uncertainty in context of scale



a) Smoothness

b) Geometric (datum free/local) properties

c) Position and orientation

Increasing scale →

Relative accuracy / precision

Absolute accuracy / bias

Instrument/scanner

Geometry/Object

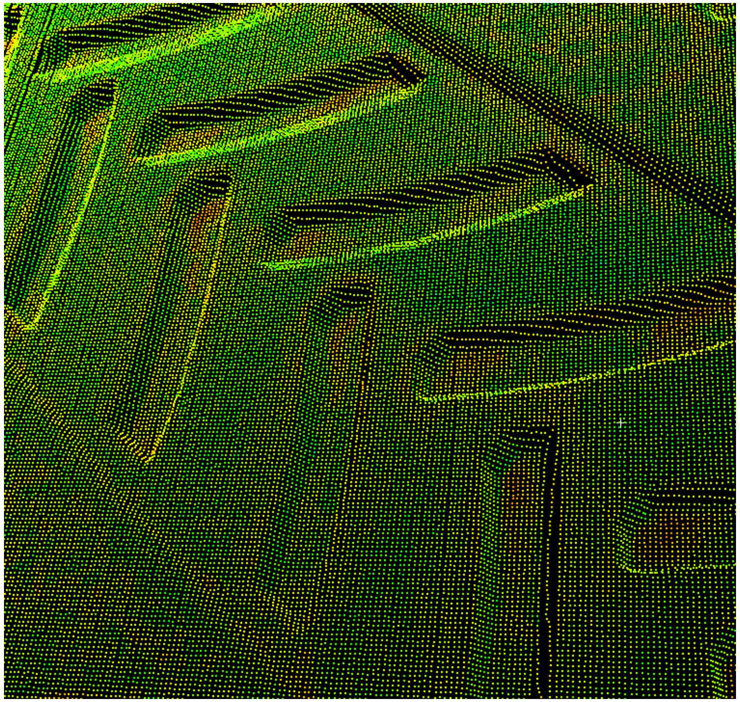
Atmosphere

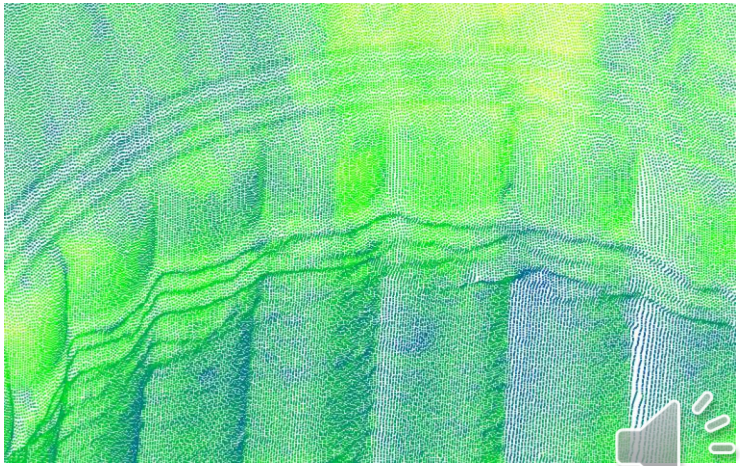
Geo-referencing static

Geo-referencing mobile

a) Smoothness

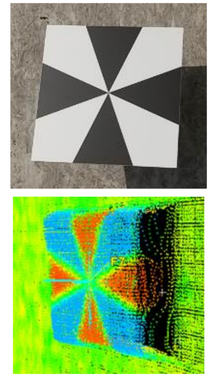
- **Reference:**
  - Object assumed to be smoother than uncertainty
- **Quantity:**
  - Precision of points in local neigborhood
- **Procedure:**
  - Estimating smoothness by averaging neighbored points
  - E.g., plane fitting, ...

