Ex. 04: Bundle Adjustment

Submission: 18.12.2020
Points: 25

In this exercise you need to derive the normal equations of the bundle adjustment described in the lecture. You're given the initial guess of the position of *four identical calibrated cameras* as well as *eight object points*, where each point is observed in each camera. Apart from that no further observations were made. Assume the rotation matrices R_j to be given with $R_j = I_3$. Therefore the rotations κ, θ, ω are not included as unknowns in the bundle adjustment. Furthermore, assume all cameras to be *euclidian cameras*. The calibration matrix of this camera type is defined by the camera constant c and its principal point $[x_h, y_h]^T$.

1. **(2)** Write down the observation equation of the bundle adjustment. How many observations do you get for each observed point?

2. **(3)** How many observations and unkowns are involved in the configuration described above? Determine the redundancy of the system.

3. **(8)** Given the *collinearity constraint* $\mathbf{x}'_{ij} = \lambda_{ij} P_j \mathbf{X}_i$ derive the functional model $\boldsymbol{x}'_{ij} = f(\mathbf{X}_{0j}, \mathbf{X}_i)$ of an point i oberserved in image j.

   - Start by writing down the **explicit** representation of the projection matrix. Take the assumptions described above into account.
   - In the next step transform the model into euclidean coordinates by eliminating the unkown scale factor λ_{ij}.
   - Simplify the equations to the greatest possible extent.
   - Write down the final observation equation of an point i observed in image j in euclidean coordinates $\boldsymbol{x}'_{ij} = [x'_{ij}, y'_{ij}]$.

   Note that $\mathbf{x}'_{ij}$ describes a point in homogenous coordinates whereas $\boldsymbol{x}'_{ij}$ desribes a point in euclidean coordinates.

4. **(9)** Linearize the functional model f at the initial values.

   - For this purpose write down the partial derivatives of the function $f(\mathbf{X}_{0j}, \mathbf{X}_i)$ w.r.t. the unkowns.
   - Given the partial derivatives write down the **explicit** representation of the matrices C_{ij} and B_{ij} described in the lecture.

5. **(3)** Can you solve the normal equations $A^T \Sigma^{-1} A \Delta x = A^T \Sigma^{-1} \Delta l$ given the configuration described above? Substantiate your answer and if appropriate give further details about what kind of and how many additional observations are required.