Chapter 5
Transforming Geocentric Coordinates to Official Coordinates

Module MGE-01: Coordinate Systems

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WS 2020/21

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Agenda

1. Geocentric coordinate systems
2. Need for official coordinates
3. Official coordinates in Germany
4. Transformation of GNSS coordinates to official coordinates
5. Transformation of TLS coordinates to official coordinates

Conventional Terrestrial Reference System (CTRS)

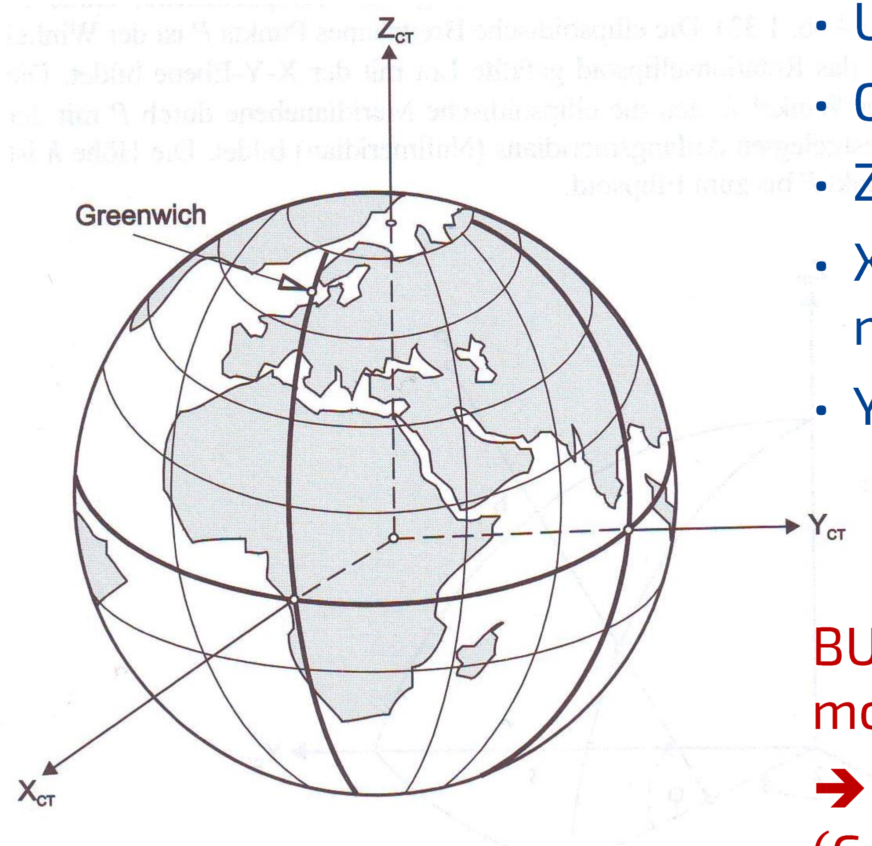


Fig.: Global geozentrisches Koordinatensystem
Source: Bauer, 2011

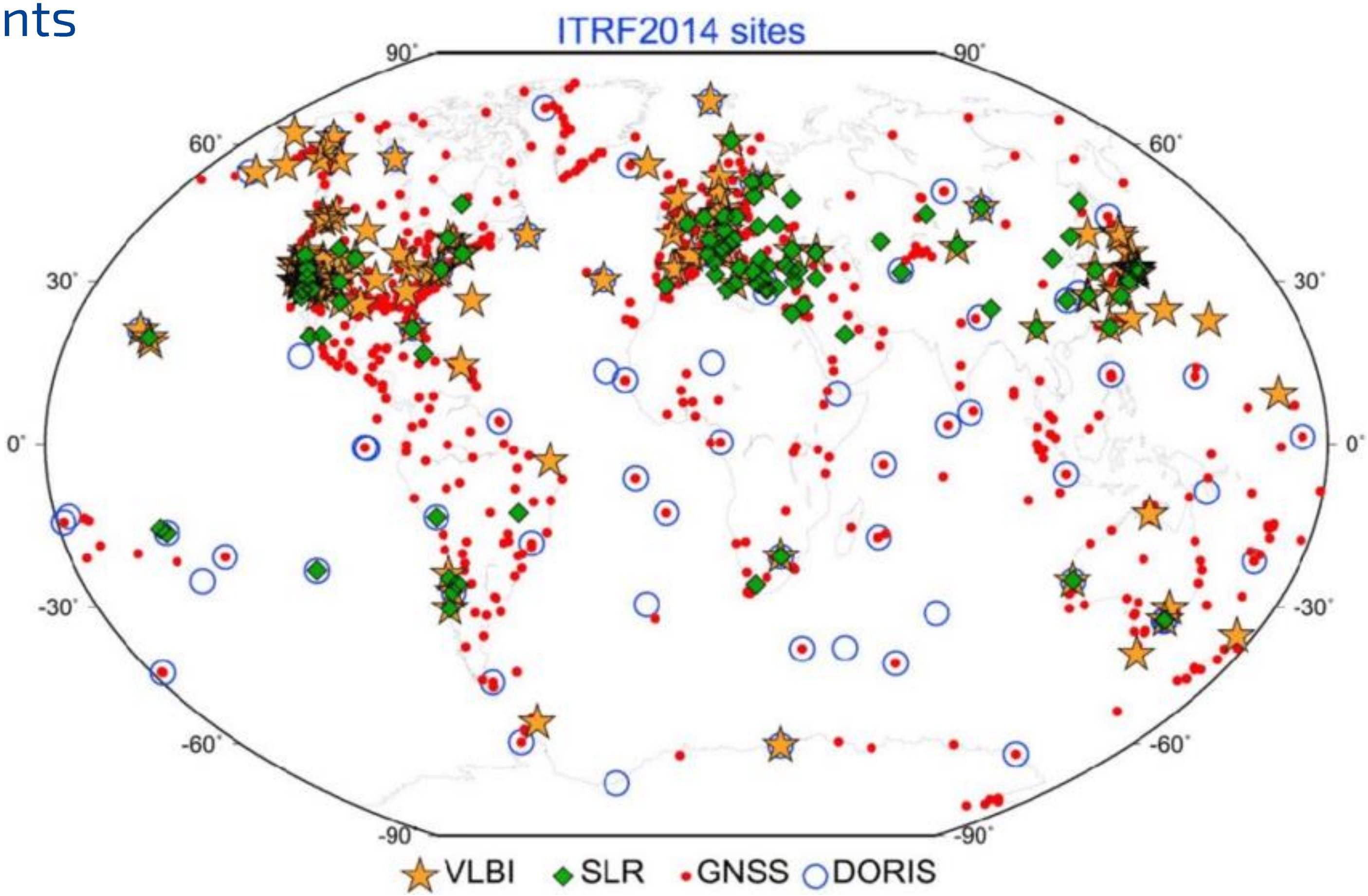
- Global geocentric system (ECEF, Earth Centered Earth Fixed)
- Used to describe the position on Earth
- Origin: Center of Mass of the Earth
- Z-axis through rotation axis
- X-axis is intersection between Greenwich meridian and equatorial plane
- Y-axis completes right handed system

BUT: rotation axis is not constant (polar motion)

➔ z-Axis is going through the CTP (Conventional Terrestrial Pole, the average of the Earth pole between 1900 and 1905)

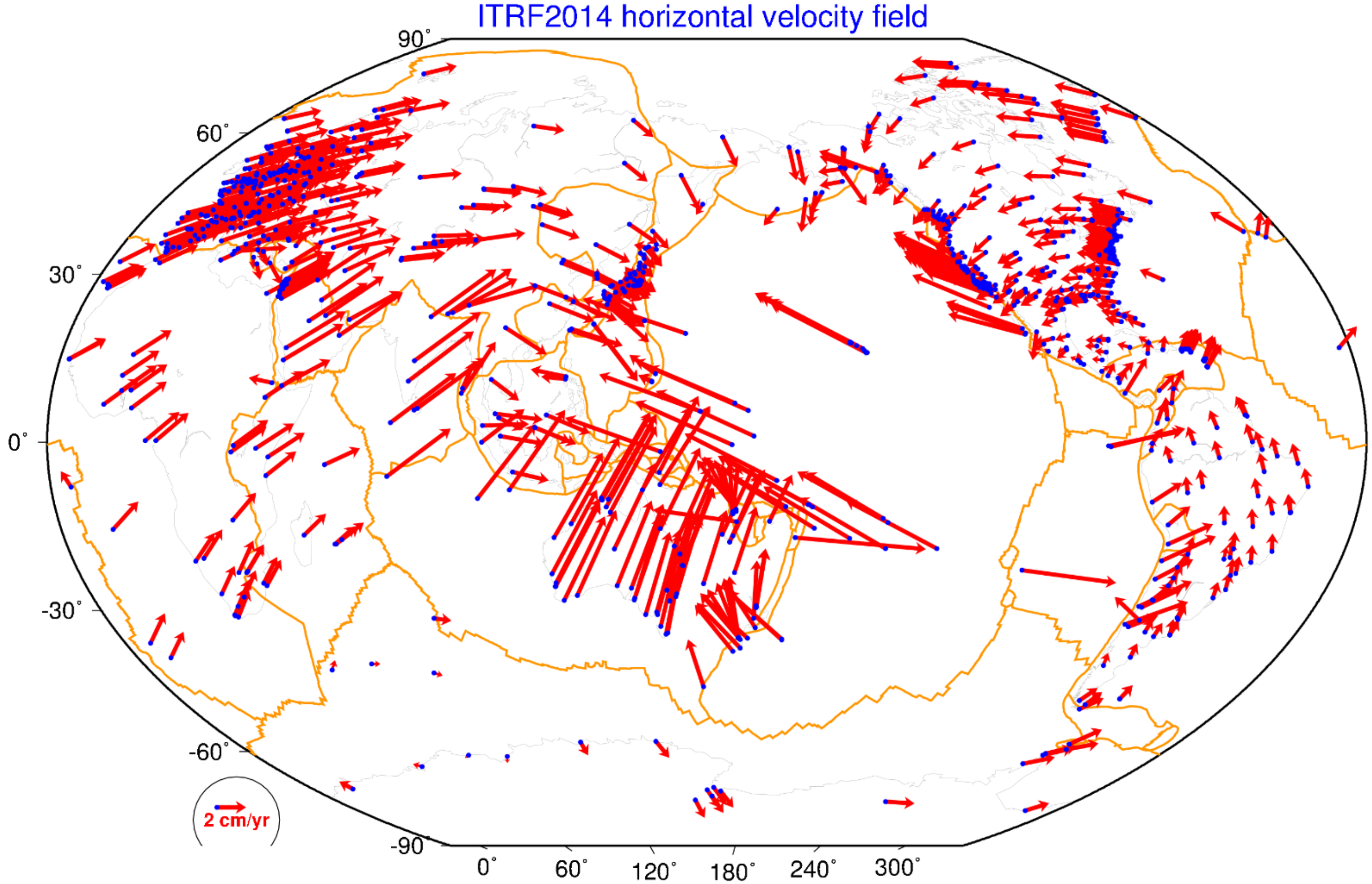
- Reference Frames are realizations of Reference Systems
- Axis and origin are not physically accessible AND change in time

=> the system is implemented by providing the coordinates at a number of dedicated points



Source: gpsmet.agt.bme.hu

- Because of the motion of the Earth crust, the points have to be updated regularly and a velocity of the points is also given



Source: gpsmet.agt.bme.hu

Examples of CTRF

- **ITRFyy**
  - International Terrestrial Reference Frame, calculated in the year yy
  - Maintained by the Scientific Community
  - Current version is ITRF2014
  - Accuracy 5-15mm and 2-3mm/year
- **ETRS89**
  - European Terrestrial Reference Frame
  - Frozen ITRF89
  - Definition of official coordinates in Germany
- **WGS84**
  - World Geodetic System
  - Developed by Department of Defense (US), Military System
  - Also specifies a reference ellipsoid, a set of fundamental constants and an Earth gravity model
  - Adapted to the ITRF, differences are in the order of decimeters
  - Definition of GPS Satellites and corresponding user coordinates

Examples of CTRF

- **ITRFyy**
  - International Terrestrial Reference Frame, calculated
  - –
  - –
  - –
- **ETI**
  - –
    Reference Frame
  - Frozen ITRF89
  - Definition of official coordinates in Germany

- **WGS84**
  - World Geodetic System
  - Developed by Departement
  - … d
  - … r of … decimeters
  - Definition of GPS Satellites and corresponding user coordinates

- Similarity Transformation between coordinate systems:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \lambda \underbrace{\boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)}_{\boldsymbol{R}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

- Since rotations quite small, approximation is valid:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

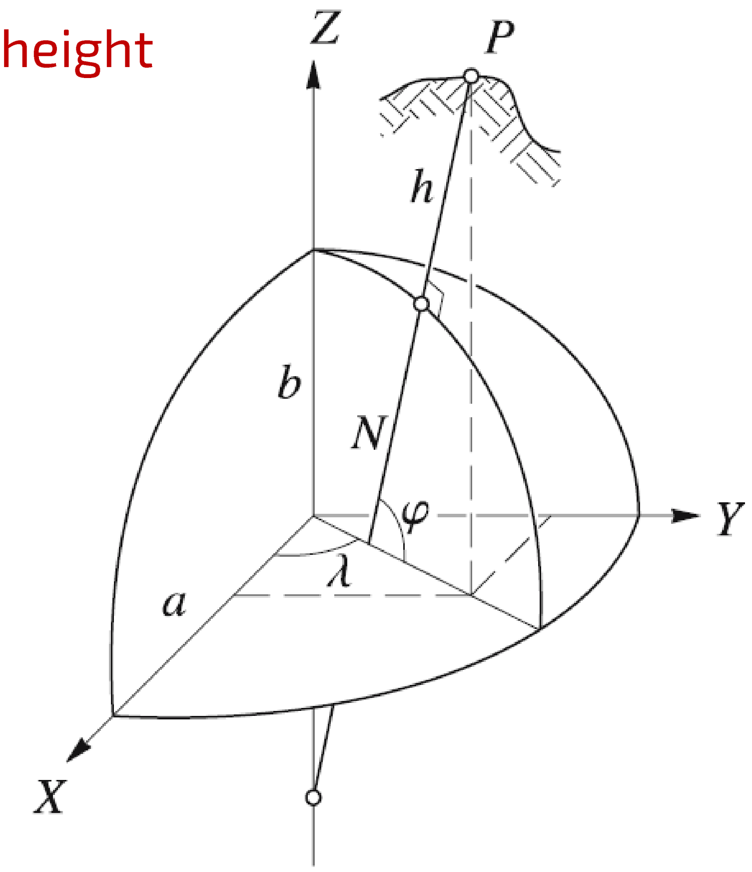
Conversion to Ellipsoidal Coordinates

- Ellipsoidal coordinates are easier to interpret on earth
- Ellipsoidal coordinates need the definition of an reference ellipsoid (e.g. GRS80, WGS84, Bessel)
- λ… longitude, φ… latitude, h… ellipsoidal height
- $\boldsymbol{\lambda, \varphi, h \Rightarrow X, Y, Z}$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (N(\varphi)+h)\cdot\cos\varphi\cdot\cos\lambda \\ (N(\varphi)+h)\cdot\cos\varphi\cdot\sin\lambda \\ \left(\frac{b^2}{a^2}N(\varphi)+h\right)\cdot\sin\varphi \end{bmatrix}$$

$$N(\varphi) = \frac{a^2}{\sqrt{a^2\cdot\cos^2\varphi + b^2\cdot\sin^2\varphi}}$$

