September 2020

How can we transform station coordinates from the terrestrial to the celestial frame? [or the other way round …]

Michael Schindelegger

schindelegger@igg.uni-bonn.de

UNIVERSITÄT BONN

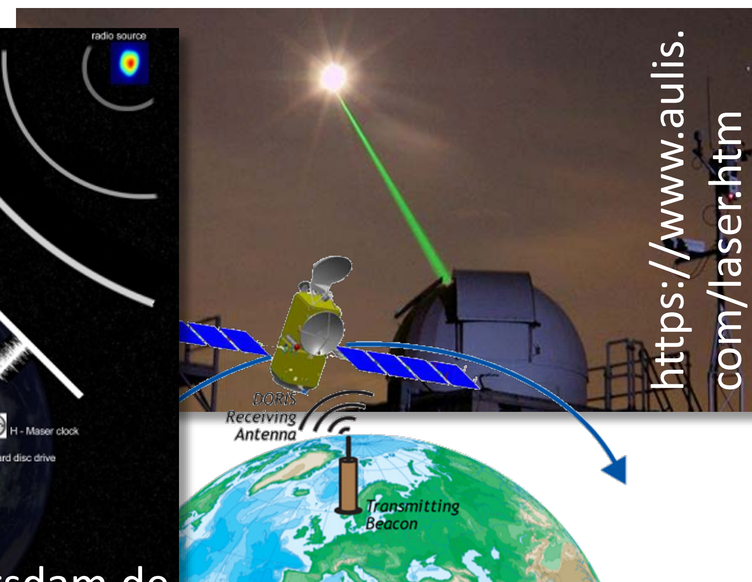
igg

Institut für Geodäsie und Geoinformation

Some motivation

Observations to/from objects in space:

- Analyzing observations for certain target parameters requires transformation celestial ⟷ terrestrial frame
- ⟹ Consideration of **Earth's variable rotation**



Outline

Contents of the lecture:

- Earth rotation – an overview
- Conventional reference systems
- Transformation between celestial and terrestrial reference systems

---

Main reference:

IERS Conventions (2010). Gérard Petit and Brian Luzum (eds.). (IERS Technical Note ; 36) Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2010. 179 pp., ISBN 3-89888-989-6.

https://www.iers.org/IERS/EN/Publications/TechnicalNotes/tn36.html

Overview

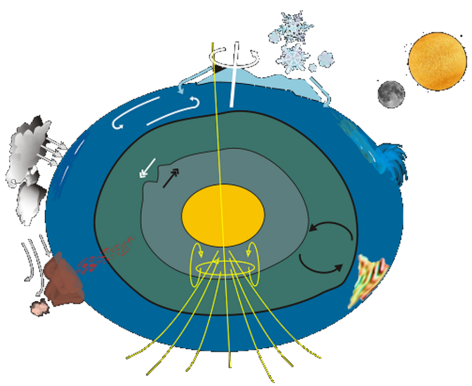
Three aspects of a **variable rotation**:

- Earth's angular velocity is not constant
- Orientation of the reference (e.g., rotation) axis varies with respect to space
- Earth as a body changes its position with respect to this reference axis

Why does the Earth not rotate …

… uniformly around an invariant axis?

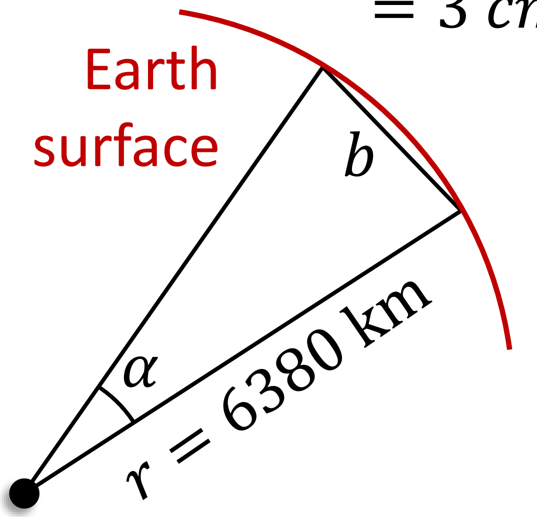
- The Earth‘s figure is not a sphere but rather corresponds to an oblate spheroid
- Axes of rotation and maximum inertia (= figure axis) do not coincide
- The Earth is neither rigid nor homogenous (deformations of solid parts, atmosphere, ocean, etc. …)



Implications for …

- Measurement of time
- Positioning and navigation
  - On Earth ($1\,ms \triangleq 45\,cm$ at the equator / $1\,mas \triangleq 3\,cm$)
  - In space: spacecraft tracking
- Geodesy: reference frame, gravity field
- Earth system research
  - Interaction and feedback effects:
    - Ocean, atmosphere, cryosphere
    - Mantle, Earth's core

$$b = \alpha[rad] \cdot r = 3\,cm$$



$\alpha = 1\,mas$

We seek quantities to describe …

- The changing **orientation of the rotation axis with respect to a celestial reference frame**

  → precession/nutation

- The **changing position of the rotation axis relative to the body-fixed reference frame**

  → polar motion

- **Variations in the rotation rate** (= angular velocity) **with respect to an atomic frequency standard**

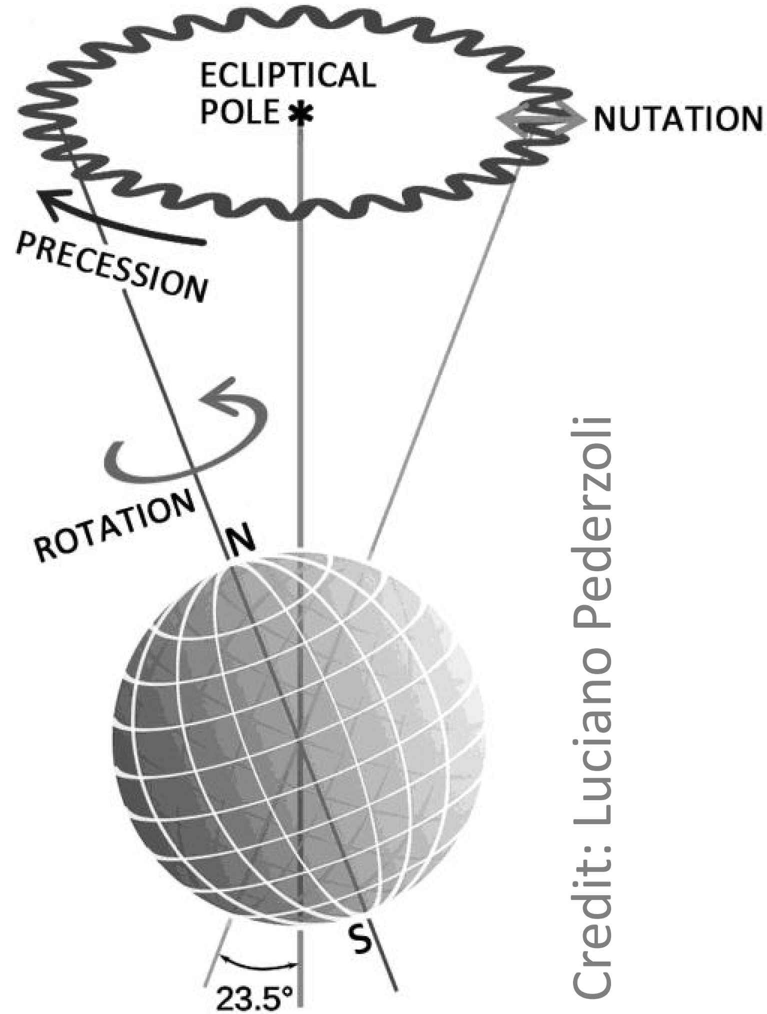
  → dUT1 (or ΔLOD)