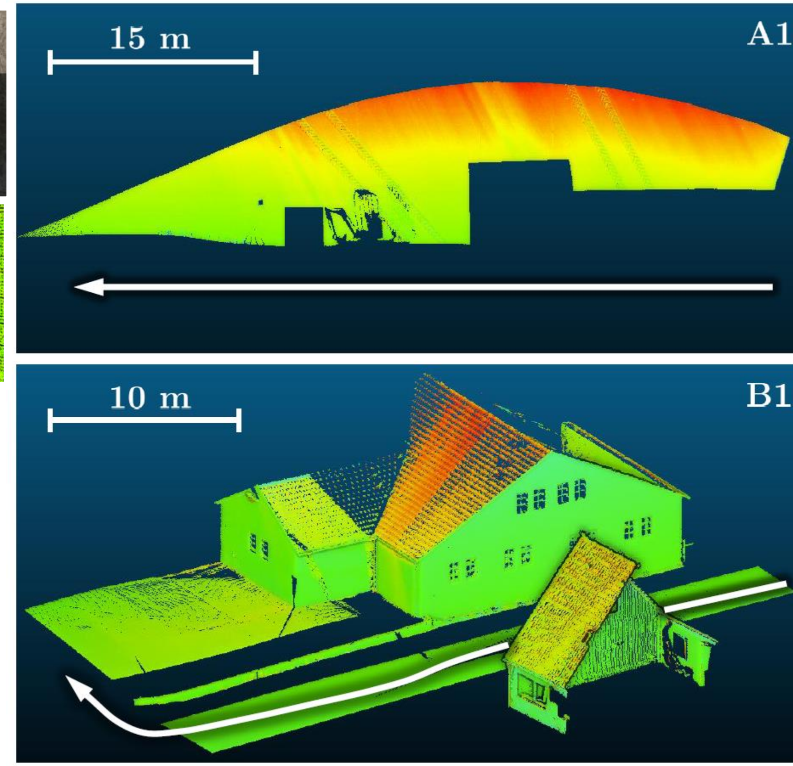


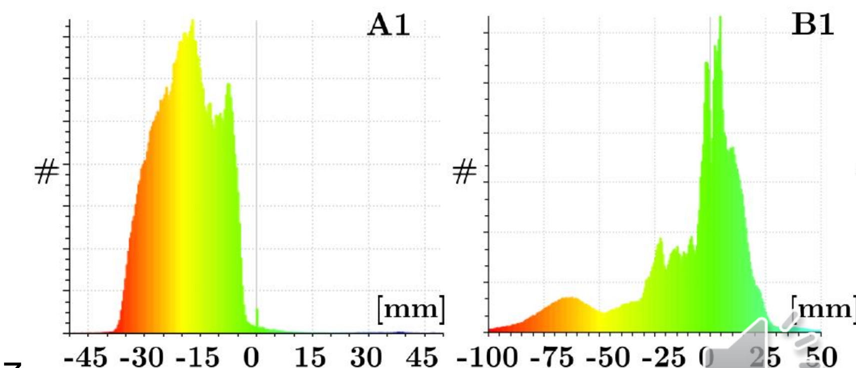


b) Geometric properties

- **Reference:**
  - Target coordinates
  - Point clouds of scanned object
- **Quantity:**
  - Differences between distances of targets
  - Relative differences between point clouds of objects
- **Procedure:**
  - Vector estimation
  - Point cloud comparison
  - Parameter analysis of plane fitting or similar