… transverse radius of curvature



Hofmann-Wellenhof et al. (2008), Fig.8.1

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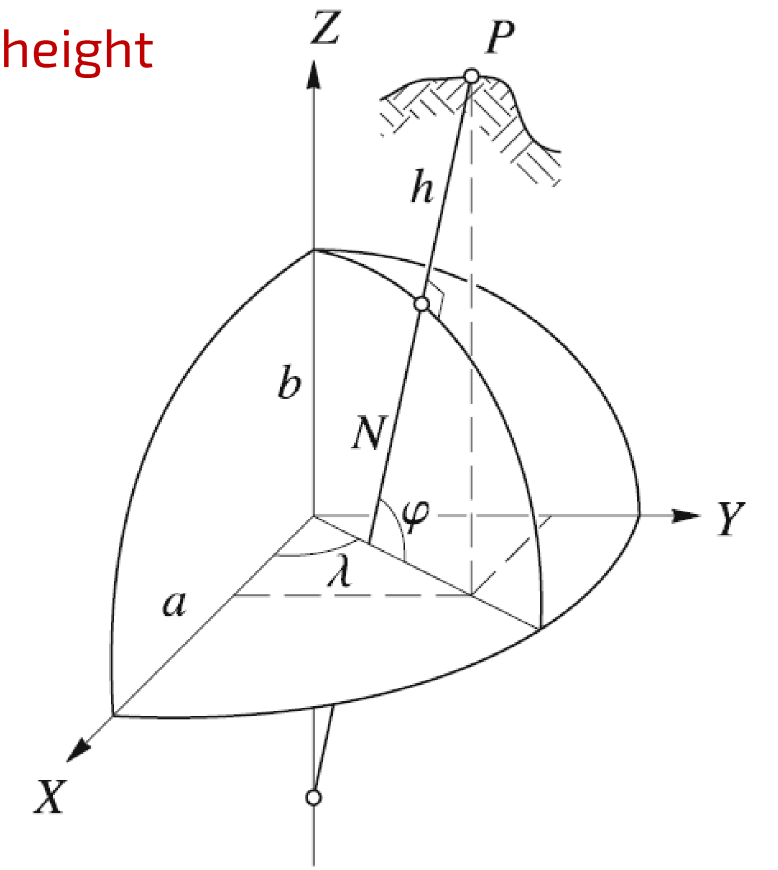
- λ… longitude, φ… latitude, h… ellipsoidal height

- $\boldsymbol{X, Y, Z \Rightarrow \lambda, \varphi, h}$

$$\begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}\cdot\left(1-\frac{a^2-b^2}{a^2}\cdot\frac{N(\varphi)}{N(\varphi)+h}\right)^{-1}\right)\\ \tan^{-1}\left(\frac{Y}{X}\right)\\ \frac{\sqrt{X^2+Y^2}}{\cos\varphi}-N(\varphi)\end{bmatrix}$$

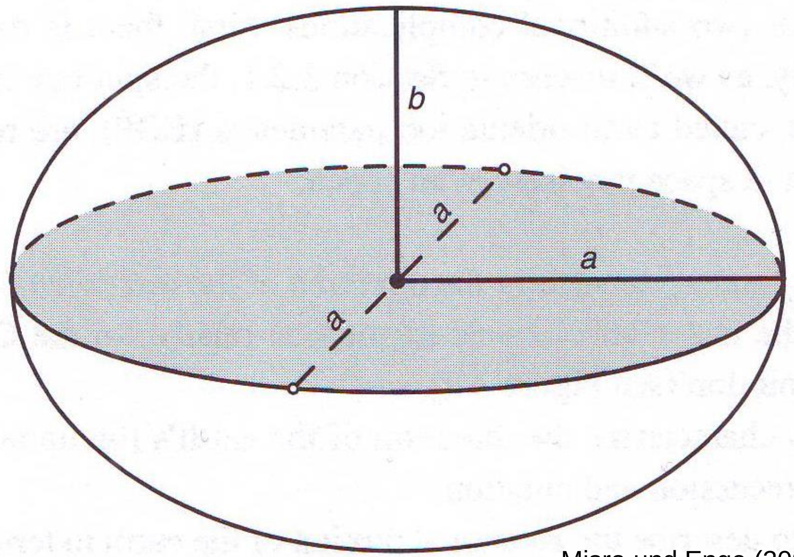
- Iterate: $N(\varphi) = \frac{a^2}{\sqrt{a^2\cdot cos^2\,\varphi+b^2\cdot sin^2\,\varphi}}$



Hofmann-Wellenhof et al. (2008), Fig.8.1

Reference Ellipsoids

- A reference ellipsoid has to be defined in combination with a reference frame
- $e^2 = \frac{a^2 - b^2}{a^2}$; $f = \frac{a-b}{a}$
- e… eccentricity
- f… flattening
- a… semi-major axis
- b… semi-minor axis



Misra und Enge (2001), Fig. 3.3

	a	1/f
WGS84	6378137.000	298.25722356300
GRS80	6378137.000	298.25722210088
Bessel	6377387.155	299.15281285000
PZ90.11	6378136.000	298.25783930300

Summary

- Each coordinate is related to
  - Geodetic datum: ITRFyy, ETRS89, WGS84, ...
  - Reference surface: none = 3d Cartesian, Ellipsoid, Sphere, Plane

- GPS coordinates are given with geodetic datum WGS84 as 3D Cartesian coordinates (= no reference surface)
  - X, Y, Z

- Often, they are transformed to ellipsoidal coordinates for better interpretation (= Ellipsoid reference surface)
  - λ... longitude, φ... latitude, h... ellipsoidal height

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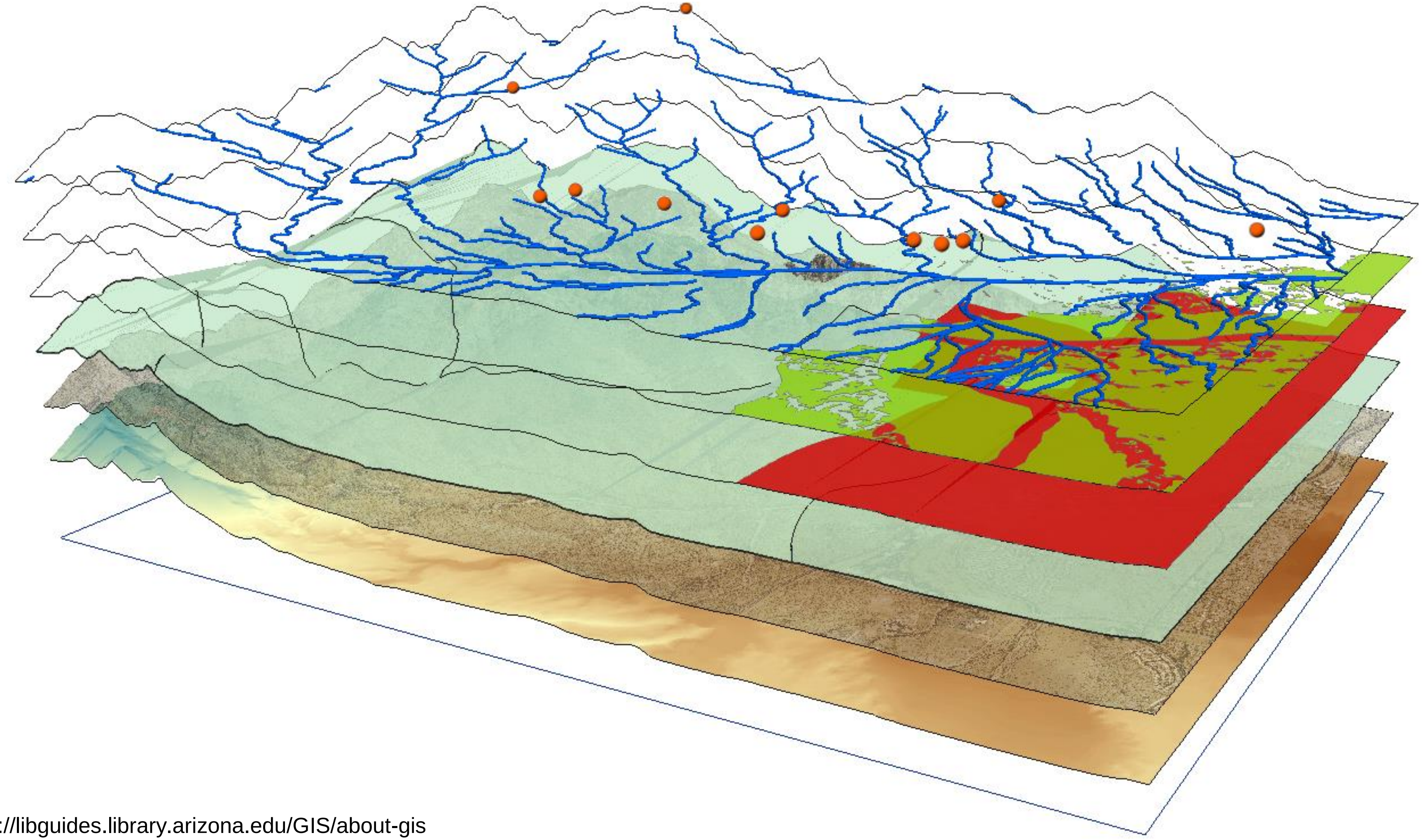
Real estate

- Cadaster: only positional coordinates (2D), metric, stable

Source: LAiV-MV

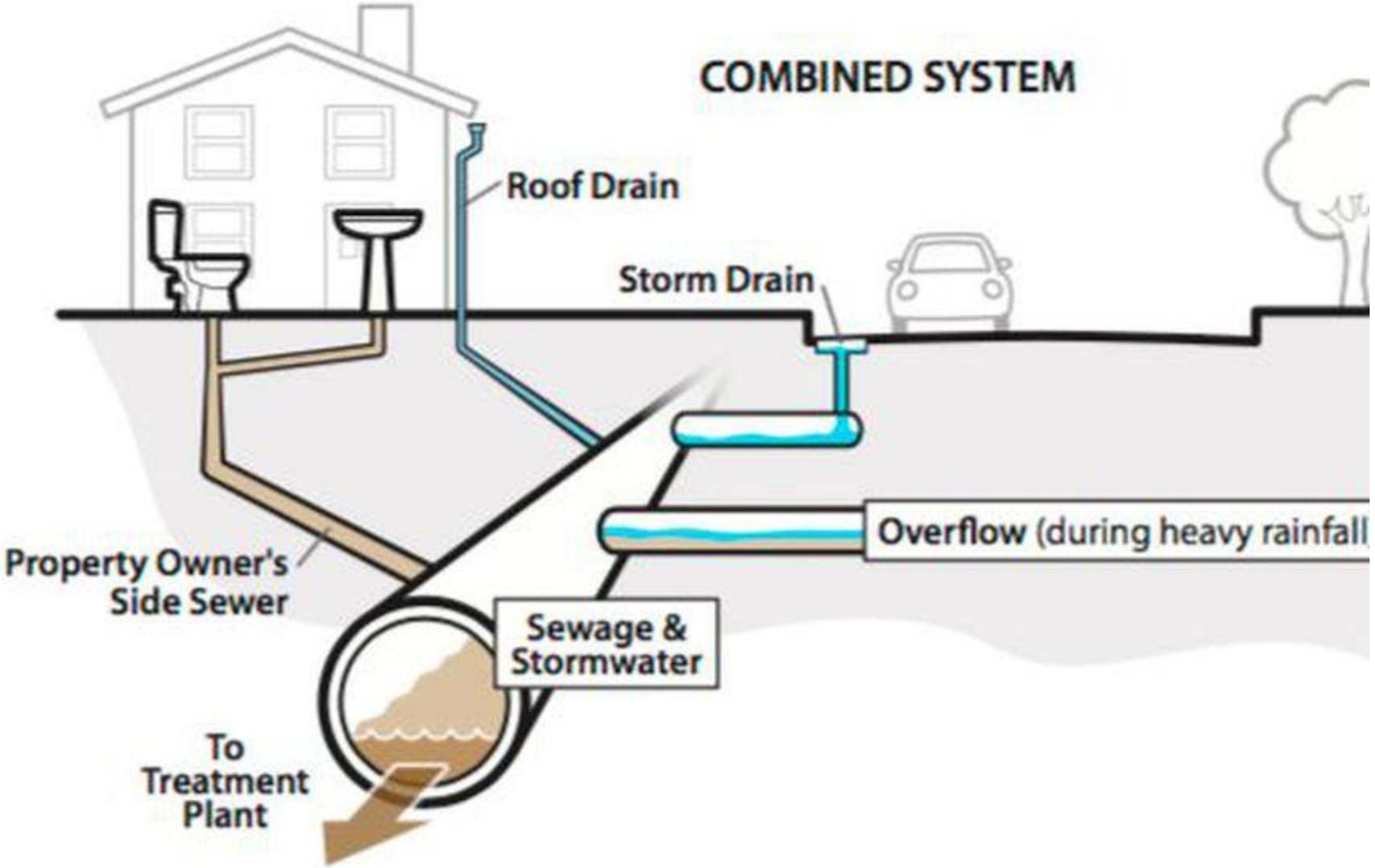
- Positional coordinates (+ height), metric, stable