⟹ **5 Earth Orientation Parameters**

Earth Orientation Parameters

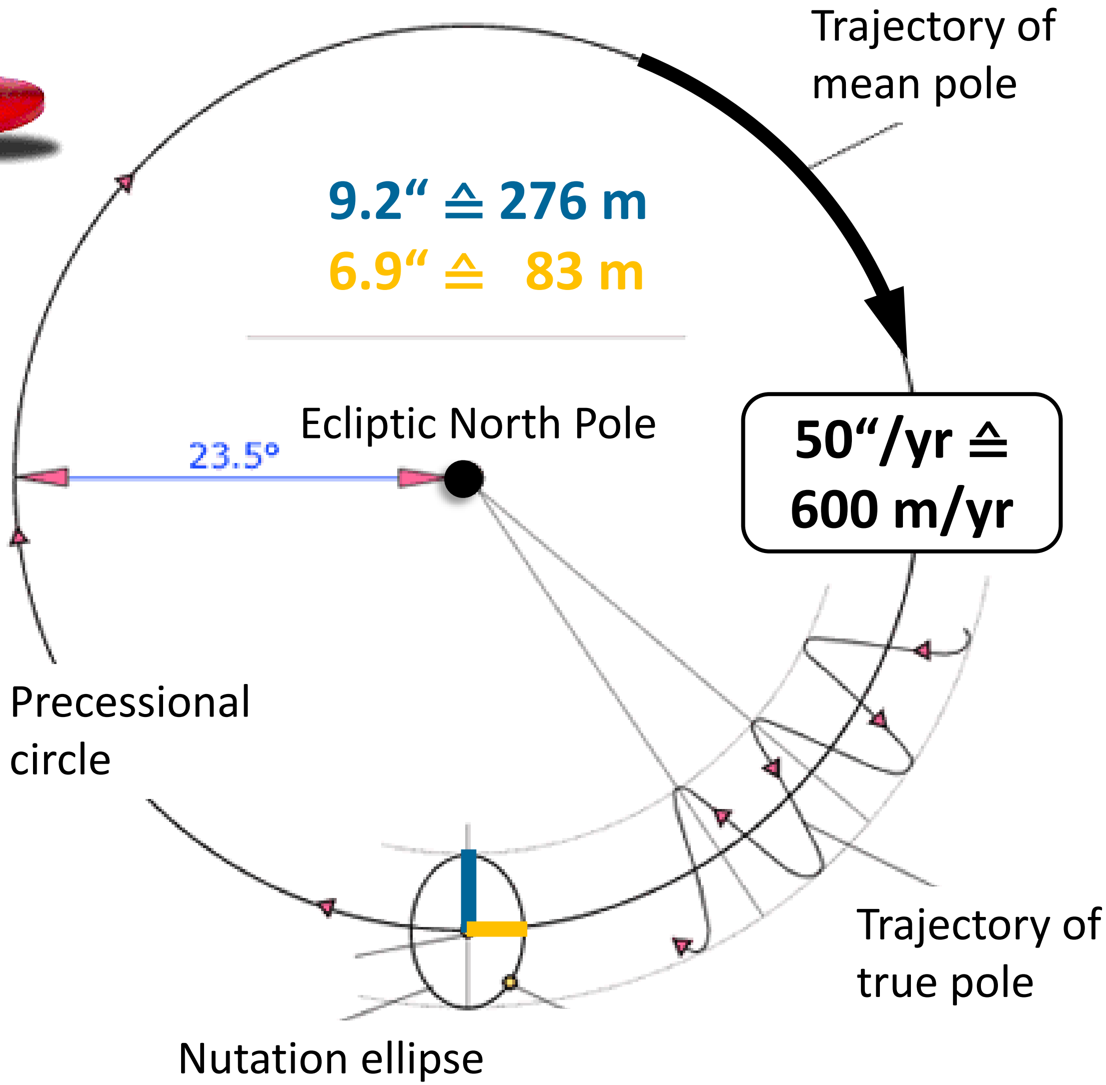
Precession und nutation:

- Response of the oblate Earth to **luni-solar torques**
- Formal separation ⟶
- Precession (long-period):
  - Circular movement of rotation axis around ecliptic pole
  - 1 revolution in 25800 yr
- Nutation (short-period):
  - Smaller oscillations



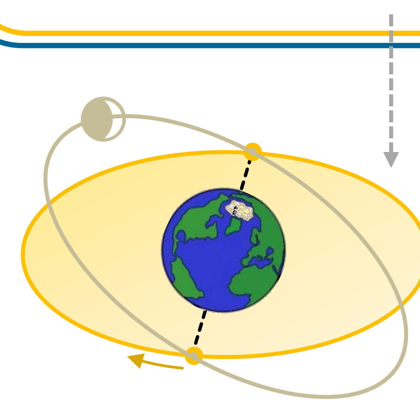
Credit: Luciano Pederzoli

Precession/nutation



Precession period also called Great” or “Platonic Year”

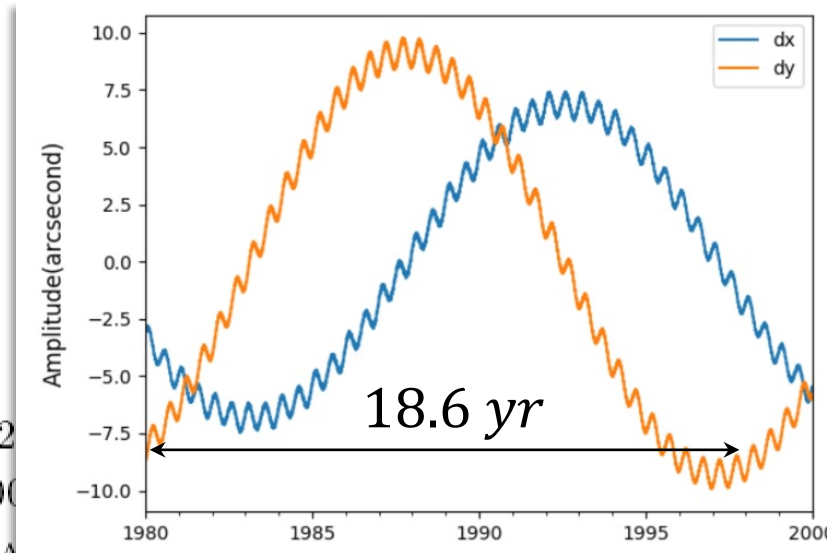
Nutation period with largest amplitude: 18.6 yr (Regression of the lunar node)



Precession/nutation

P/N can be predicted almost perfectly $\Longrightarrow$

- Typical **series expansion**:



$$
\begin{aligned}
X = \quad & -0.01661699'' + 2004.19174288''t - 0.4272 \\
& -0.19862054''t^3 - 0.00004605''t^4 + 0.0000 \\
& + \textstyle\sum_i [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{A} \\
& + \textstyle\sum_i [(a_{s,1})_i t \sin(\text{ARGUMENT}) + (a_{c,1})_i t \cos(\text{ARGUMENT})] \\
& + \textstyle\sum_i [(a_{s,2})_i t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i t^2 \cos(\text{ARGUMENT})] \\
& + \cdots ,
\end{aligned}
$$

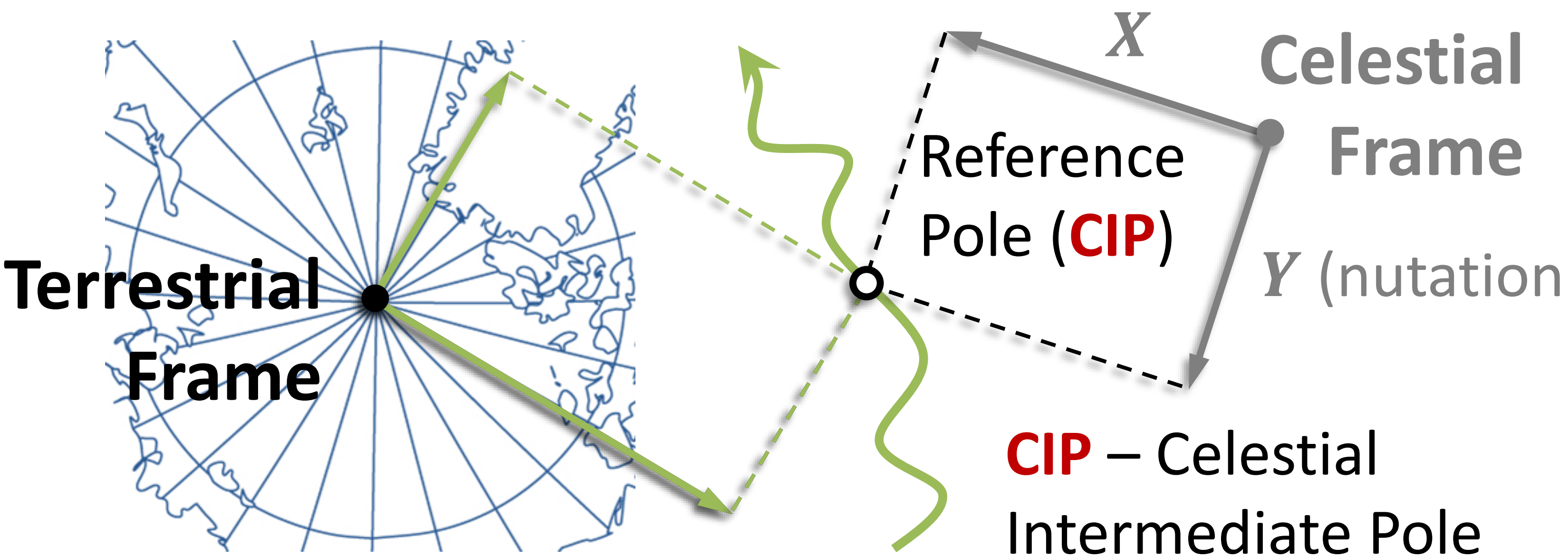
$$
\begin{aligned}
Y = \quad & -0.00695078'' - 0.02538199''t - 22.40725099''t^2 \\
& +0.00184228''t^3 + 0.00111306''t^4 + 0.00000099''t^5 \\
& + \textstyle\sum_i [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] \\
& + \textstyle\sum_i [(b_{c,1})_i t \cos(\text{ARGUMENT}) + (b_{s,1})_i \; t \sin(\text{ARGUMENT})] \\
& + \textstyle\sum_i [(b_{c,2})_i t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i t^2 \sin(\text{ARGUMENT})] \\
& + \cdots ,
\end{aligned}
$$