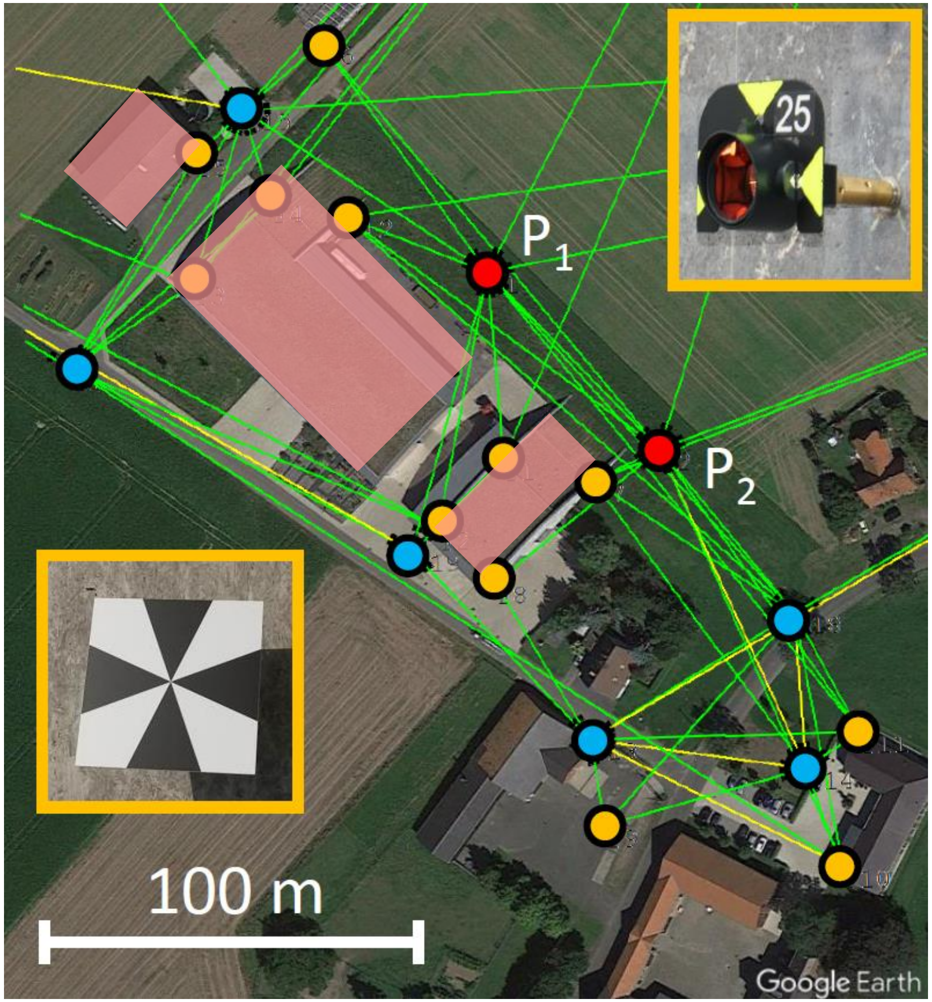




Courtesy: Erik Heinz

c) Position and orientation

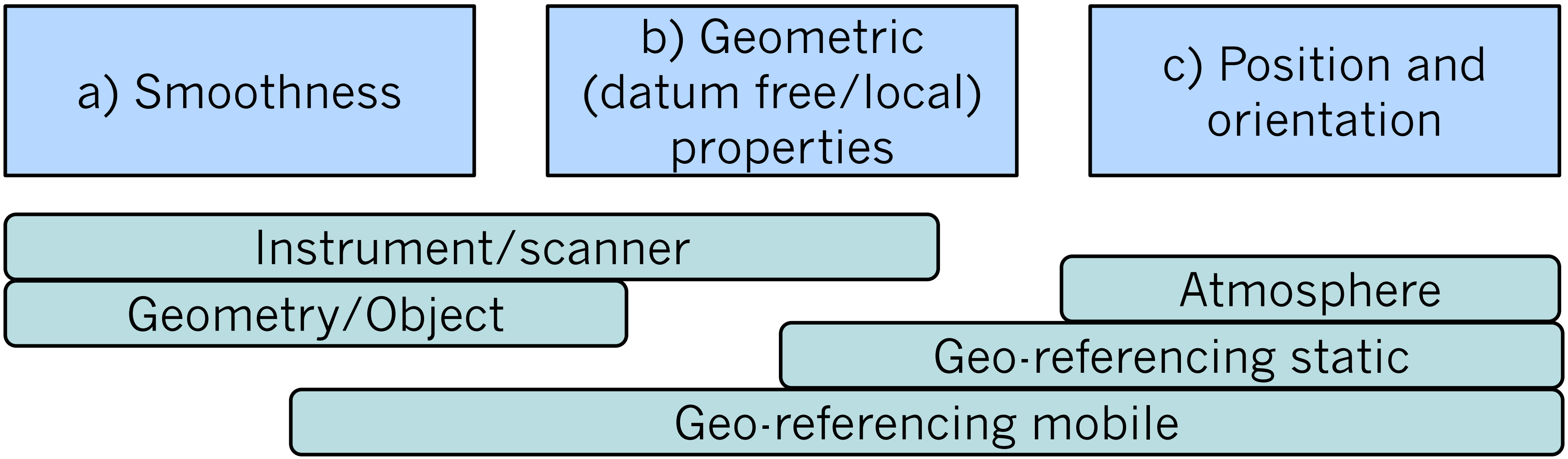
- **Reference:**
  - Superordinate coordinate system materialized by signalized targets and digitalized objects
- **Quantity:**
  - Differences between positions of targets
  - Absolute differences between point clouds of objects
- **Procedure:**
  - Coordinate transformations
  - Point cloud comparison
  - Parameter analysis of plane fitting or similar



Courtesy: Erik Heinz

Identification of error sources

- If we quantify the uncertainty with the aforementioned methods, can we also identify the error causing sources and their actual transmission to build a VCM?