http://libguides.library.arizona.edu/GIS/about-gis

Agency of urban planning

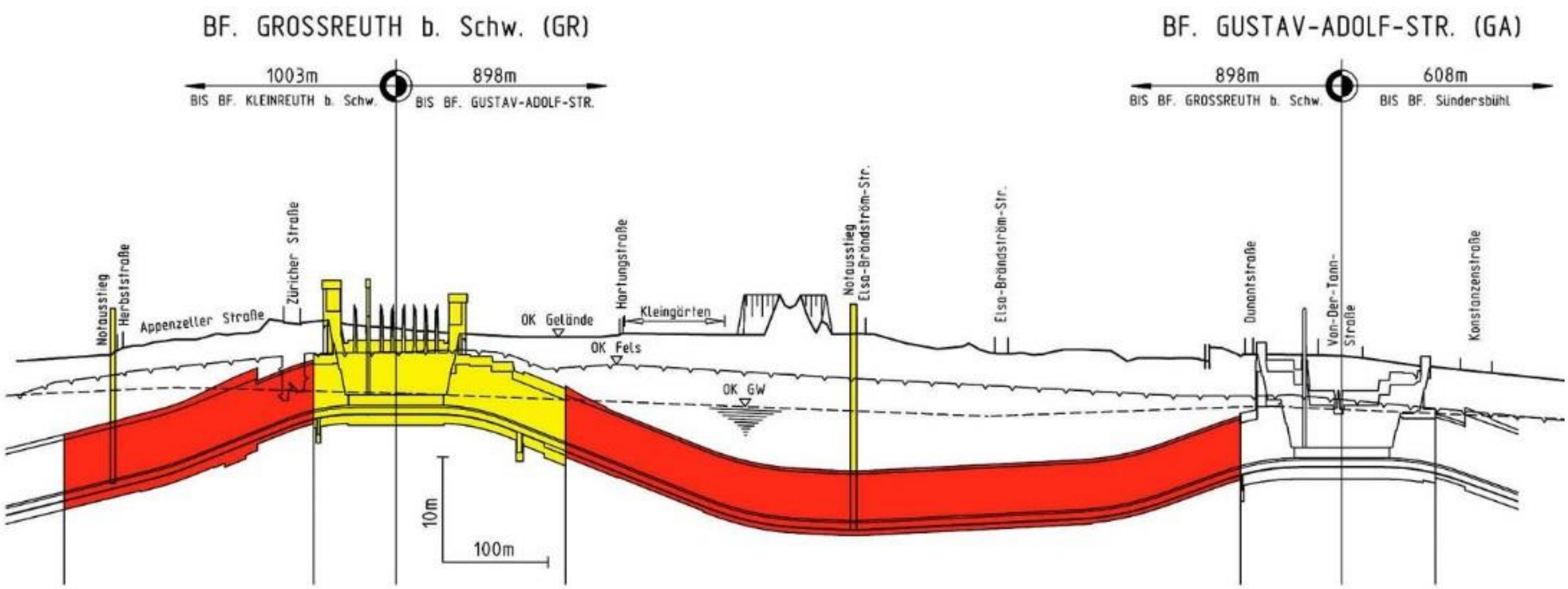
- Heights: physical interpretable => no water flows between two points with the same height



Source: City of Seattle

Civil Engineering

- Positional coordinates and height, metric, stable
- Heights: physical interpretable => no water flows between two points with the same height



Source: City of Münster

Summary

- Official coordinates need to be **separated by position (2D) and height**
- Position needs to be **metric** => no longitude and latitude
- Height needs to be **physical** => no ellipsoidal height
- Official coordinates need to be **stable**, i.e., no coordinate change due to global earth processes => no ITRF / WGS84 for Europe

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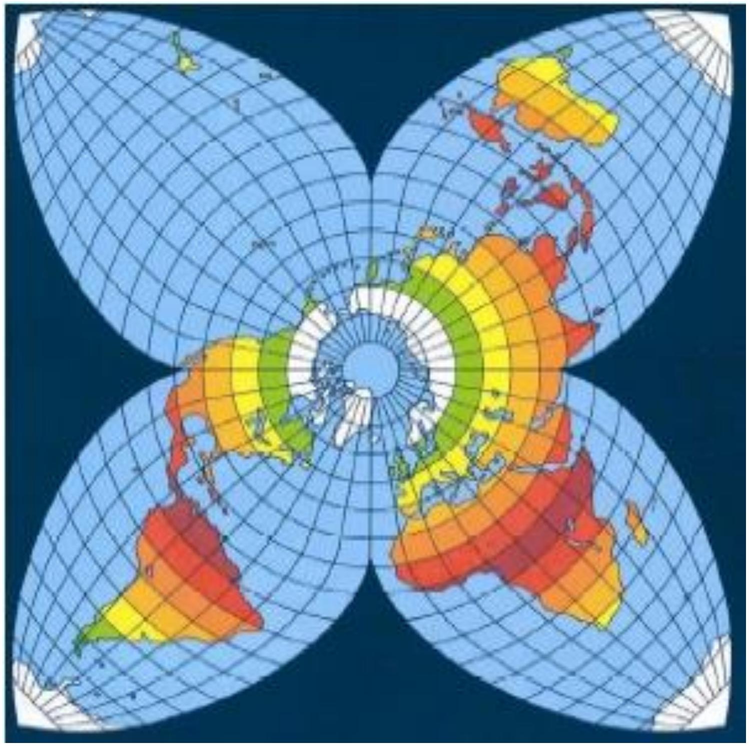
How to flatten the earth?