$\{X, Y\}$ are two of the EOP and called **nutation angles**

Earth Orientation Parameters

Precession/Nutation ⟶ polar motion**:**

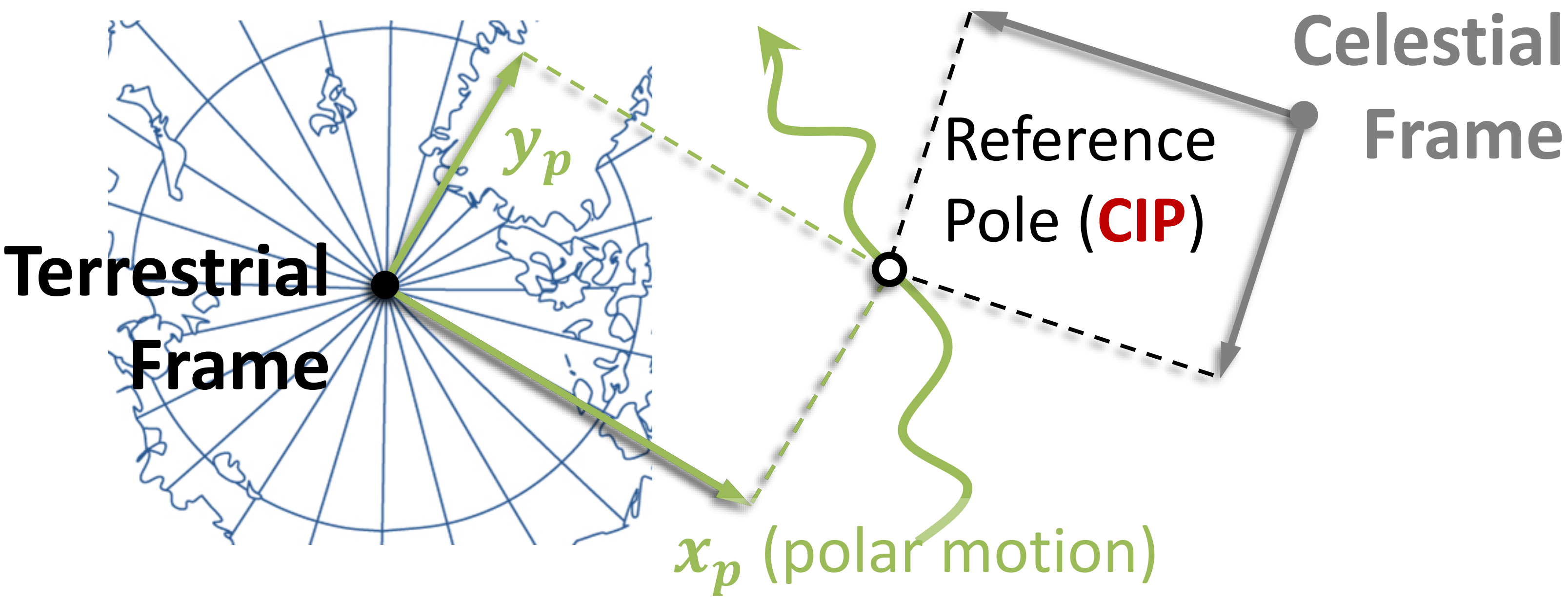
- What is our reference axis?
- Actual **pole of rotation** exhibits large short-term variability in the celestial frame ⟶ not suited



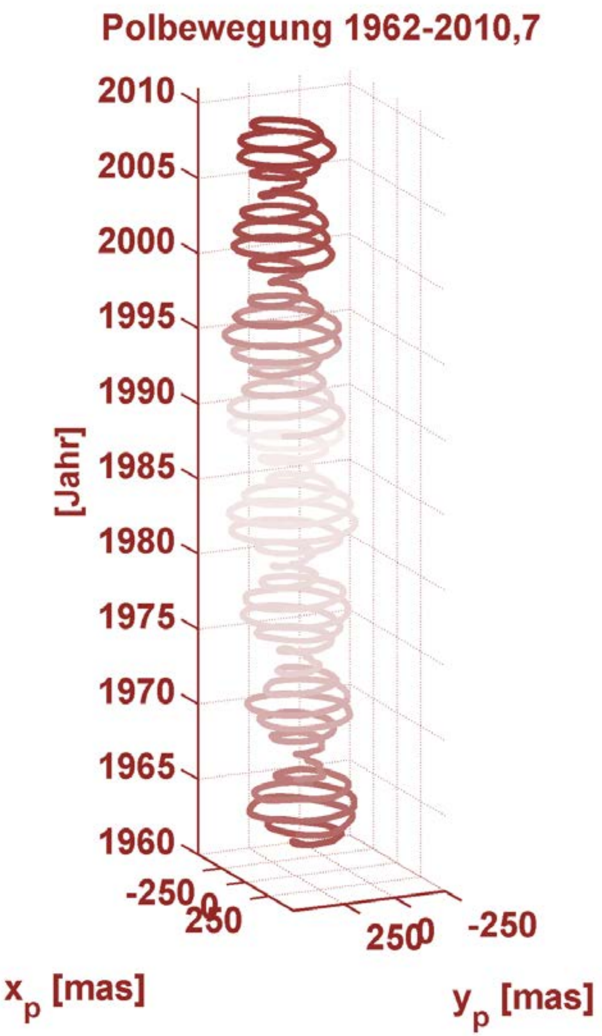
Earth Orientation Parameters

Precession/Nutation ⟶ polar motion:

- Reality: we use a conventional reference pole = **CIP**
- Intermediate pole in the eventual transformation
- Polar motion angles $\{x_p, y_p\}$ also refer to CIP



Polar motion – Geophysical effects



„1st order" pole trajectory:

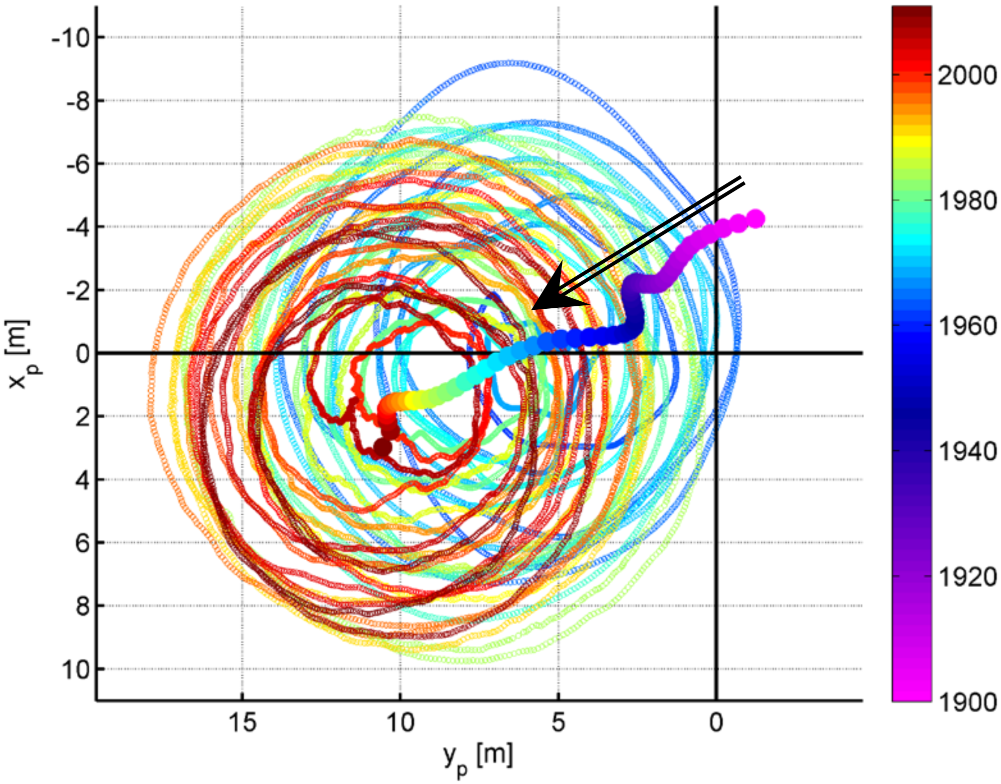
- Chandler Wobble
  - Free oscillation of the Earth
  - Period ~ 433 days
- Annual Wobble
  - Forced oscillation, main driver (~75%) are annual pressure over continental areas

$\Rightarrow$ Beat with a period of 6.3 yr and peak amplitudes of 9 m

Polar motion – Geophysical effects

Secular polar motion:

Pole trajectory 1962.0 – 2010.7

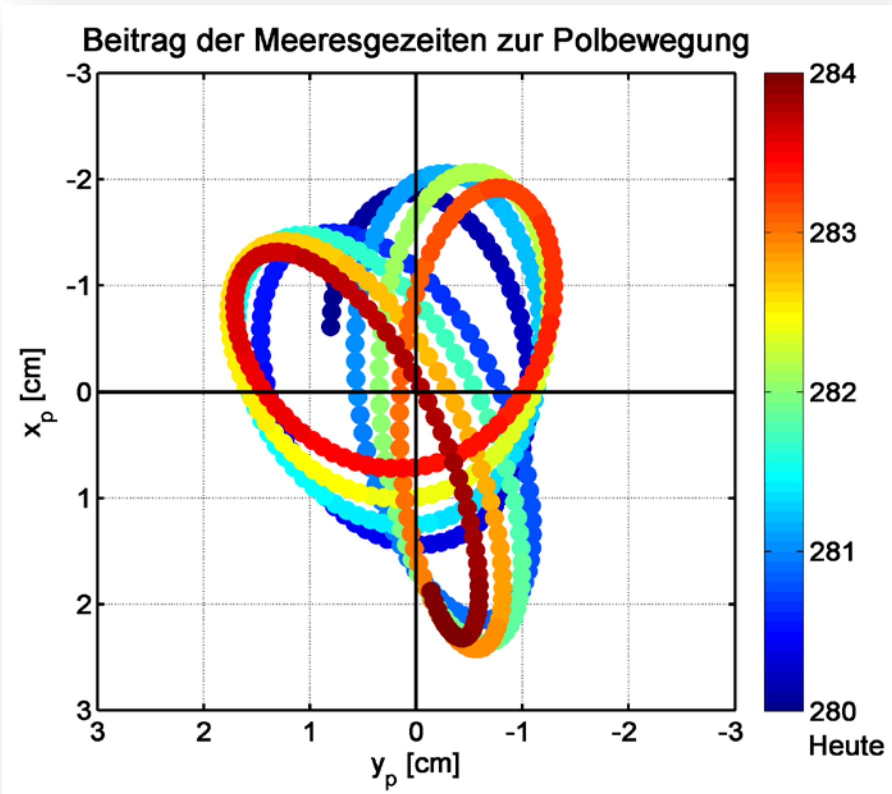


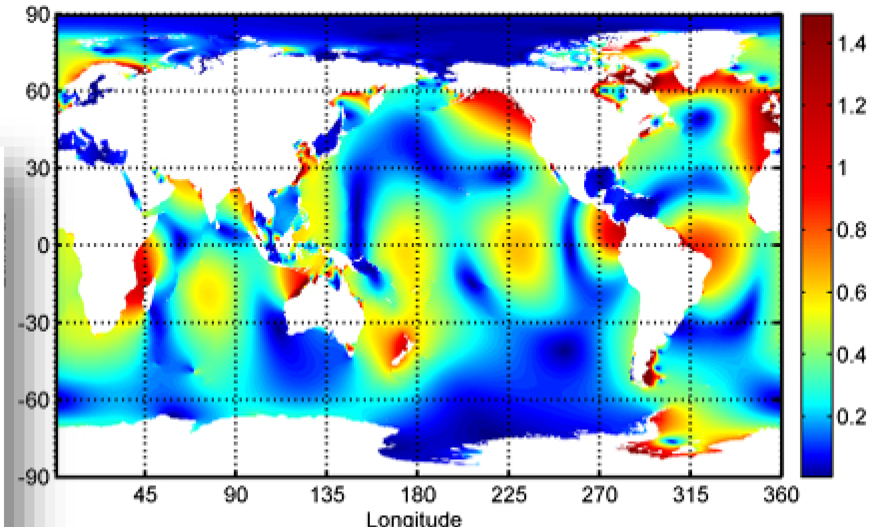
- (Linear) drift of the reference pole:
  - $10\ cm/yr$ towards $78°\ W$
  - Main cause: glacial isostatic adjustment
- Sudden changes since 2005 may have other causes (ice sheet melt, terrestrial water …)

Polar motion – Geophysical effects

Diurnal (daily) and **semi-diurnal** (half-daily) effects:

$\sim 3\ cm$ displacement of the CIP at the Earth's surface



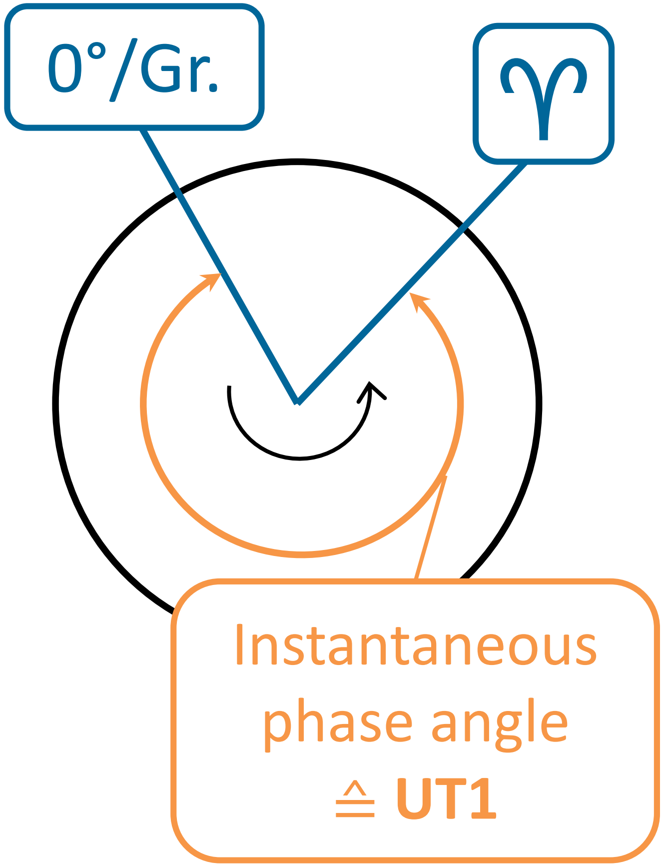


Example: $\mathbf{M_2}$ amplitude (m) in the global ocean. Other partial tides: $\mathbf{S_2}$, $\mathbf{K_1}$, $\mathbf{O_1}$ …

Earth Orientation Parameters

5th EOP ⟶ **Variations in the rate of rotation**

- Understanding of time scales required, particularly UT1 (**Universal Time** – corrected for polar motion effects)
- Phase angle of a terrestrial object w.r.t. a reference point on a sphere
- For more information, see: https://www.timeanddate.com/time/universal-time.html



Universal Time – ΔLOD

Universal Time UT1, changes in length-of-day

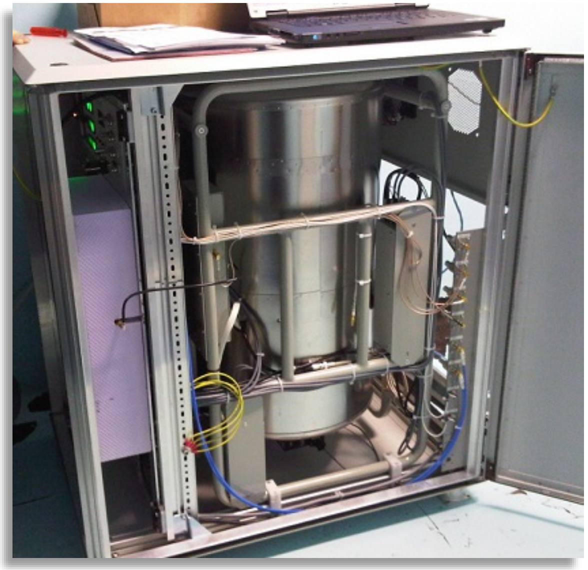
- Variations of the rotation rate can be expressed as deviations of UT1 from an atomic time scale (such as UTC) …

  UT1-UTC = dUT1

- … or as perturbations of the nominal rotation rate $1/86400\ sec$

  ΔLOD (Excess of length-of-day)