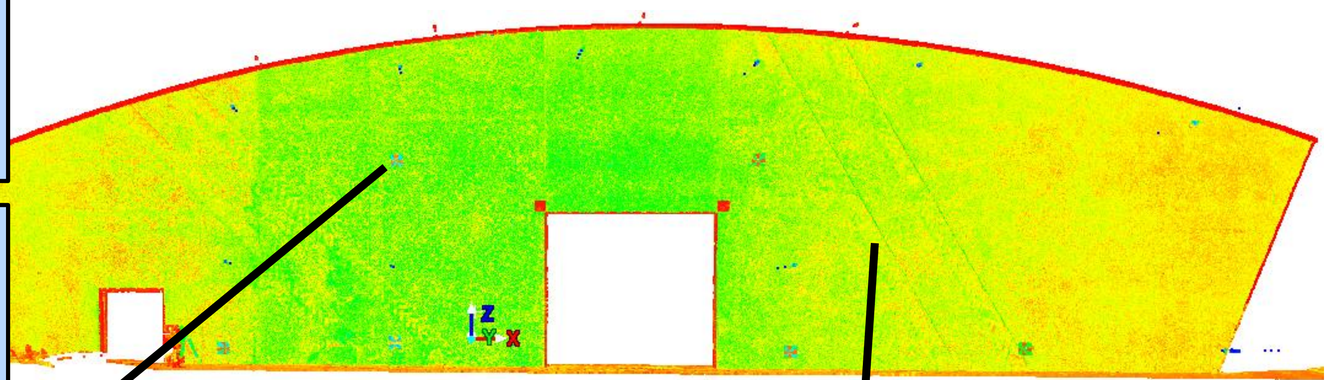
- We might connect the uncertainty to general sources, as e.g. the instrument or the geo-referencing
- Explicitly identifying individual errors requires special setups to be able to separate them

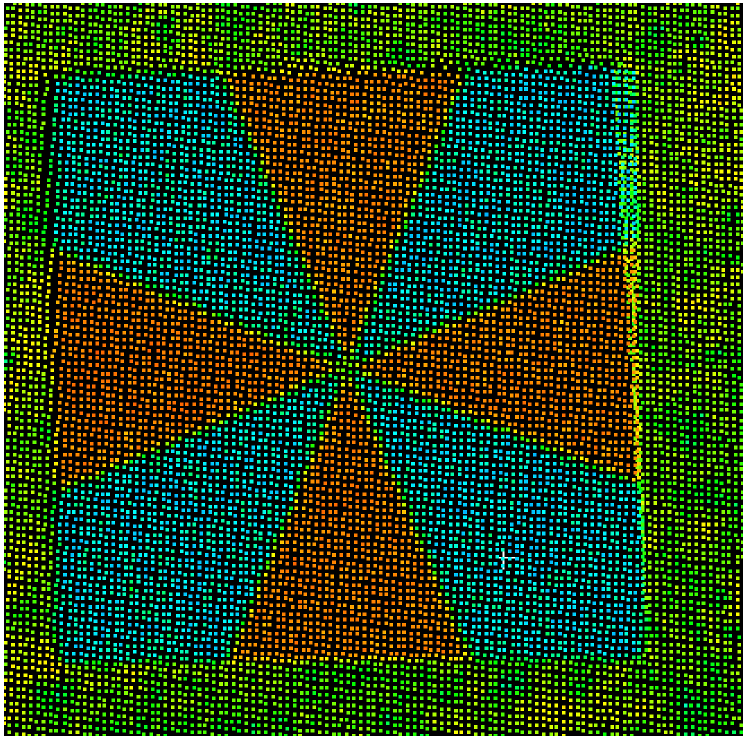
=> Designing an evaluation environment for mobile mapping systems