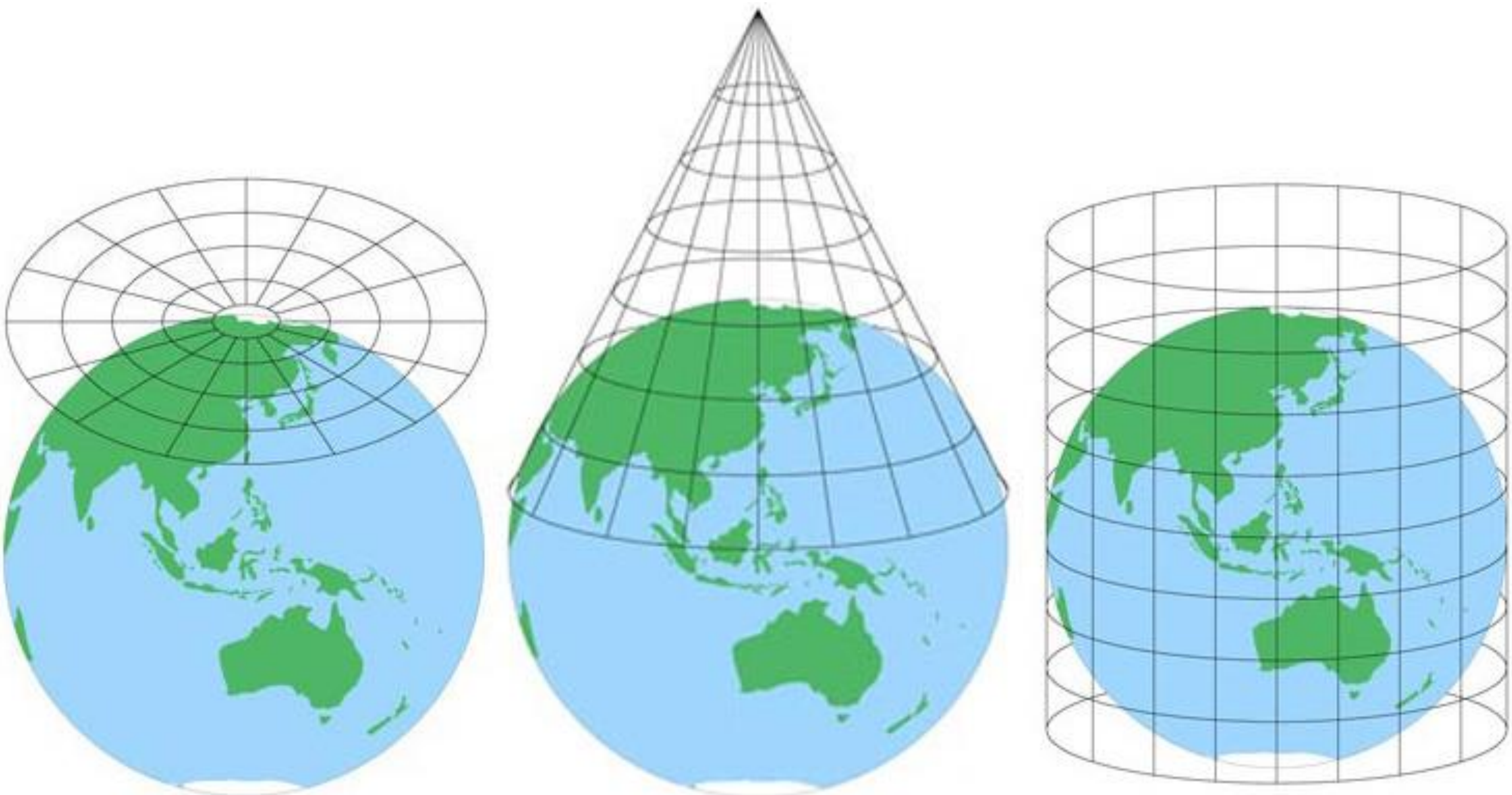


Projection of 3D coordinates in 2D plane =>

Not applicable without distortion

Map projections

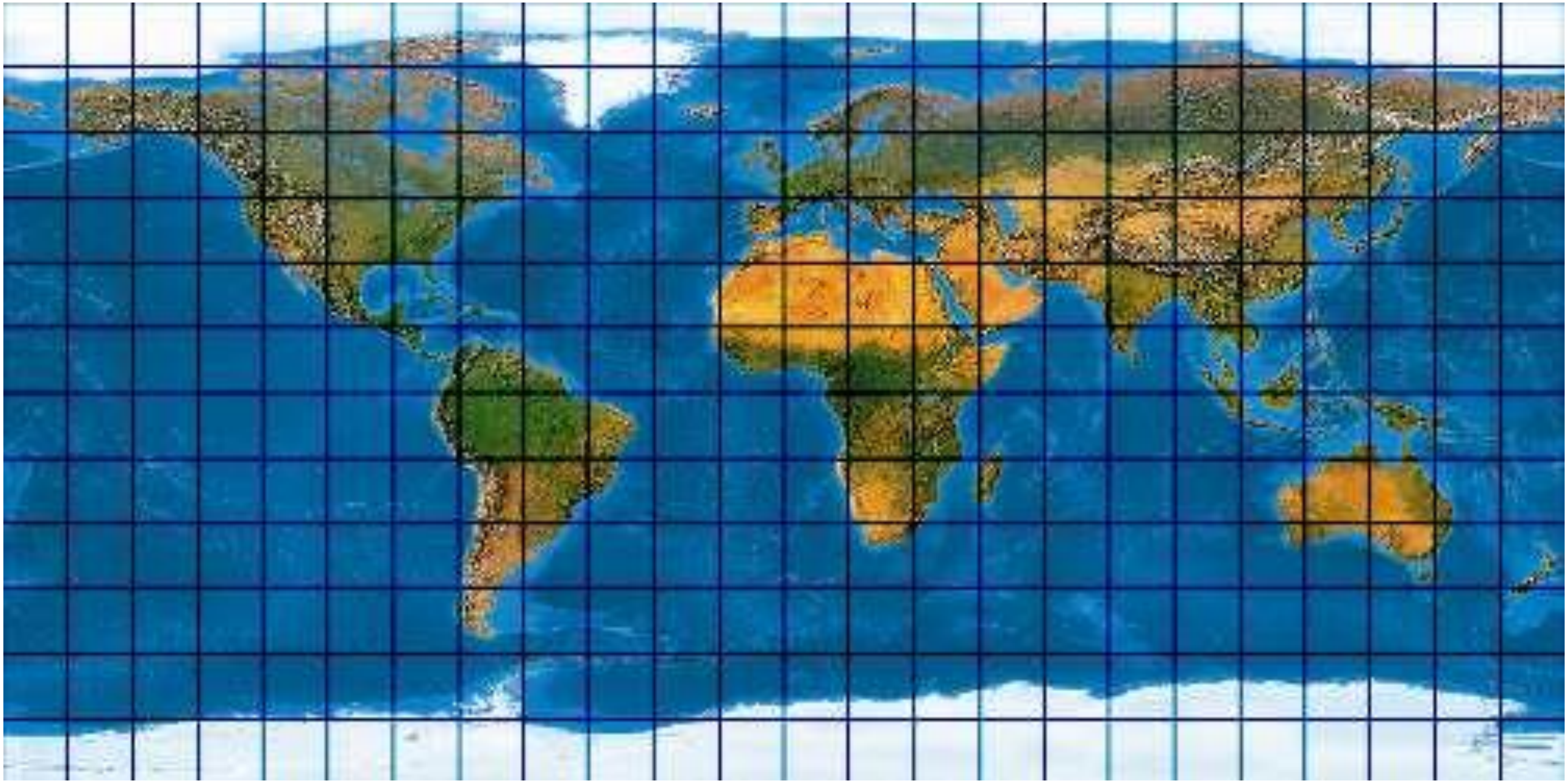
azimuthal conical cylindrical



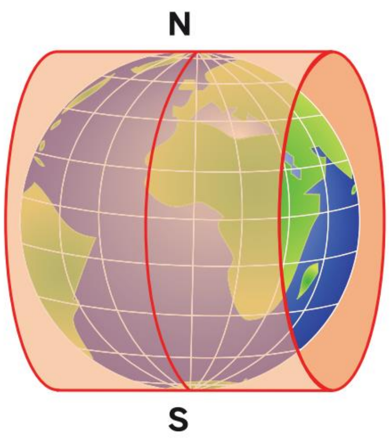
www.atlasandboots.com

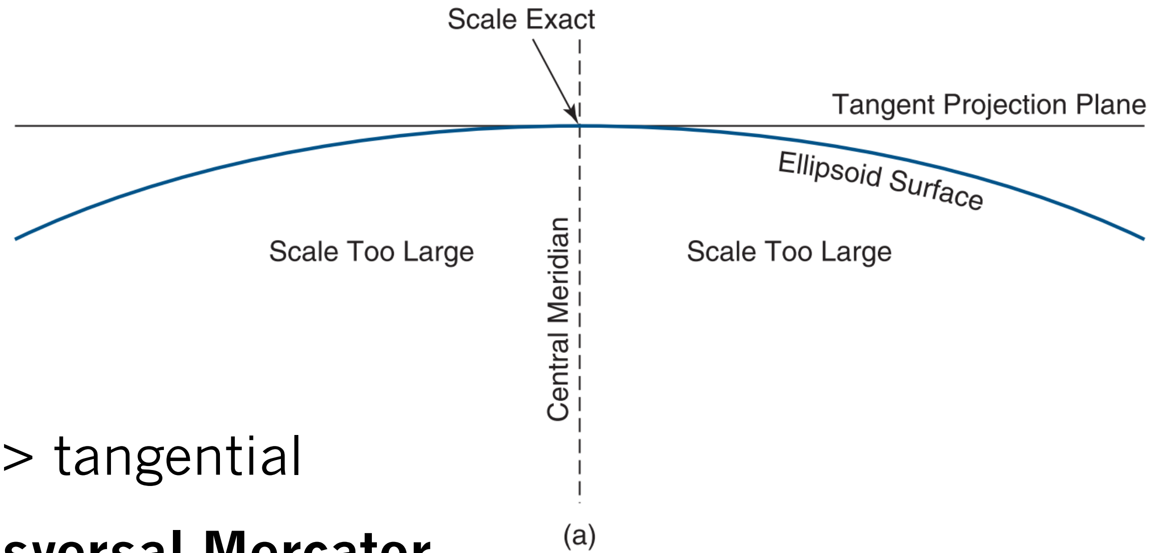
Cylindrical map projection

- Rectangular, planar coordinate system
- East axis, north axis



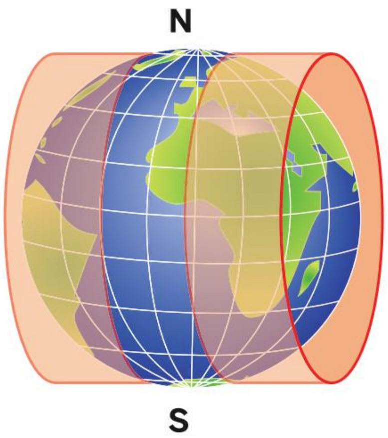
Realization of map projection

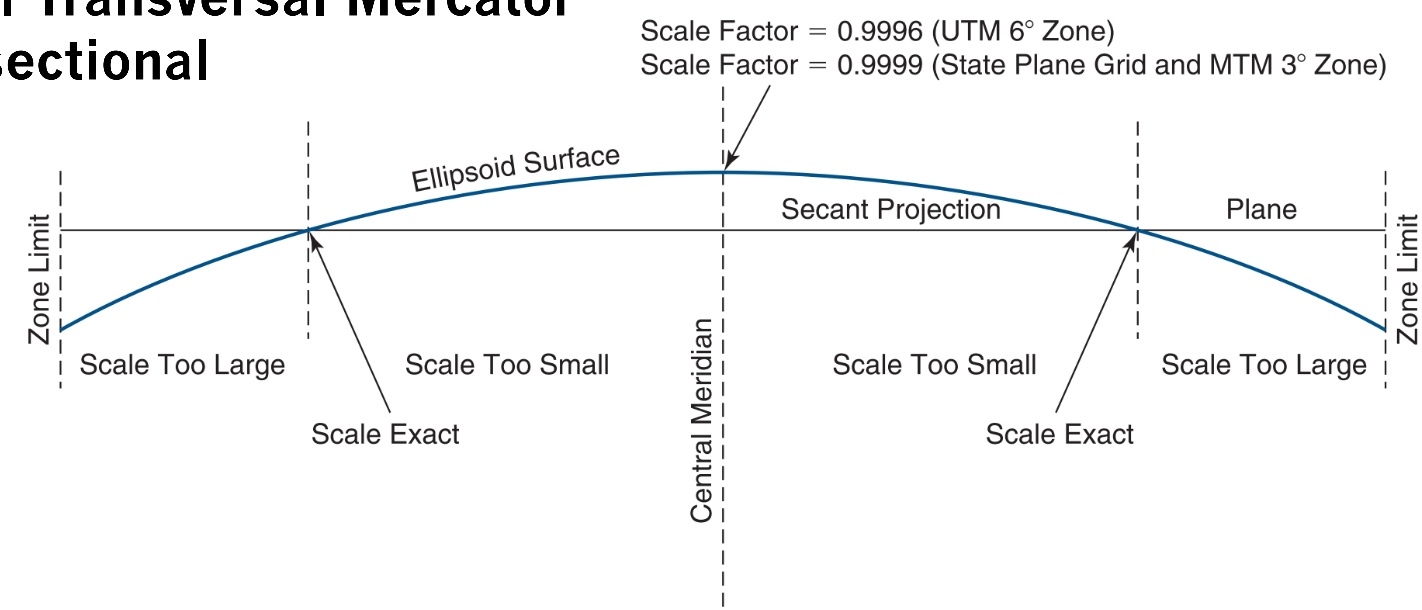




Above: Gauß-Krüger => tangential

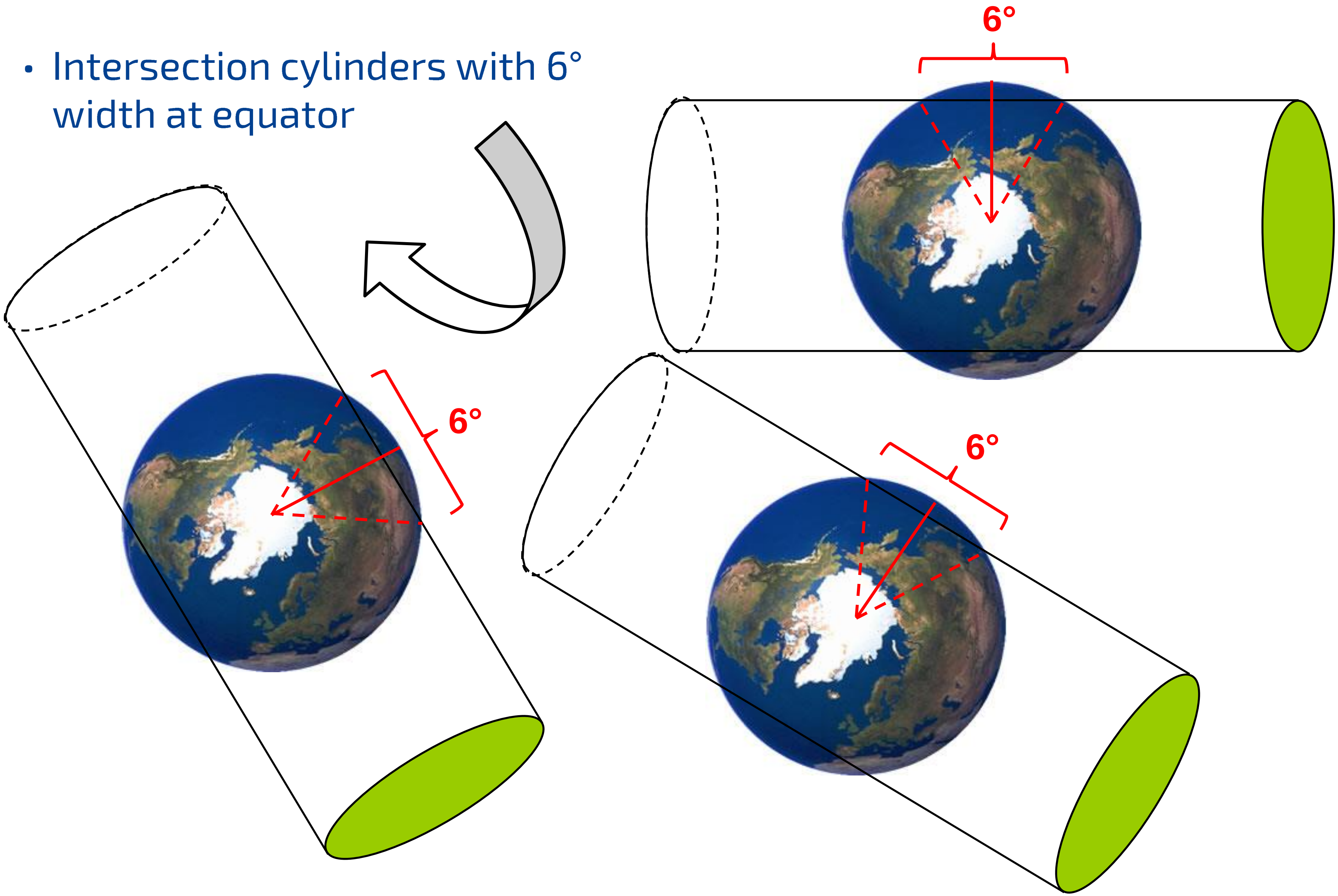
Below: **Universal Transversal Mercator (UTM) => intersectional**



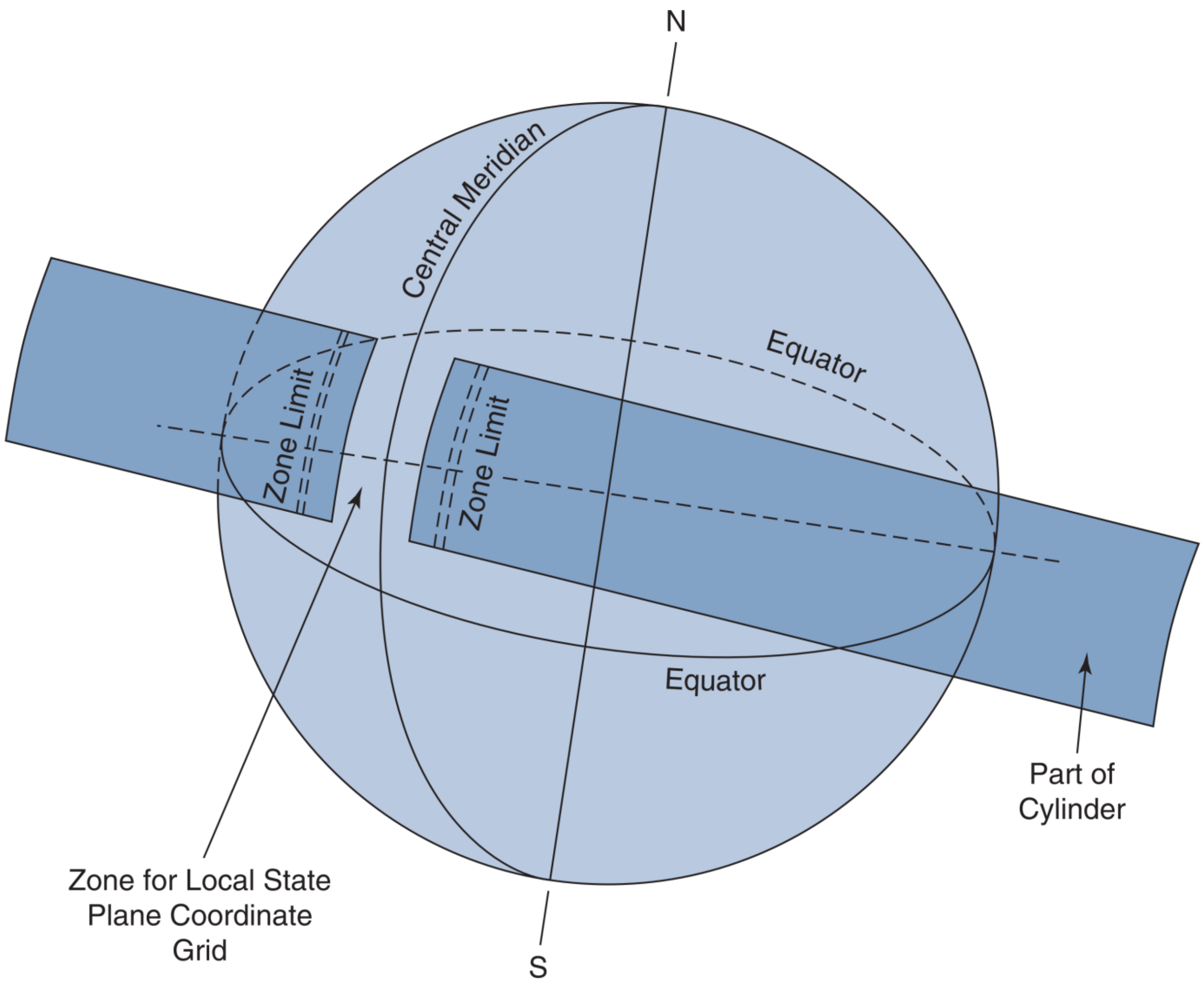


UTM projection: Top view

- Intersection cylinders with 6° width at equator

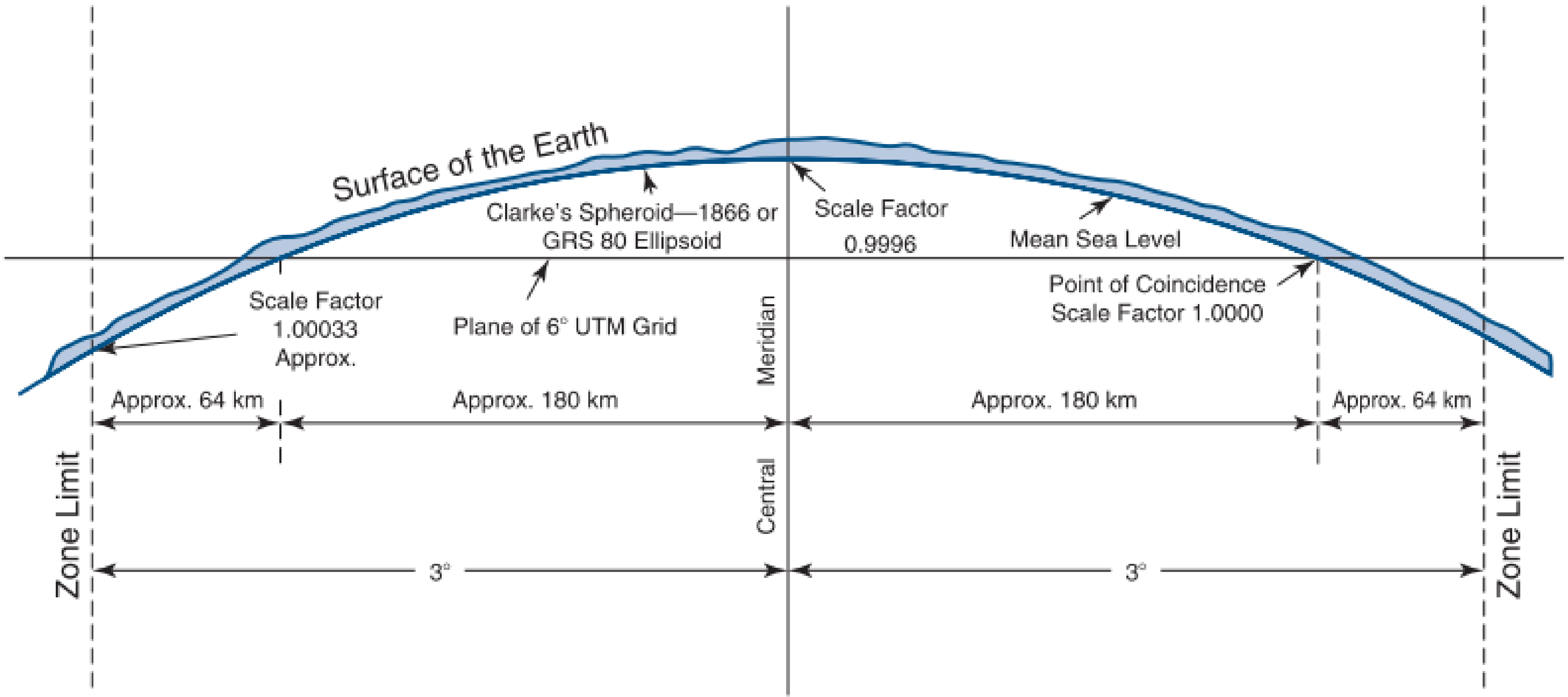


UTM projection: Side view



Source:
Kavanagh
(2014)