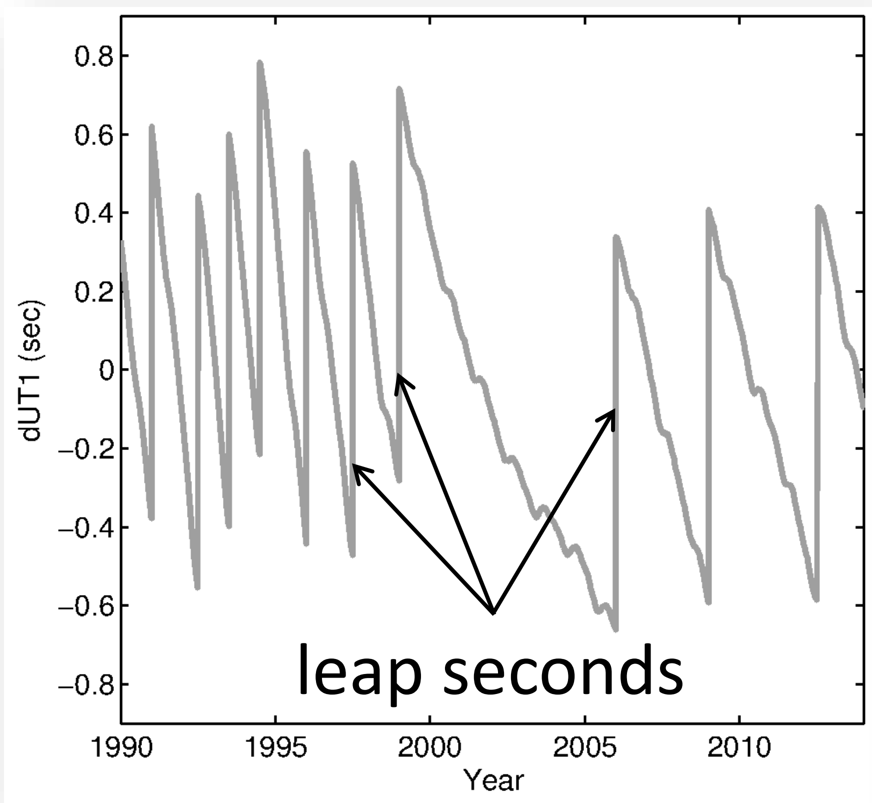
Atomic frequency standard



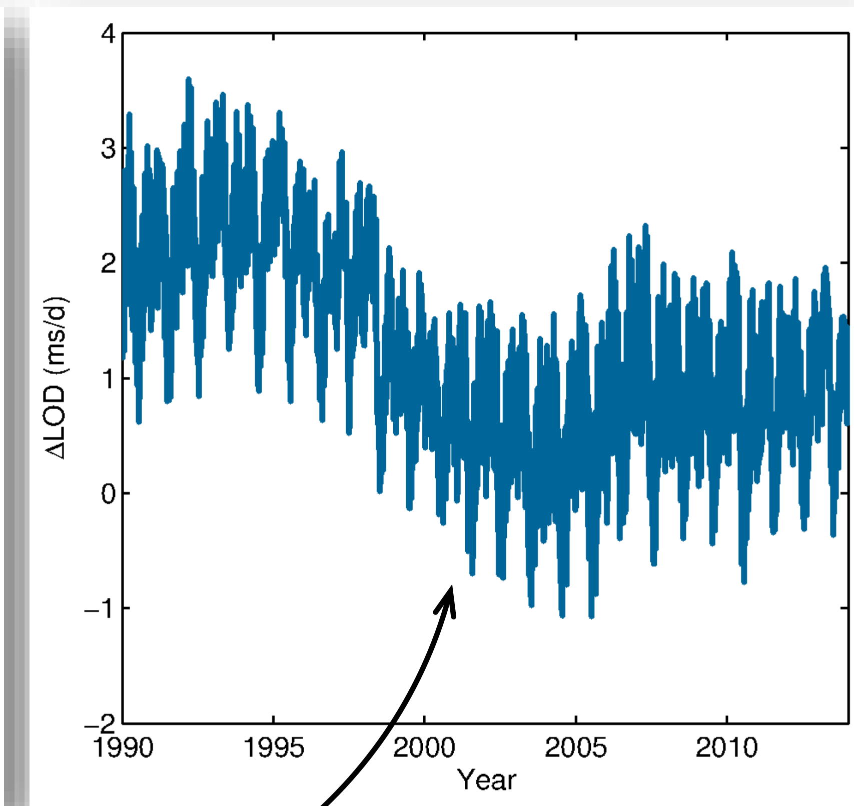
$$\Delta\mathrm{LOD} = -\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{dUT1}$$

Universal Time – ΔLOD

UT1-UTC < 0.9 s

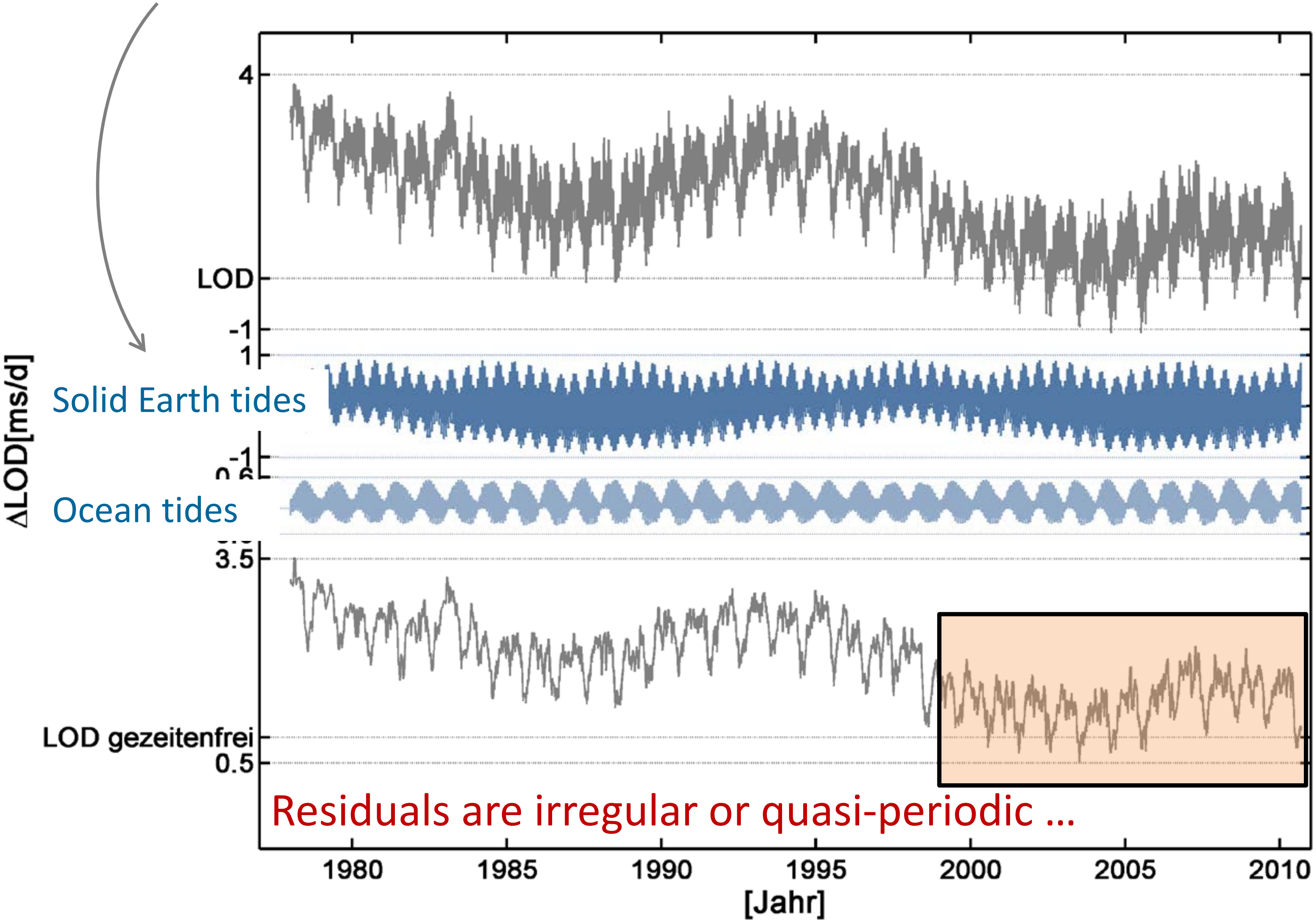


ΔLOD (ms)



UT1-UTC without jumps → differentiation

ΔLOD – Geophysical effects



Outline

Contents of the lecture:

- Earth rotation – an overview
- Conventional reference systems
- Transformation between celestial and terrestrial reference systems

---

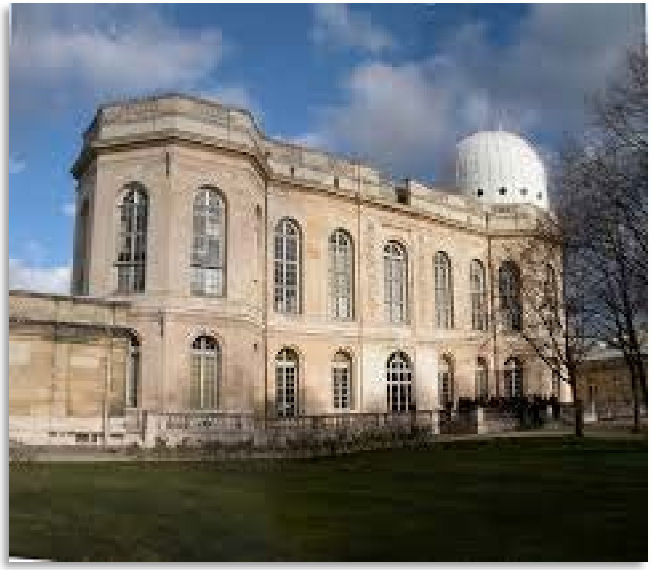
- **Reference System ⟷ Reference Frame**
- **Earth Orientation Parameters**

$$\{X, Y, x_p, y_p, \mathrm{dUT1}\}$$

Reference systems

International Terrestrial Reference System

- **ITRS** is geocentric and co-rotates with the Earth
- Unit length is the meter (SI)
- Orientation of axes $\{x, y, z\}$ follows **conventions**:
  - Requirement that $\{x, y, z\}$ are aligned with a previous realization: BIH at epoch 1984.0
  - Arbitrary definition