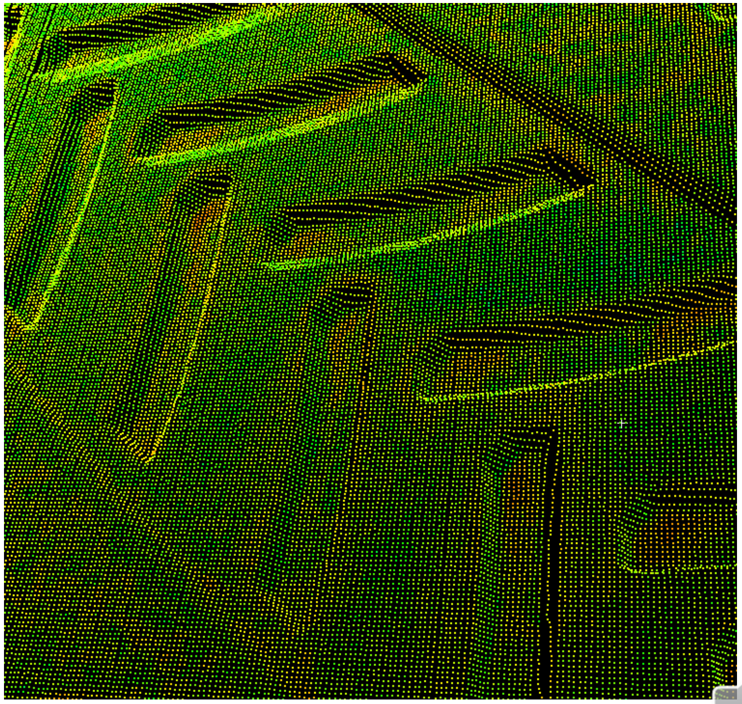
Uni Bonn: reference point clouds

a) Smoothness

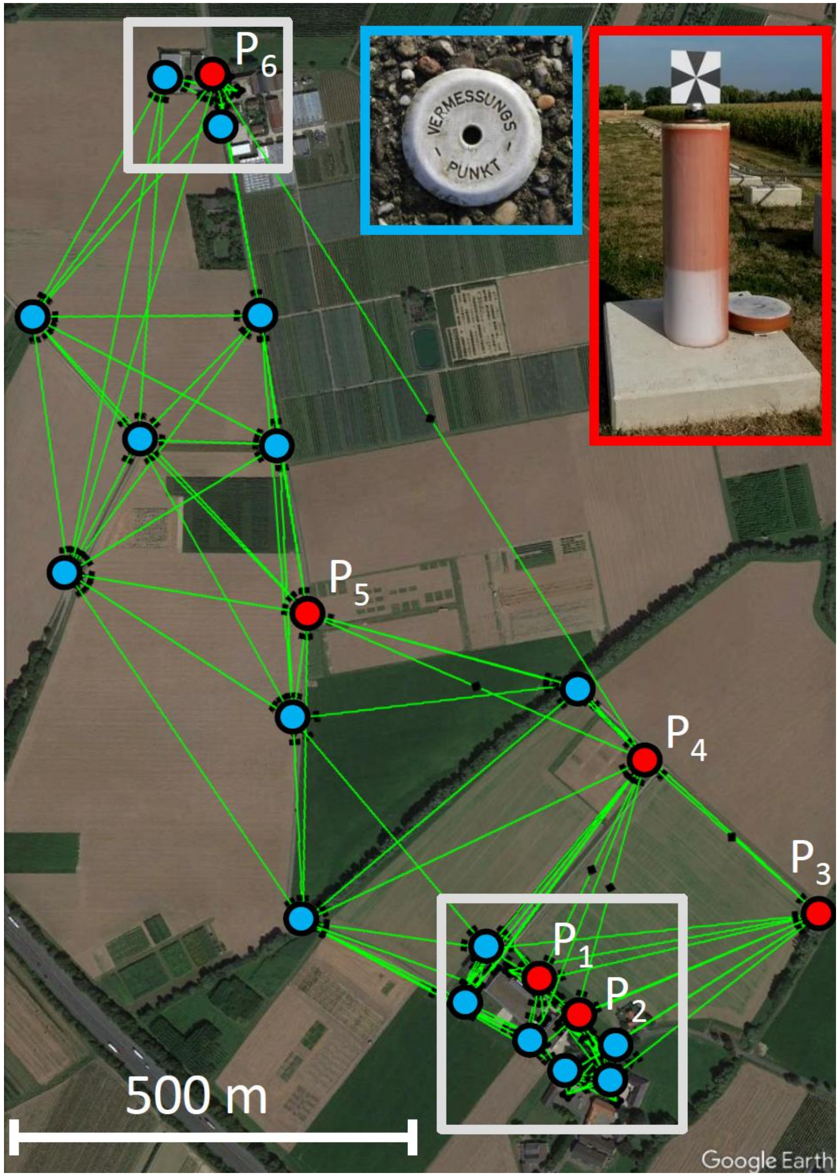
b) Geometric (datum free/local) properties