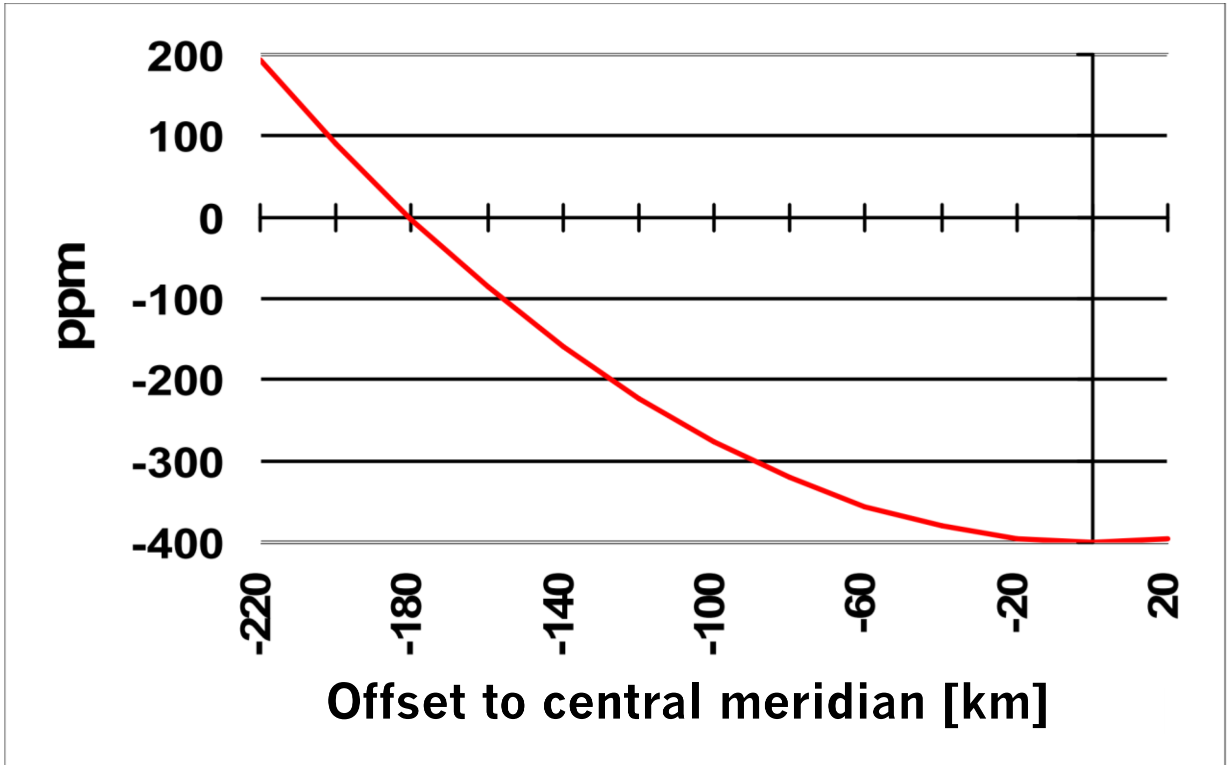
UTM projection: scale



Source: Kavanagh (2014)

Distortion due to projection

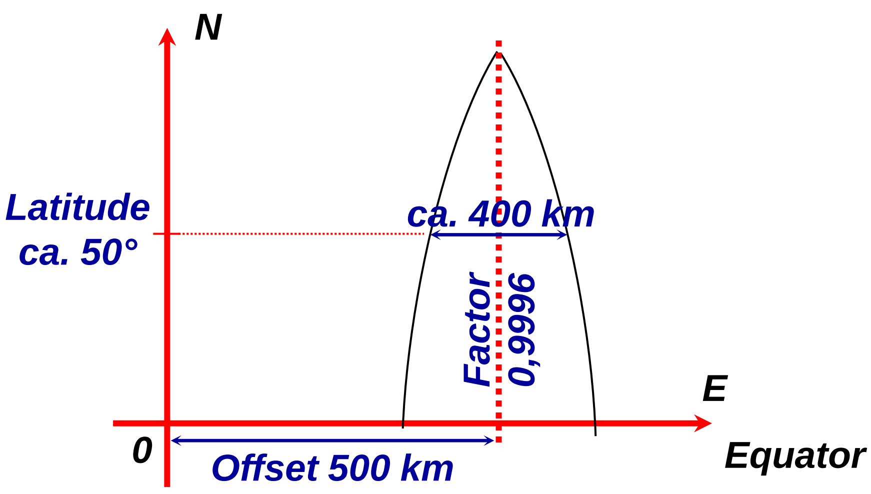
Offset to central meridian [km]	Distortion in distance [mm] (distance 100 m, height = 0 m above GRS80 ellipsoid)
0	- 40
100	- 28
180	0
200	+ 9



Source: Benning (2009)

UTM characteristics

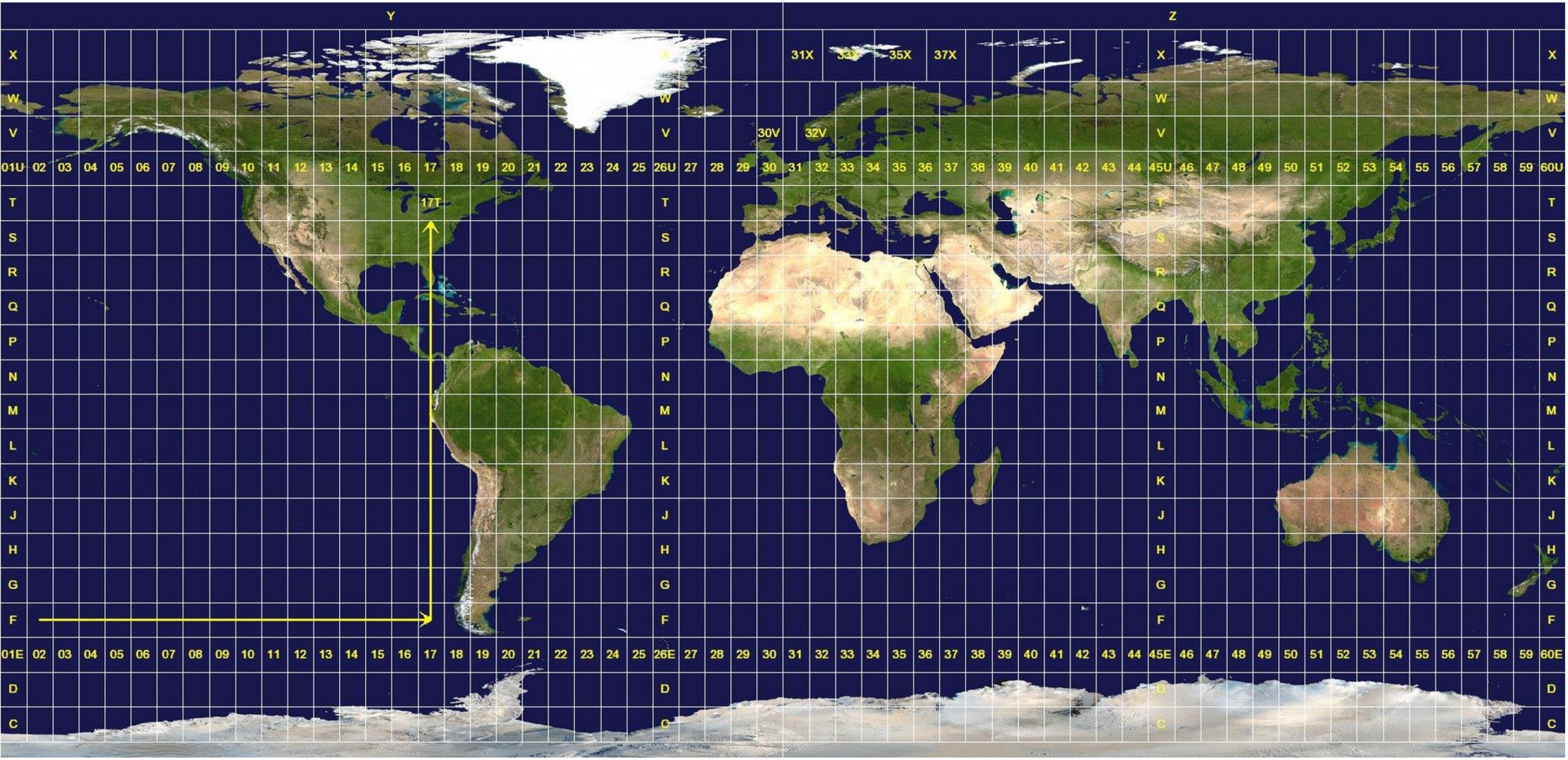
- Zone is 6° wide
- Projection is isogonal / equal of angle
- Northing value of the equator: N = 0 m
- Easting value of each central meridian: E = 500‘000 m
- Scale factor at central meridian: 0.9996
- Zone numbering commences with 1 in the zone 180°W to 174°W and increases eastward to zone 60 at the zone 174°E to 180°E
- Projection limits of latitude 80°S to 80°N



East [m]	32 364939
North [m]	5621299

UTM projection: world grid

- 60 stripes of 6° each = 360°



UTM projection: European grid

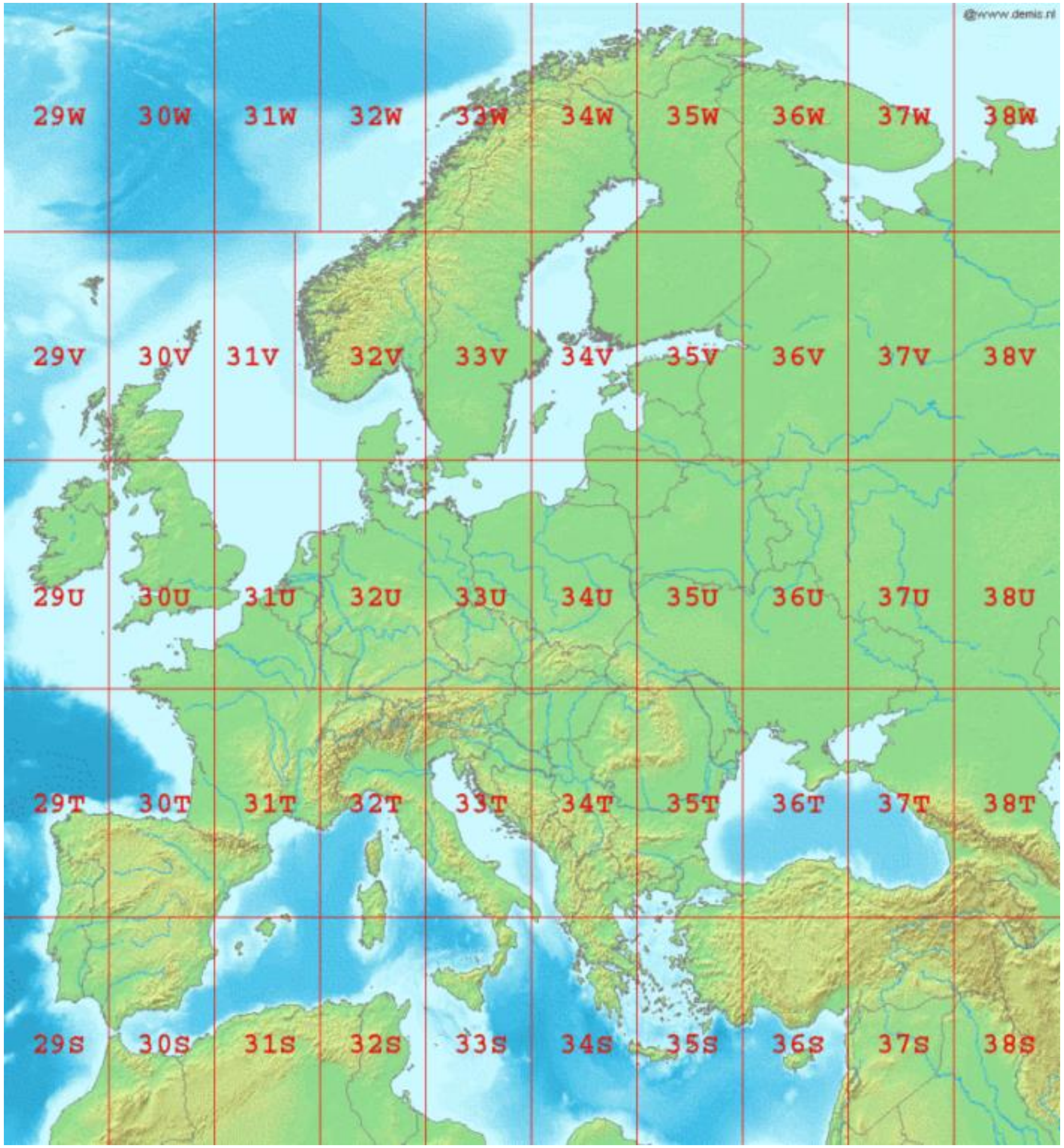


Fig.: UTM Zones in Europa
Source: www.wikipedia.org

UTM projection: German grid

- Germany: stripes 31, 32, 33
- North Rhine-Westphalia: only stripe 32 with central meridian at 9°