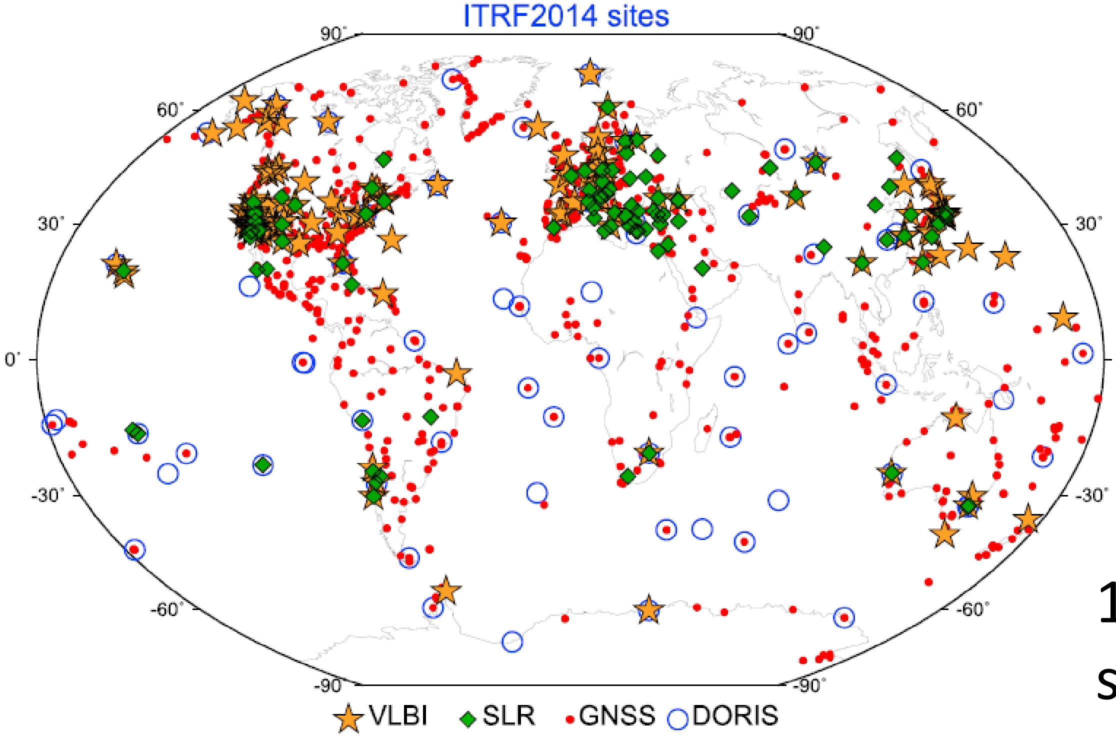


Bureau International de l'Heure (BIH) at Paris Observatory

Reference systems

International Terrestrial Reference Frame

- **ITRF** = realization of ITRS by virtue of positions and velocities of space-geodetic stations:



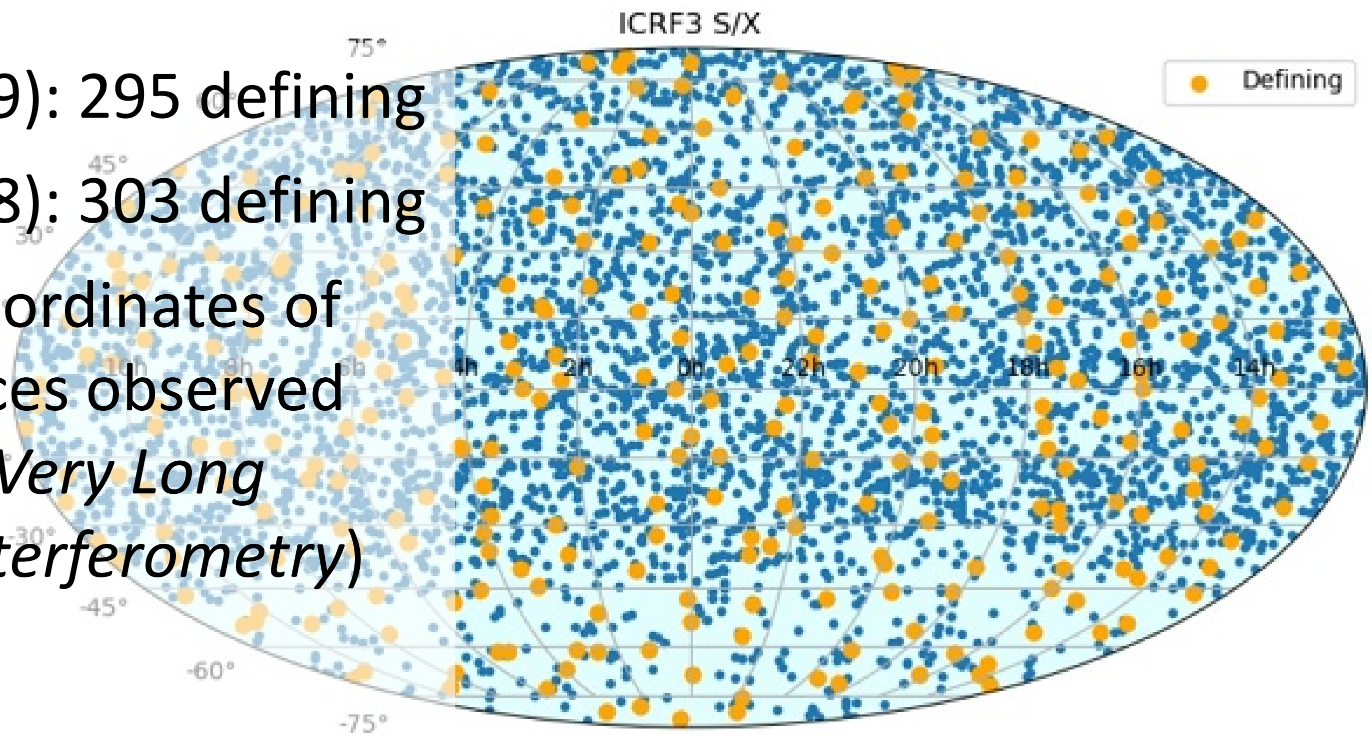
- ITRF94
  ITRF96 …
  ITRF2008
  ITRF2014

1499 observing stations at 975 sites

Reference systems

Internat. Celestial Reference System/Frame

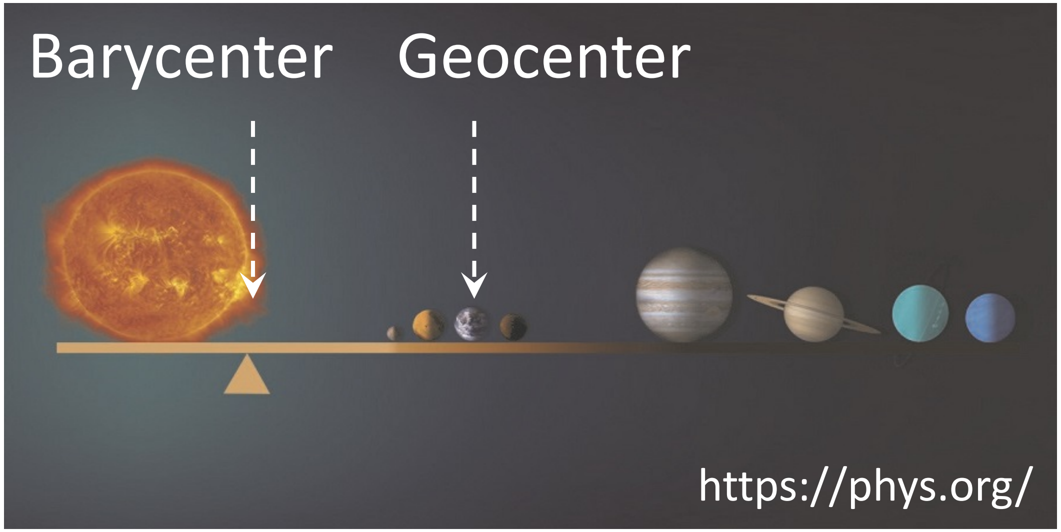
- Both definition (**ICRS**) & realization (**ICRF**) constrained by stable “defining” radio sources:
  - **ICRF2** (2009): 295 defining
  - **ICRF3** (2018): 303 defining
  - Celestial coordinates of these sources observed with **VLBI** (*Very Long Baseline Interferometry*)

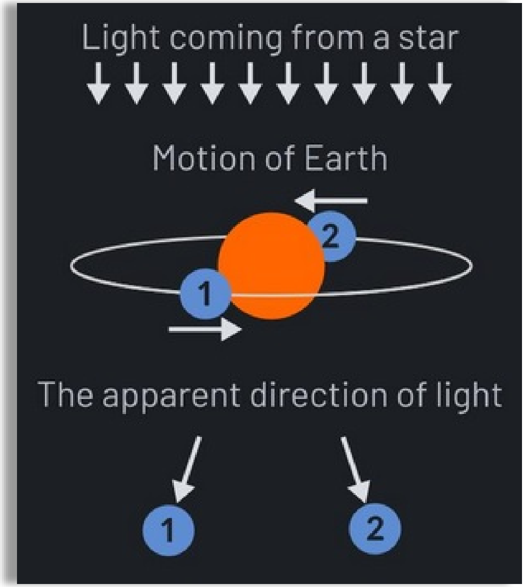


Reference systems

Internat. Celestial Reference System/Frame

- **ICRS** is barycentric, but in the following …
- We assume that it is geocentric and the necessary **corrections** have been applied, e.g., aberration





https://flatearth.ws/stellar-aberration

Outline

Contents of the lecture:

- Earth rotation – an overview
- Conventional reference systems
- Transformation between celestial and terrestrial reference systems