Uni Bonn: reference point field



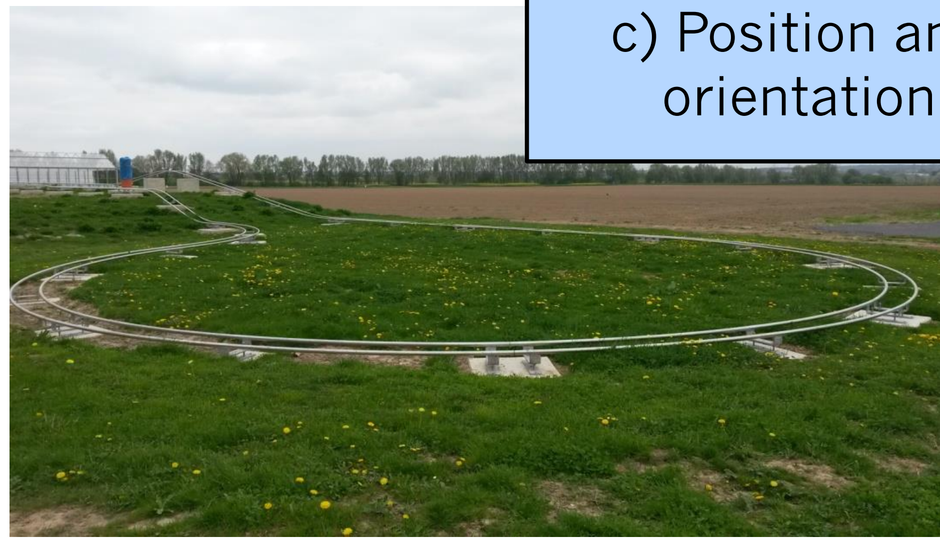
Courtesy:
Erik Heinz

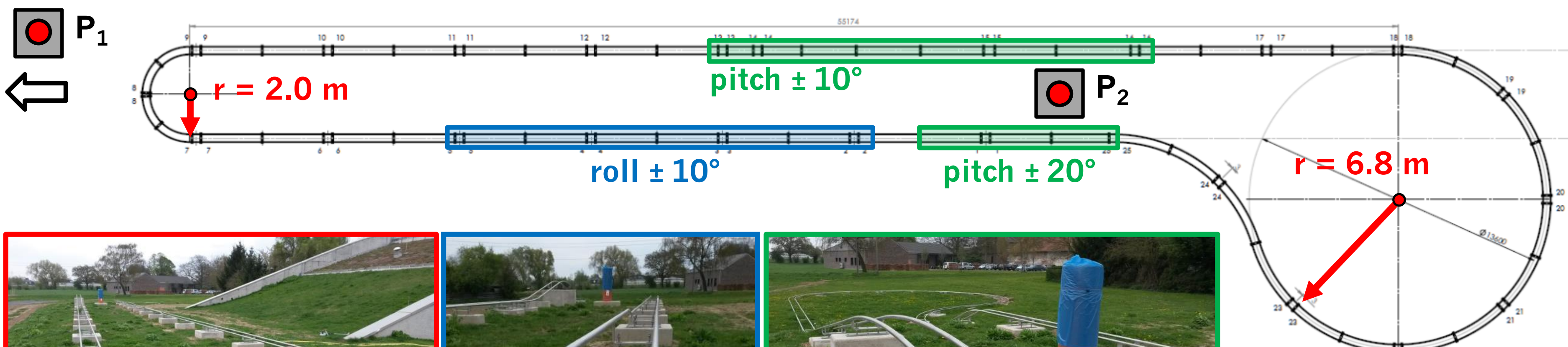
- Refrence points: pillars (# 6), bolts at houses (# 15), nails on ground (# 16)
- Point uncertainties: mm to sub-mm
- ETRS89/UTM32
- Measured by total station, GNSS, levelling
- Stability checked by regular deformation analysis

c) Position and orientation

Uni Bonn: reference trajectory

c) Position and orientation





Courtesy: Erik Heinz & Lasse Klinbeil

Interim summary

- Uncertainty of point clouds can be evaluated/ determined in different scales
- Based on the scale (local to global), different error sources are of varying impact for the uncertainty
- To be able to identify individual error sources within the complete uncertainty of the point cloud, a sophisticated environment needs to be materialized (pillars, structures, ...)

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Learning objectives

- How to describe the uncertainty of a point cloud within a variance-covariance matrix?
- What makes the variance-covariance matrix complex?
- How to determine the uncertainty of existing point clouds?