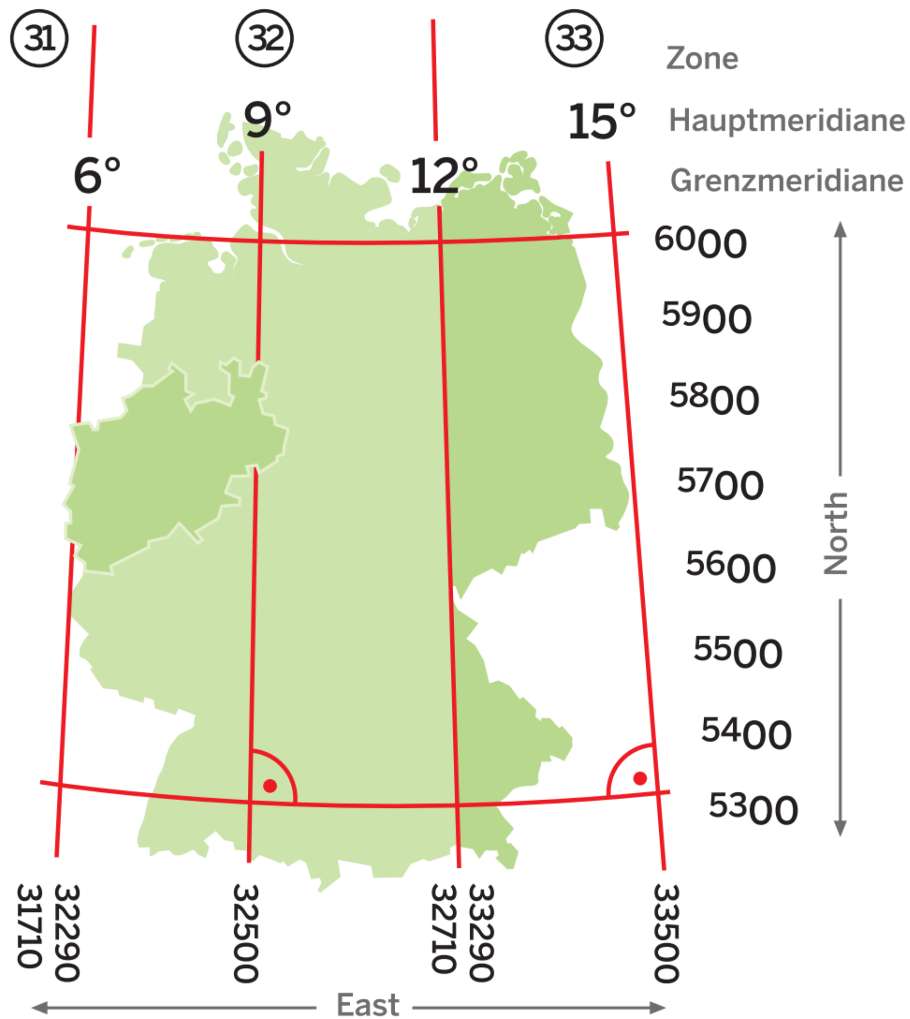


Abb. 6: Die Lage von NRW in der UTM-Zone 32

Source: Geobasis NRW

Realization of UTM

- UTM projection not directly related to any datum nor reference ellipsoid
- Countries may choose different datum and reference ellipsoids
- Germany:
  - **Datum = ETRS89**
  - **Reference Ellipsoid: GRS80 Ellipsoid**

	a	**1/f**
WGS84	6378137.000	298.2572235630
GRS80	6378137.000	298.2572221008
Bessel	6377387.155	299.1528128500
PZ90.11	6378136.000	298.2578393030

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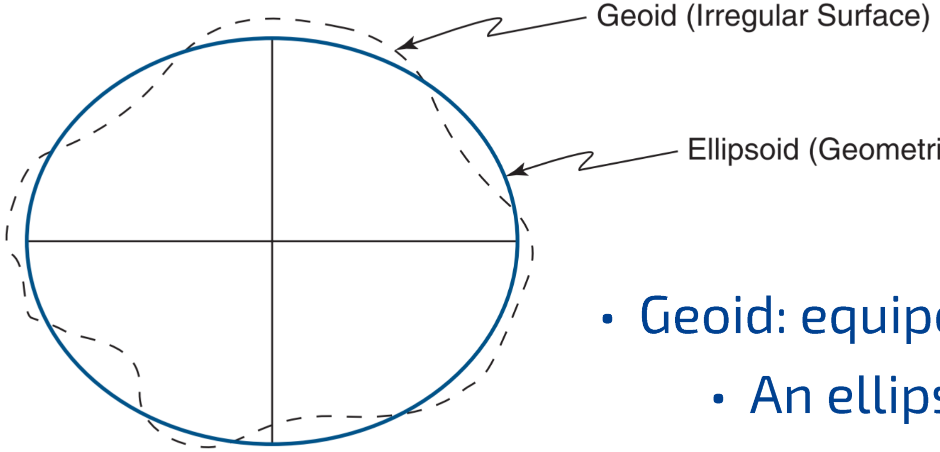
Physical heights

- Physical heights are needed

=> no water flows between two points with the same height

=> Equal to height above geoid



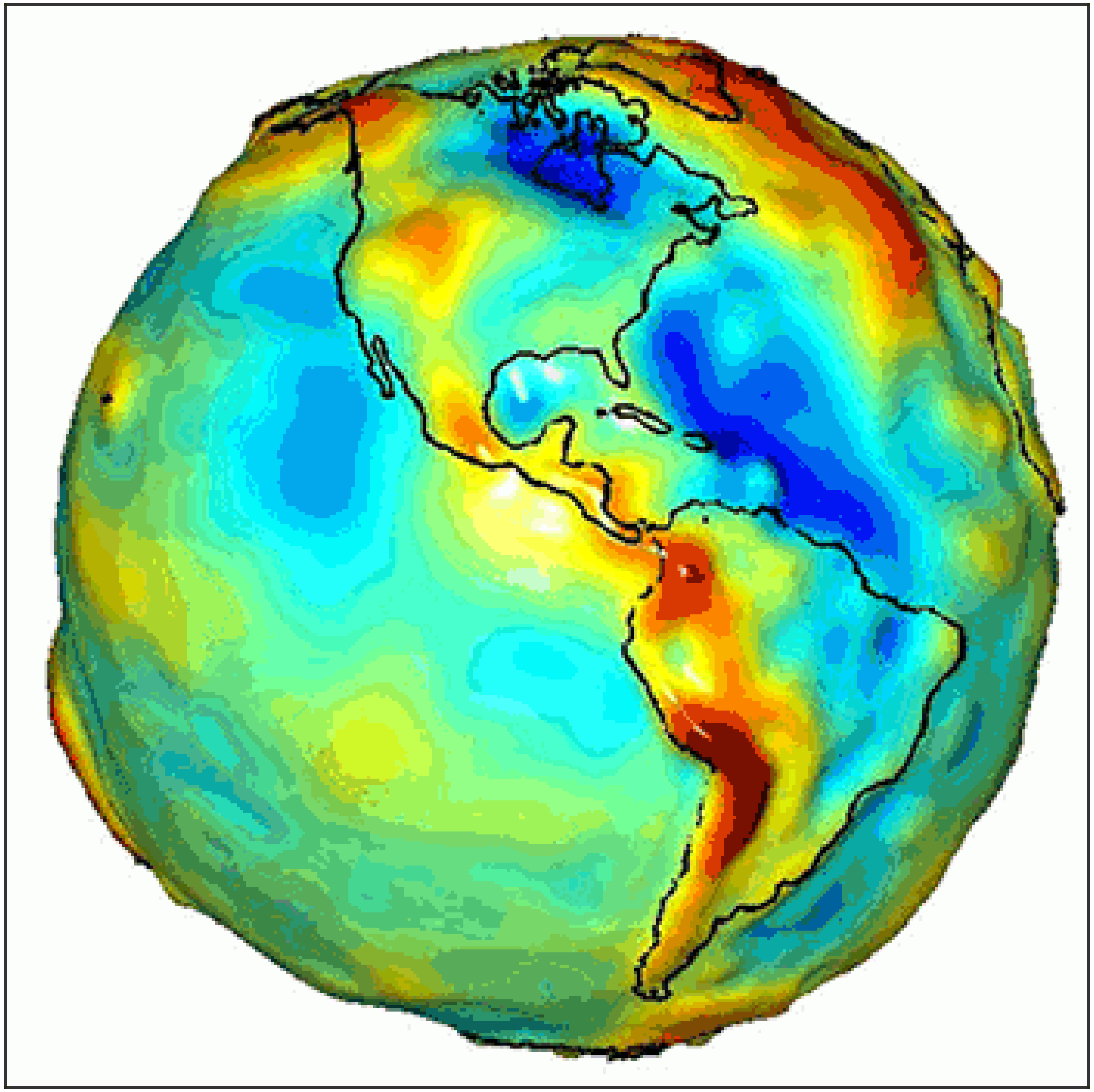
- Geoid: equipotential surface of the gravity field
    - An ellipsoid tries to approximate the geoid

- Difference between geoid and ellipsoid = Geoid undulation

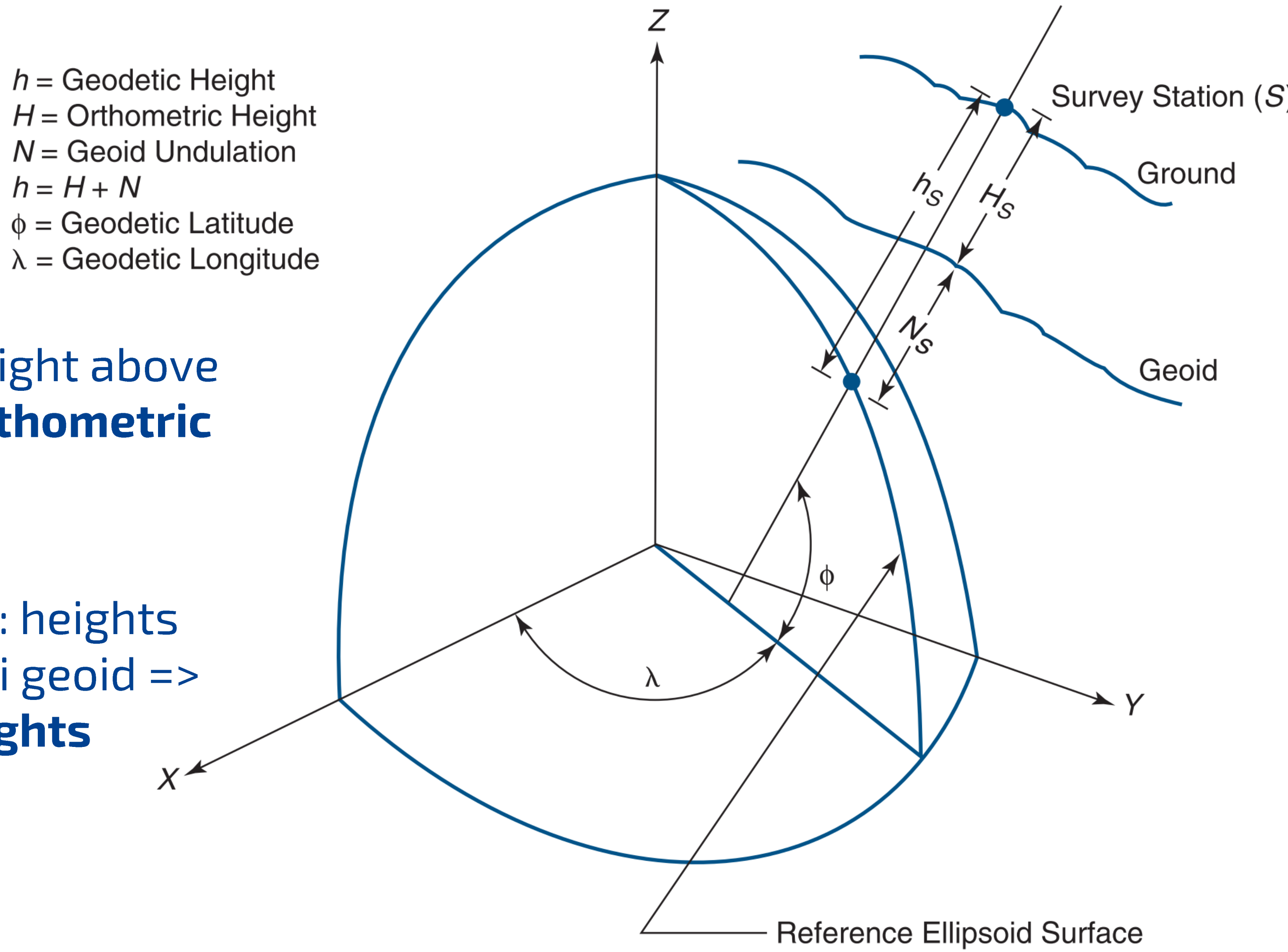
=> Geoid undulation must be known to get physical heights out of ellipsoidal heights

Source: Kavanagh (2014)

Physical heights



Physical heights



- Physical height above geoid => **orthometric heights**
- In Germany: heights above quasi geoid => **normal heights**

Source: Kavanagh (2014)

Summary

- **Positional coordinate**
  - UTM projection
  - Datum: ETRS89
  - Reference ellipsoid: GRS80

- **Height**
  - Physical heights: normal heights
  - Datum: GCG2016 (if transforming from ellipsoidal to physical)
  - Reference: Quasi geoid

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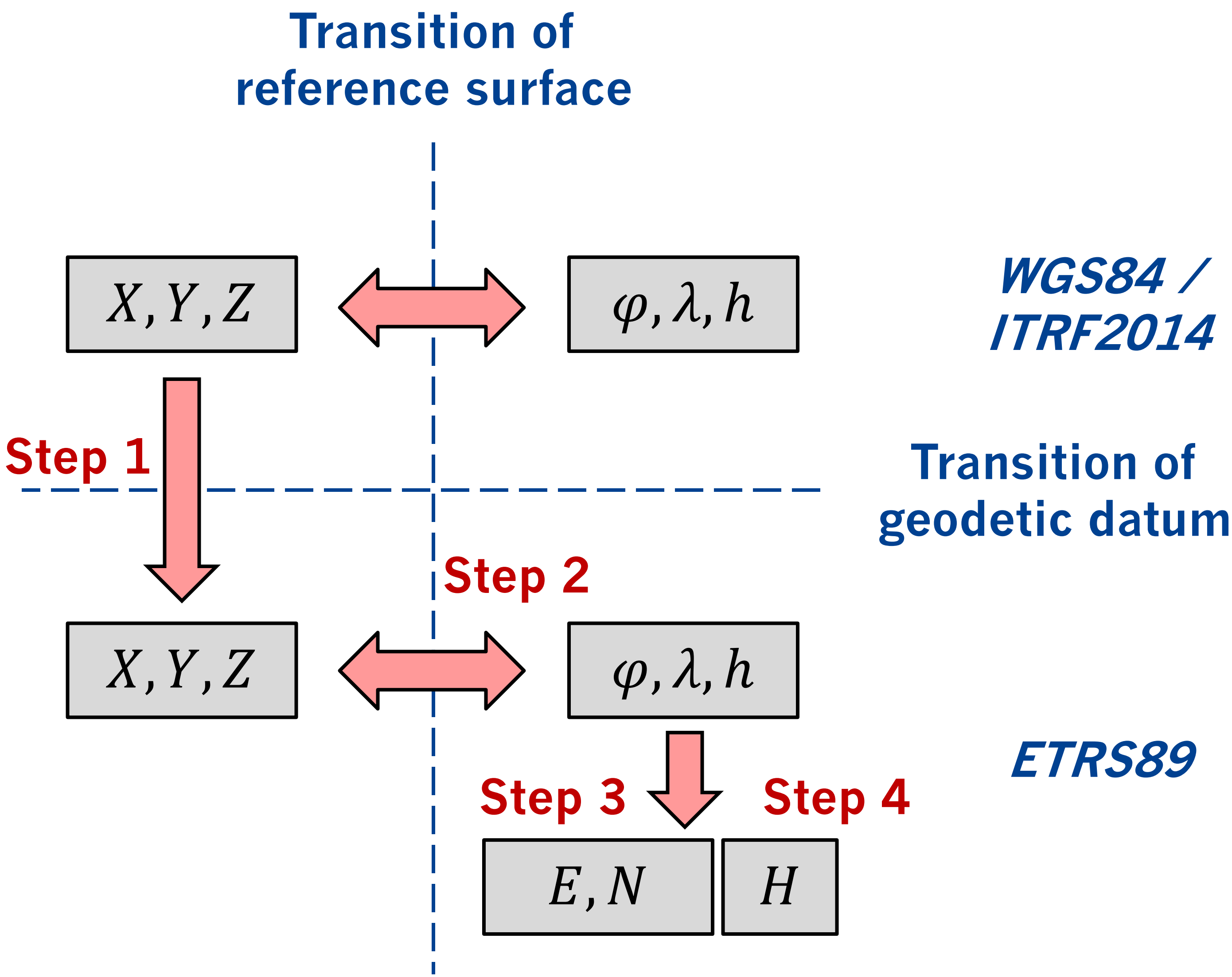
Transformation to official coordinates