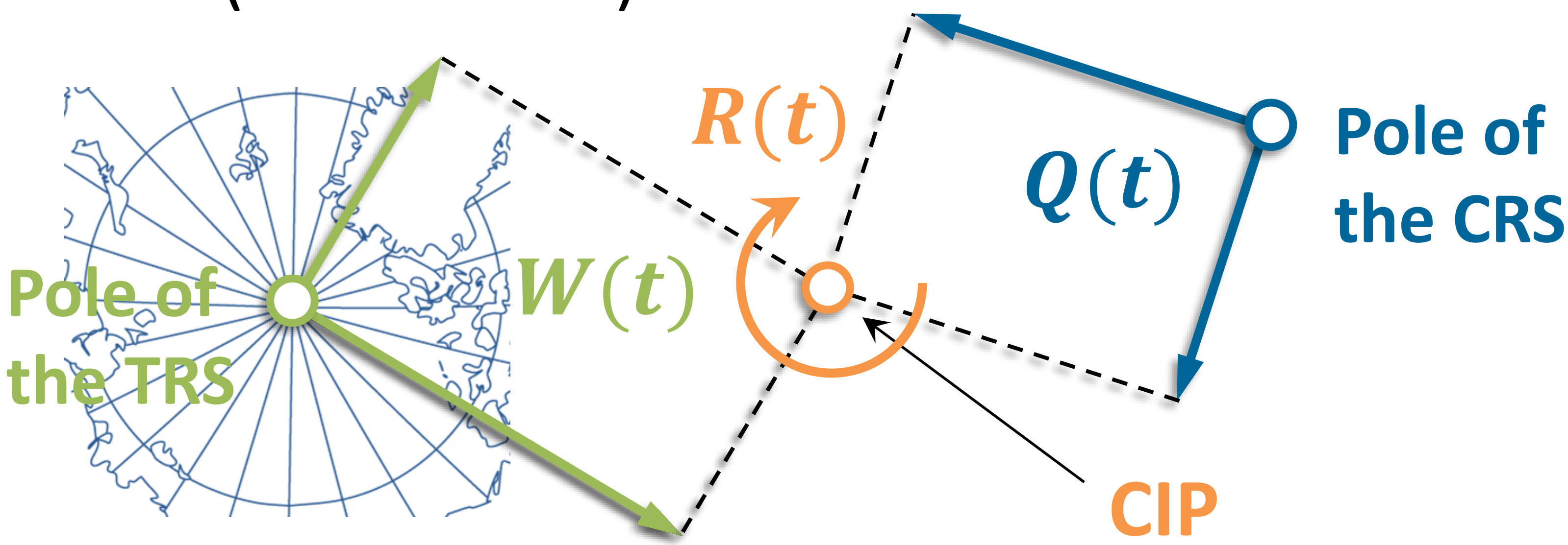
---

- Previously: transformation based on ecliptic, vernal equinox ♈, nutation in obliquity, …
- Here: **IAU 2000/2006 resolutions**

Transformation TRS ⟷ CRS

"Top view" of the IAU 2000 transformation

- Involves intermediate system(s) that are tied to the CIP (see slide 12):



$$[CRS] = Q(t) \cdot R(t) \cdot W(t) \cdot [TRS]$$

Transformation TRS ⟷ CRS

In all clarity (before addressing some details):

Rotation matrices account for … (at a particular epoch)

- $Q(t)$ … **Nutation**
- $R(t)$ … **Earth's spin**
- $W(t)$ … **Polar motion**

$$[CRS] = Q(t) \cdot R(t) \cdot W(t) \cdot [TRS]$$

or, vice versa:

$$[TRS] = \widetilde{W}(t) \cdot \tilde{R}(t) \cdot \tilde{Q}(t) \cdot [CRS]$$

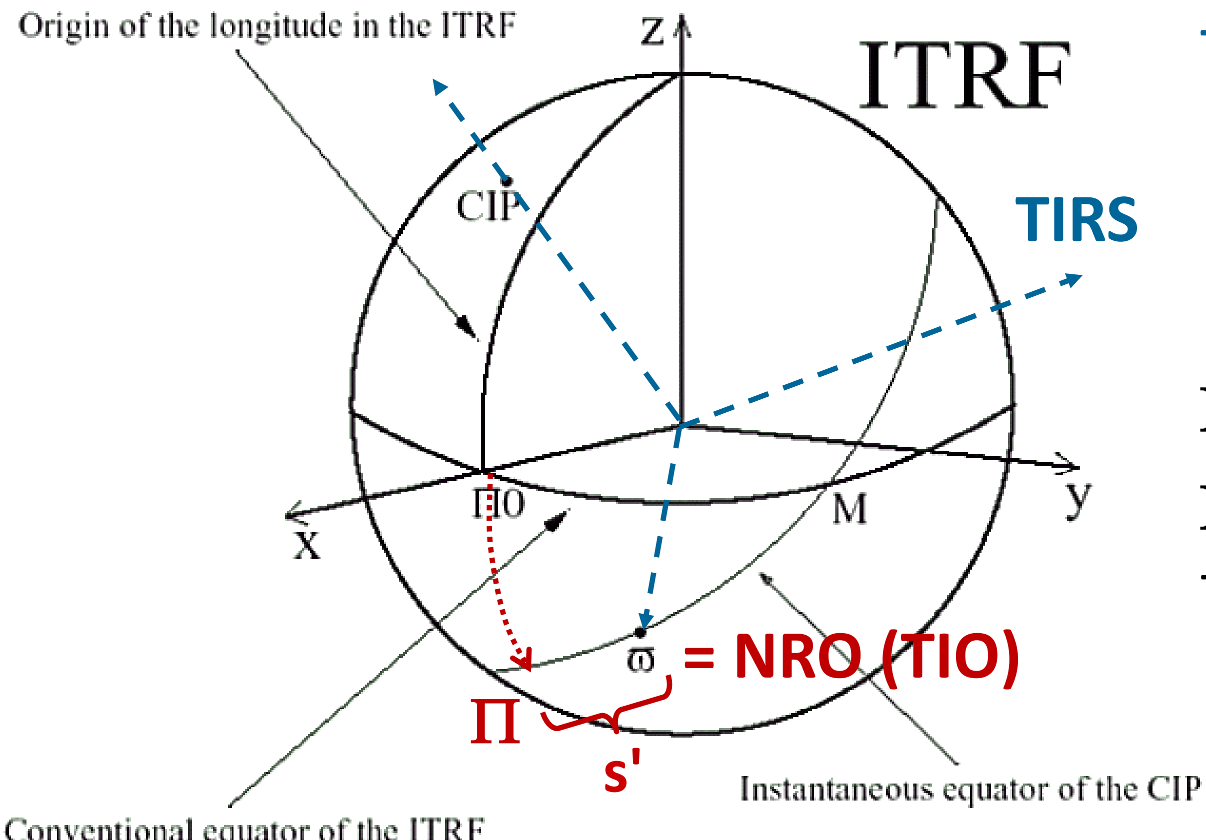
Transformation TRS $\longleftrightarrow$ CRS

Non-rotating origins (NRO):

- Kinematical definition, two NROs:
  - Celestial Intermediate Origin (CIO) $\longrightarrow$ takes over the role of the vernal equinox in the transformation
  - Terrestrial Intermediate Origin (TIO)
- NROs always on the equator of the CIP
- Their positions depend on the **history of CIP motion**
- CIP & respective NRO define 2 intermediate systems
  - Celestial Intermediate Reference System (CIRS)
  - Terrestrial Intermediate Reference System (TIRS)

Transformation TRS ⟷ CRS

NRO in the TRS → TIO or ϖ, positioned on the CIP equator using the angle **s'**



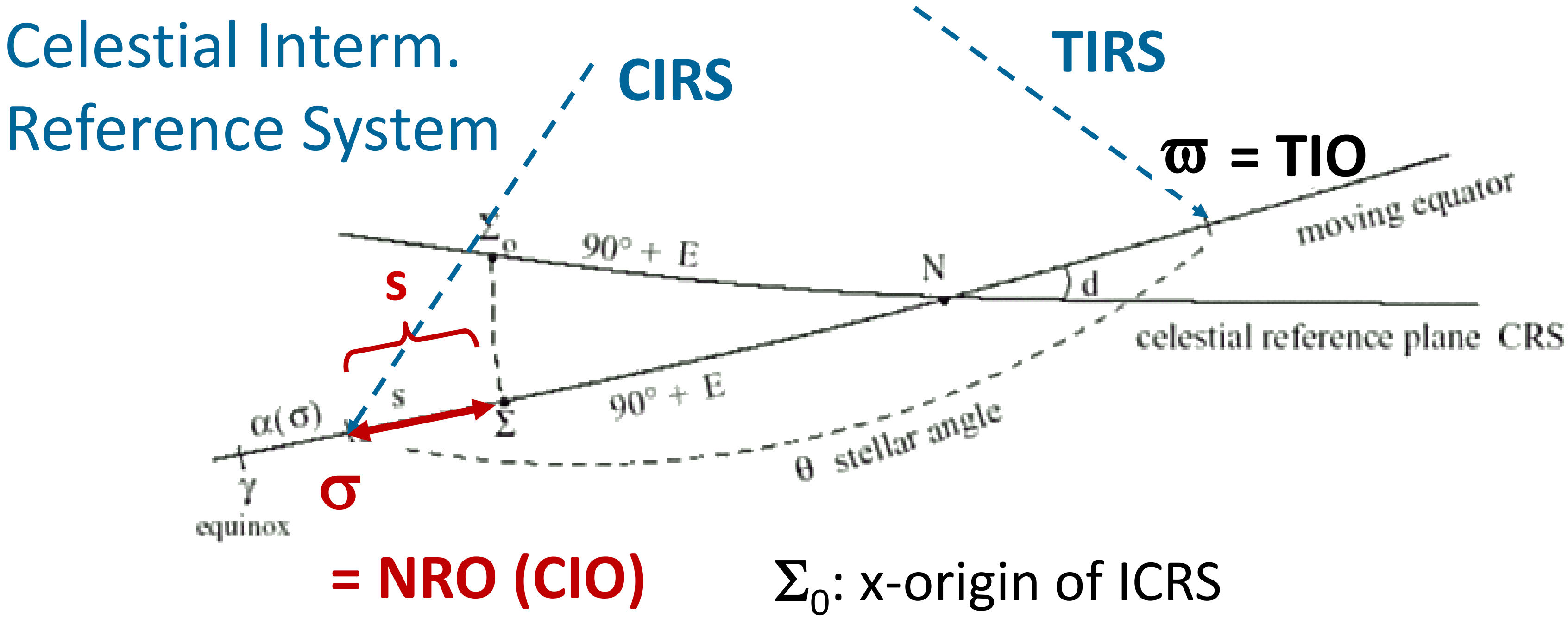
Terrestrial Interm. Reference System

Π_0: x-origin of ITRS

Π: auxiliary point on the ITRS meridian

Transformation TRS ⟷ CRS

NRO in the CRS → **CIO** or **σ**, positioned on the CIP equator using the angle **s**



Σ_0: x-origin of ICRS

Σ: auxiliary point on ICRS meridian

Transformation TRS ⟷ CRS

Precession/Nutation matrix:

$$Q(t) = R_z(-E)R_y(-d)R_z(E+s)$$

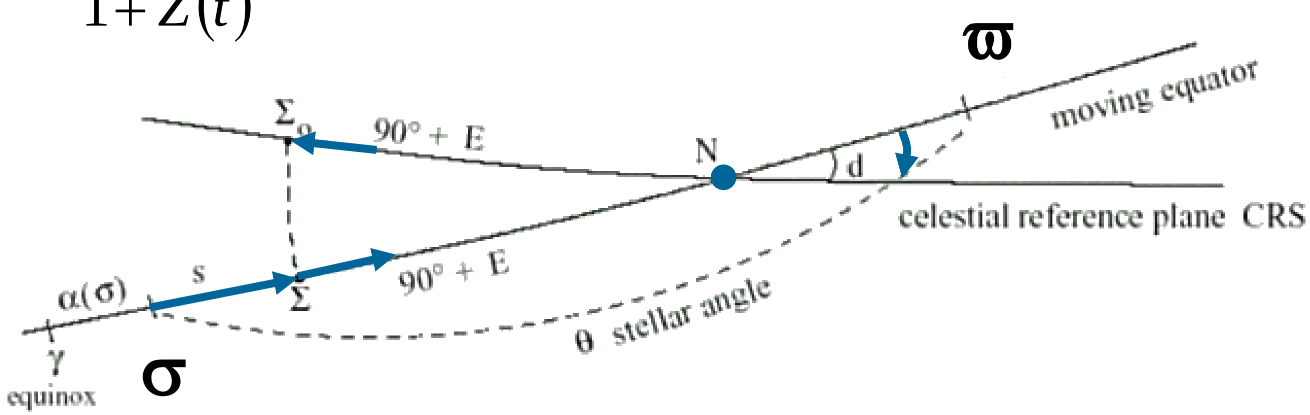
- Coordinates of the CIP in CRS: $X = \sin(d)\cos(E)$

  $Y = \sin(d)\sin(E)$

  $Z = \cos(d)$

- CIO Locator **s**:

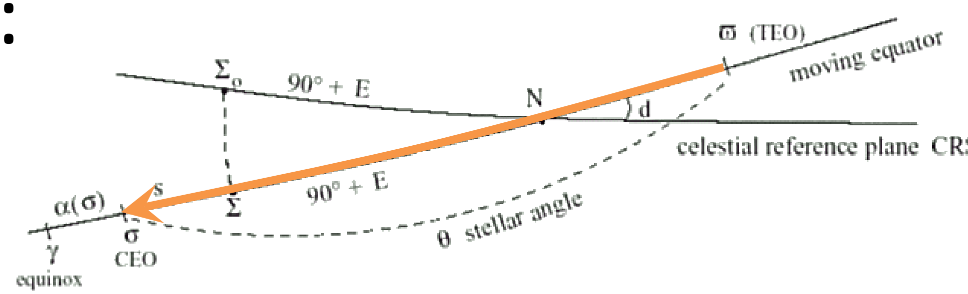
$$s(t) = -\int_{t_0}^{t} \frac{X(t)\dot{Y}(t) - \dot{X}(t)Y(t)}{1+Z(t)}\,dt + const$$



Transformation TRS ⟷ CRS

Earth rotation matrix:

$$R(t) = R_z(-ERA)$$



- ERA … **Earth Rotation Angle**
- ERA proportional to dUT1, only input UT1-UTC

$$ERA(T_u) = 2\pi(0.7790572732640 + 1.00273781191135448\, T_u)$$

$$T_u = \underbrace{mjd(\text{UT1 epoch})}\; - 51544.5$$

Modified Julian Date of the observation epoch, e.g., **1.9.2020, 16:00 UTC**:

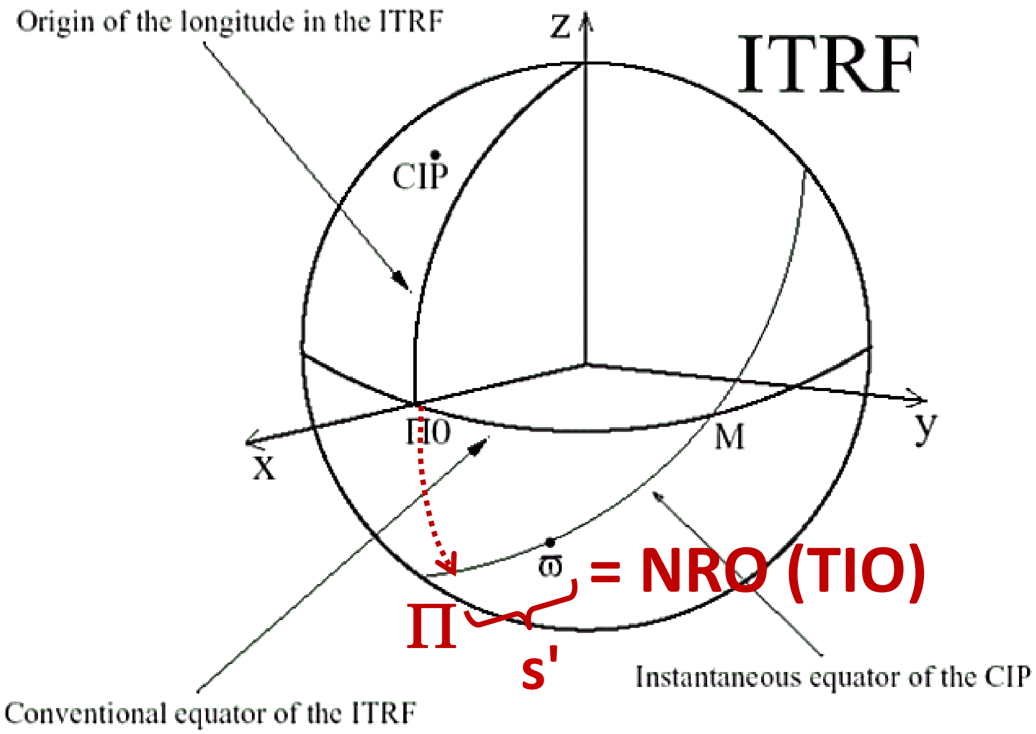
$$mjd(1.9.2020) = 59093$$

$$mjd(\text{UT1 epoch}) = 59093 + {}^{16}/_{24} + {}^{dUT1}/_{24 \cdot 3600}$$

Transformation TRS ⟷ CRS

Polar motion matrix:

$$W(t) = R_z(-s')R_y(x_p)R_x(y_p)$$



- TIO Locator **s'** reflecting CIP history (usually negligible):

$$s' = \int_{t_0}^{t} \frac{x_p \dot{y}_p - \dot{x}_p y_p}{2} dt$$

Transformation TRS $\longleftrightarrow$ CRS

Wrap-up:

- Five Earth Orientation Parameters
- Their physical meaning and practical relevance
- Importance of (global) reference systems
- Astrometry sub-routines:
  - http://www.iausofa.org/index.html (**SOFA**)
  - **NOVAS** (Naval Observatory Vector Astrometry Subroutines), in C/Fortran/Python