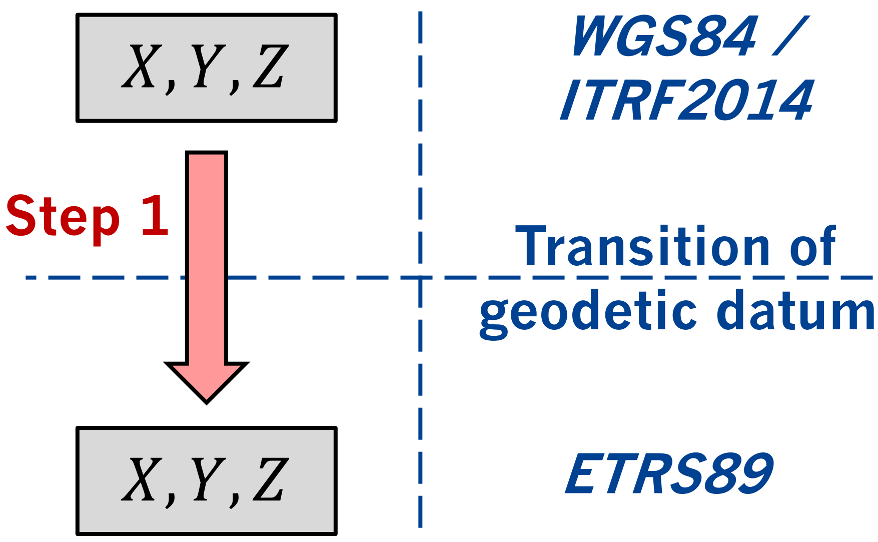
Step 1: From WGS84 to ETRS89

- Absolute positioning with GNSS:
  - Absolute coordinates X, Y, Z are given in WGS84
  - Transformation parameters between WGS84 and ETRS89 are frequently sent to GNSS receiver

  => Datum transition by selection in receiver => **X, Y, Z in ETRS89**
- Relative positioning with GNSS:
  - Baselines $\Delta X, \Delta Y, \Delta Z$ are given in WGS84
  - Similarity transformation between WGS84 and ETRS89 approx. pure translation

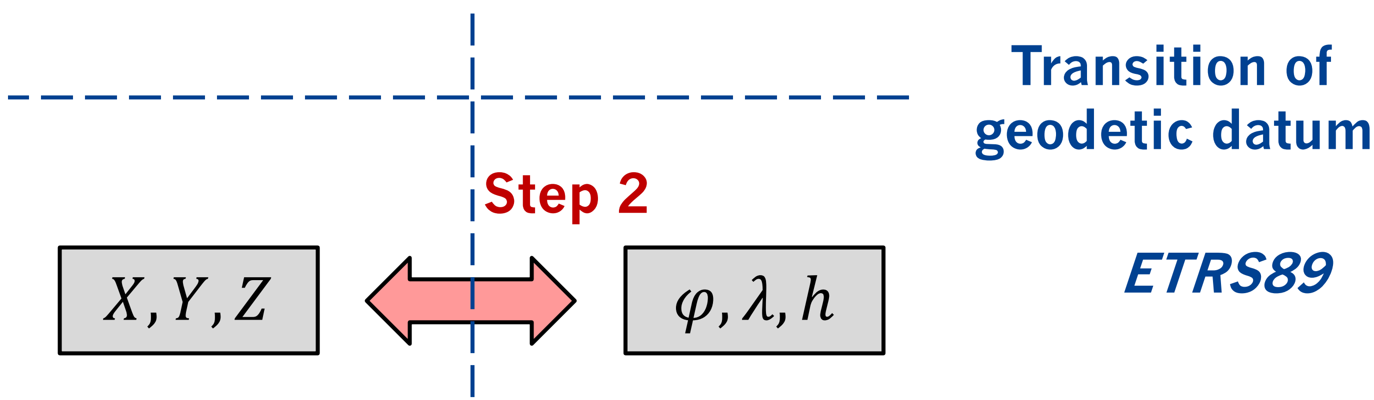
  => baselines unaffected from datum transition

  => Use master station coordinates with datum ETRS89 and add baselines $\Delta X, \Delta Y, \Delta Z$ => **$X + \Delta X, Y + \Delta Y, Z + \Delta Z$ in ETRS89**



Step 2: From X, Y, Z to φ, λ, h

- $$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}} \cdot \left(1 - \frac{a^2-b^2}{a^2} \cdot \frac{N(\varphi)}{N(\varphi)+h}\right)^{-1}\right) \\ \tan^{-1}\left(\frac{Y}{X}\right) \\ \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - N(\varphi) \end{bmatrix}$$
- Iterate: $N(\varphi) = \frac{a^2}{\sqrt{a^2 \cdot cos^2\,\varphi + b^2 \cdot sin^2\,\varphi}}$
- Use GRS80 Ellipsoid



Step 3: From λ, φ to E, N

- $\boldsymbol{E} = E_0 + [1] \cdot \Delta L + [3] \cdot \Delta L^3 + [5] \cdot \Delta L^5$
- $\boldsymbol{N} = m \cdot G + [2] \cdot \Delta L^2 + [4] \cdot \Delta L^4 + [6] \cdot \Delta L^6$
- $\Delta L = L - L_o$, with $L_o = 9°$ for North Rhine-Westfalia
- $E_0 = \left(\frac{L_o + 3°}{6°} + 30.5\right) \cdot 10^6$
- $m = 0.9996$
- $G = G_o \cdot B + G_2 \cdot \sin 2B + G_4 \cdot \sin 4B + G_6 \cdot \sin 6B$
- $[1] = \frac{m}{\rho} \cdot \bar{N} \cdot \cos B$
- $[3] = \frac{m}{6\rho^3} \cdot \bar{N} \cdot \cos^3 B \cdot (1 - t^2 + \eta^2)$
- $[5] = \frac{m}{120\rho^5} \cdot \bar{N} \cdot \cos^5 B \cdot \left(5 - 18t^2 + t^4 + \eta^2 \cdot (14 - 58t^2)\right)$
- $[2] = \frac{m}{2\rho^2} \cdot \bar{N} \cdot \cos^2 B \cdot t$
- $[4] = \frac{m}{24\rho^4} \cdot \bar{N} \cdot \cos^4 B \cdot t \cdot (5 - t^2 + 9 \cdot \eta^2)$
- $[6] = \frac{m}{720\rho^6} \cdot \bar{N} \cdot \cos^6 B \cdot t \cdot (61 - 58t^2 + t^4)$

- $\rho = \frac{180°}{\pi}$
- $\bar{N} = \frac{c}{\sqrt{1+\eta^2}}$
- $t = \tan B$
- $\eta^2 = \frac{a^2 - b^2}{b^2} \cdot \cos^2 B$
- $c = \frac{a^2}{b}$

λ = L = Longitude

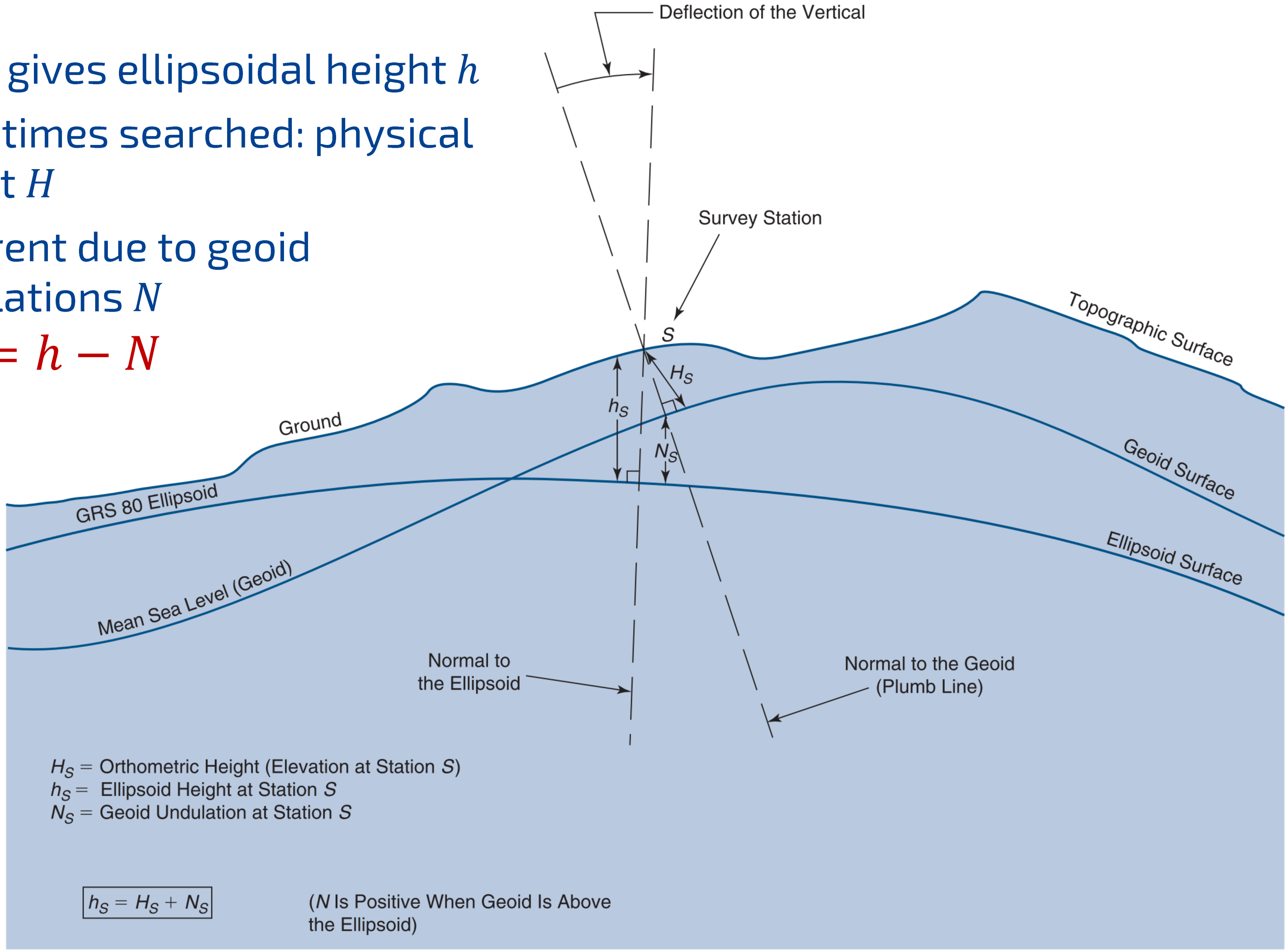
φ = B = Latitude

Parameter	GRS80 ellipsoid
G_o	111‘132.952547 m/°
G_2	-16‘038.5088 m
G_4	16.8326 m
G_6	-0.0220 m
a	6‘378‘137.000 m
b	6‘356‘752.314 m

Step 4: From h to H

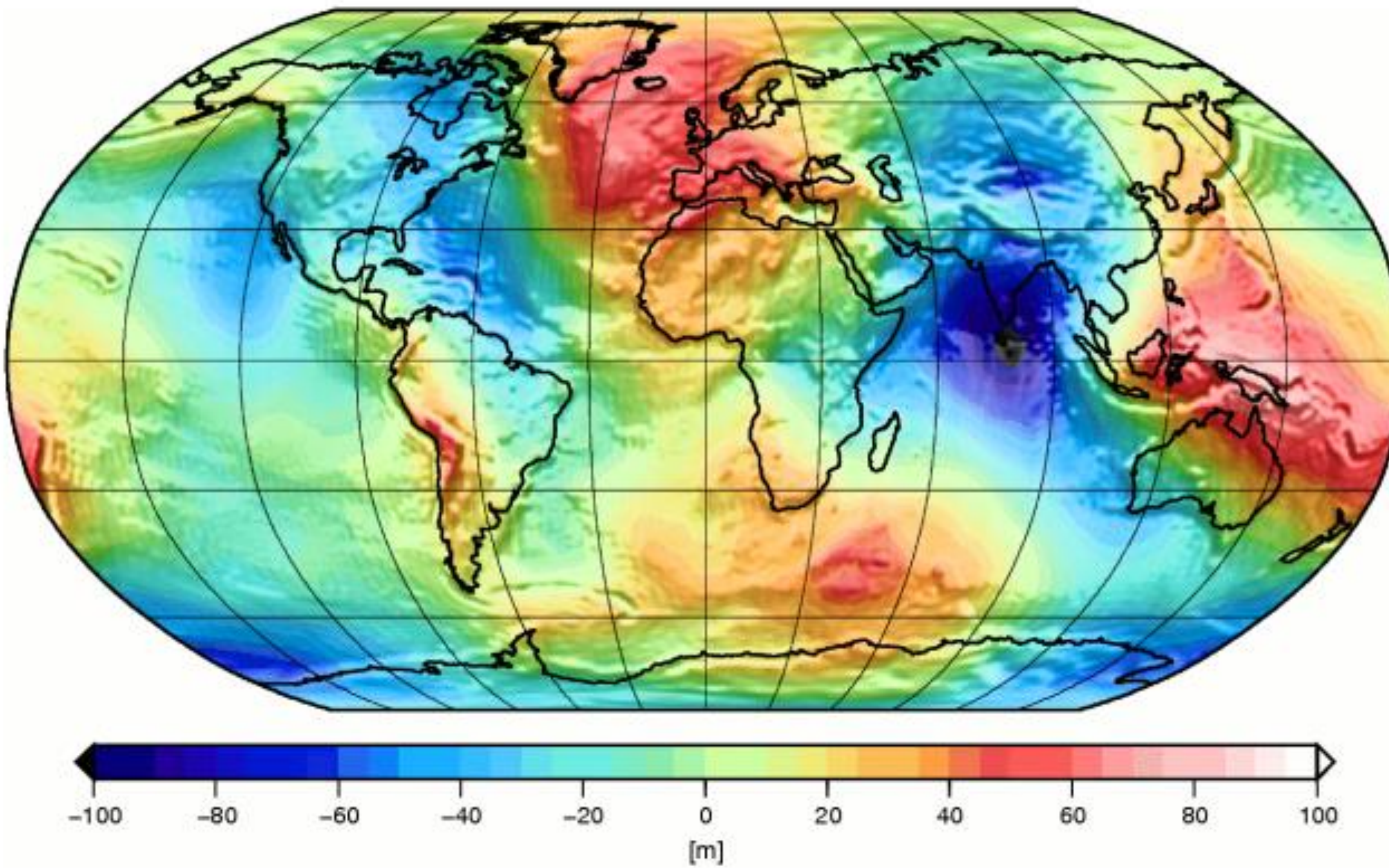
- GNSS gives ellipsoidal height h
- Most times searched: physical height H
- Different due to geoid undulations N

$\Rightarrow H = h - N$

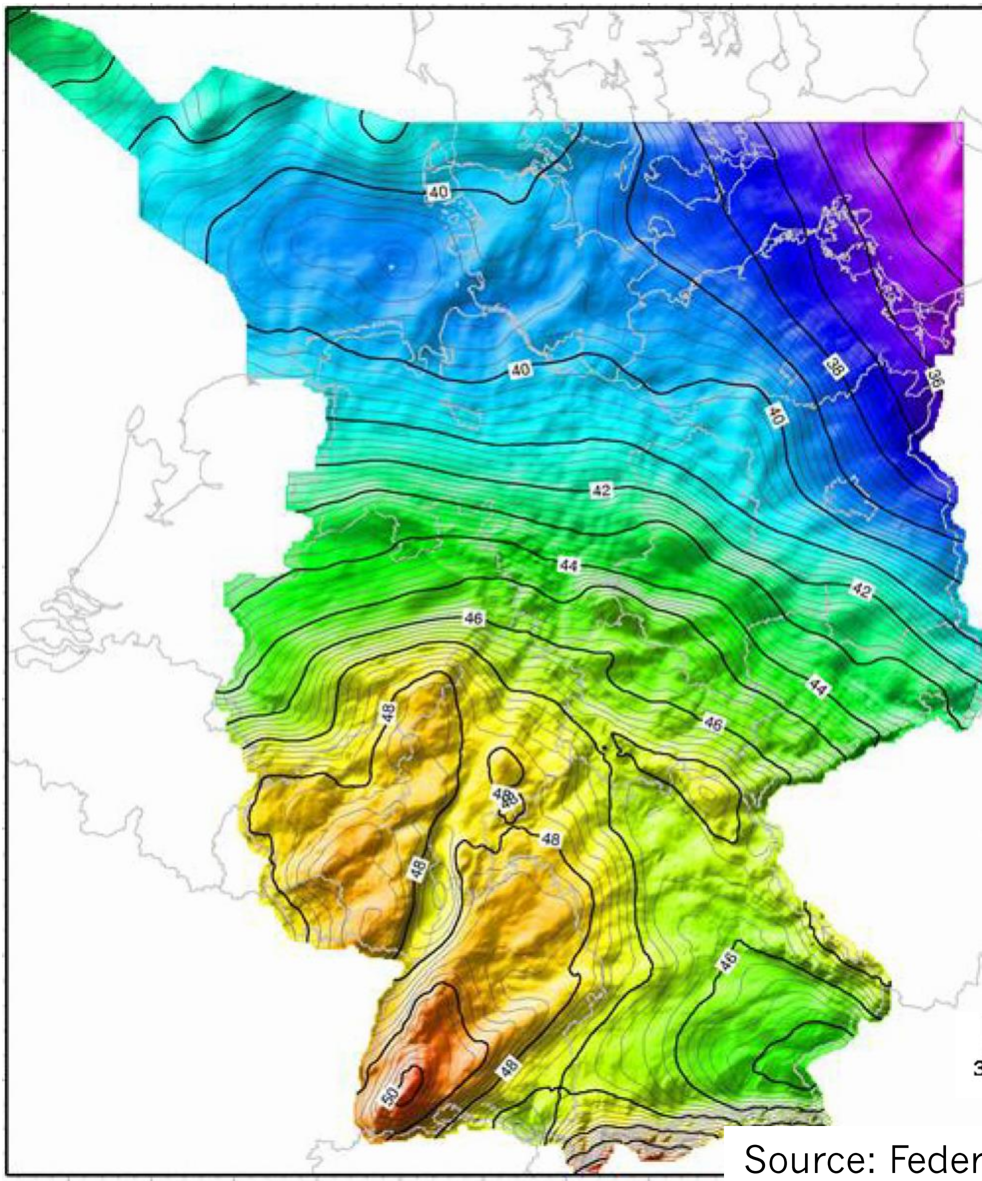


Source: Kavanagh (2014)

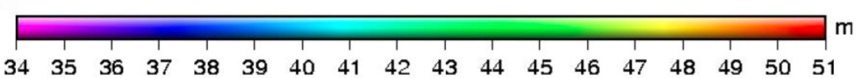
Geoid undulation



(Quasi) geoid undulation in Germany



- Federal Office for Cartography and Geodesy provides quasi geoid undulations for Germany
- Resolution: approx. 1 km x 1 km.
- Accuracy: 2-3 cm
- North Rhine-Westphalia: 42 m … 48 m
- Germany: 34 m … 50 m



Source: Federal Office for Cartography and Geodesy, Germany

Summary

- Step 1: Transform GNSS coordinate with datum WGS84 to datum ETRS89 (made by GNSS receiver that knows frequently updated transformation parameters)

=> Transition of geodetic datum

- Step 2: Transform Cartesian coordinates to ellipsoidal coordinates using GRS80 ellipsoid

=> Transition of reference surface

- Step 3: Transform latitude and longitude to UTM coordinates

=> Projection

- Step 4: Transform ellipsoidal height to physical height using the GCG2016 quasi geoid undulations

=> Transition of reference surface

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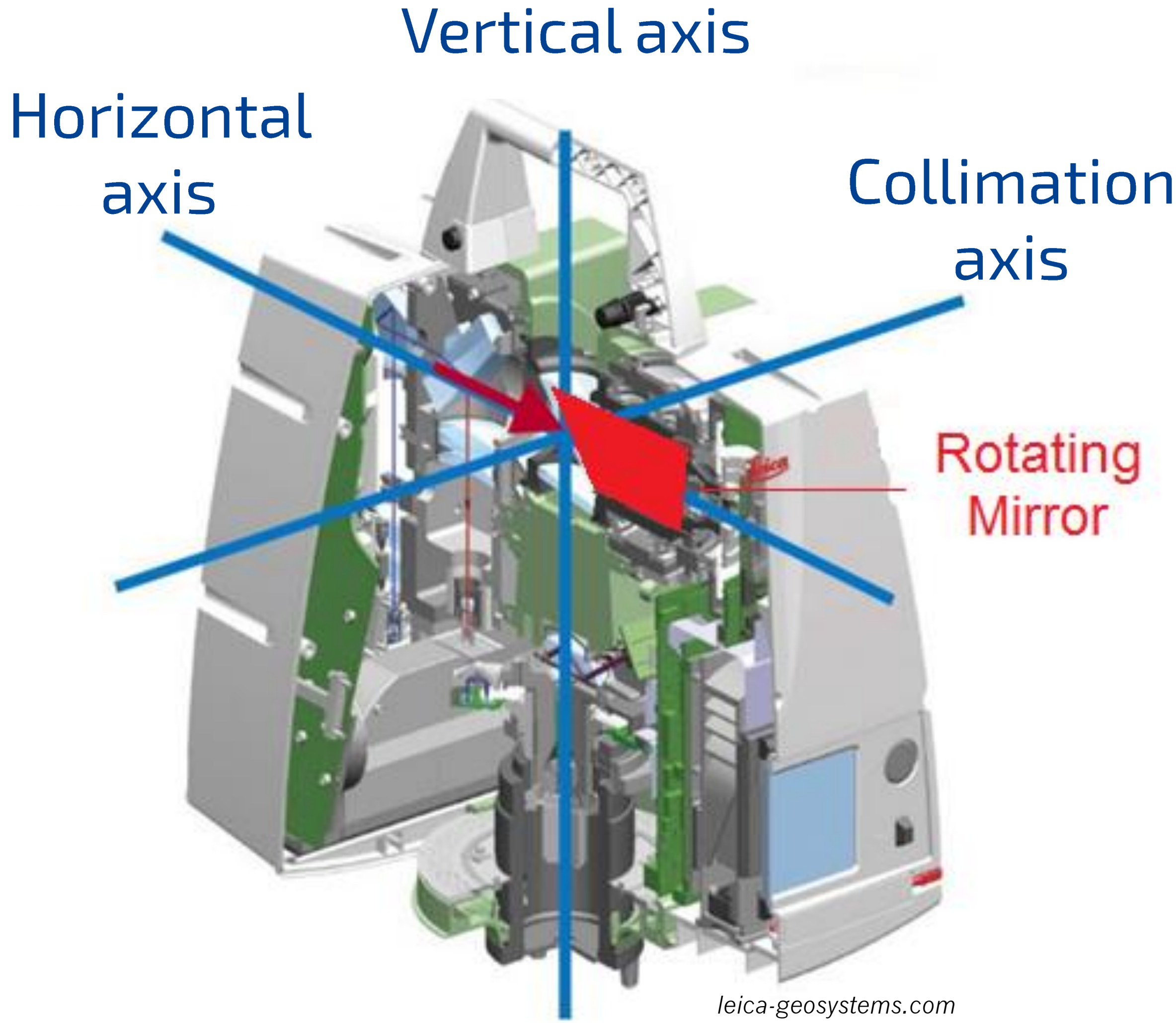
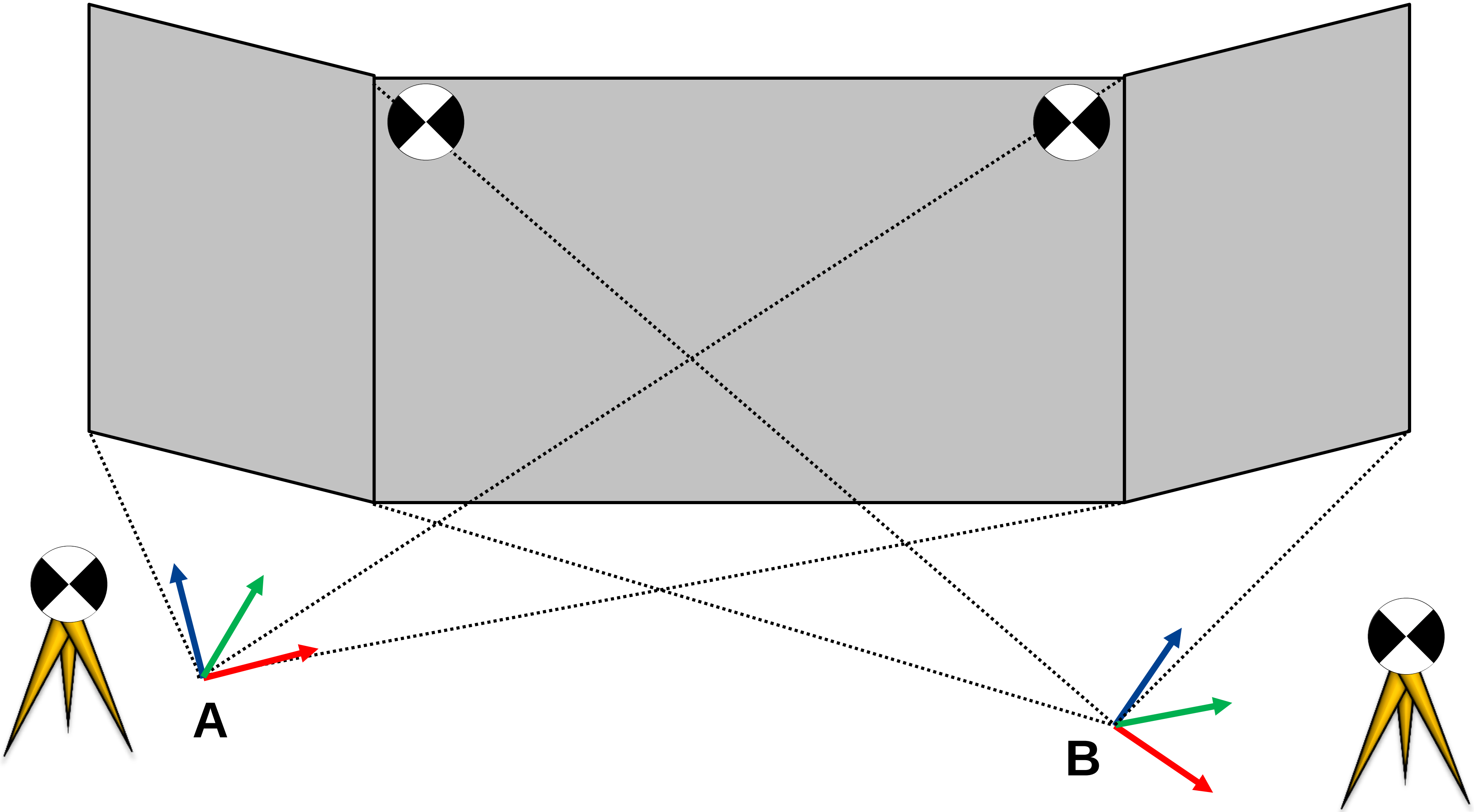
5. Transformation of TLS coordinates to official coordinates

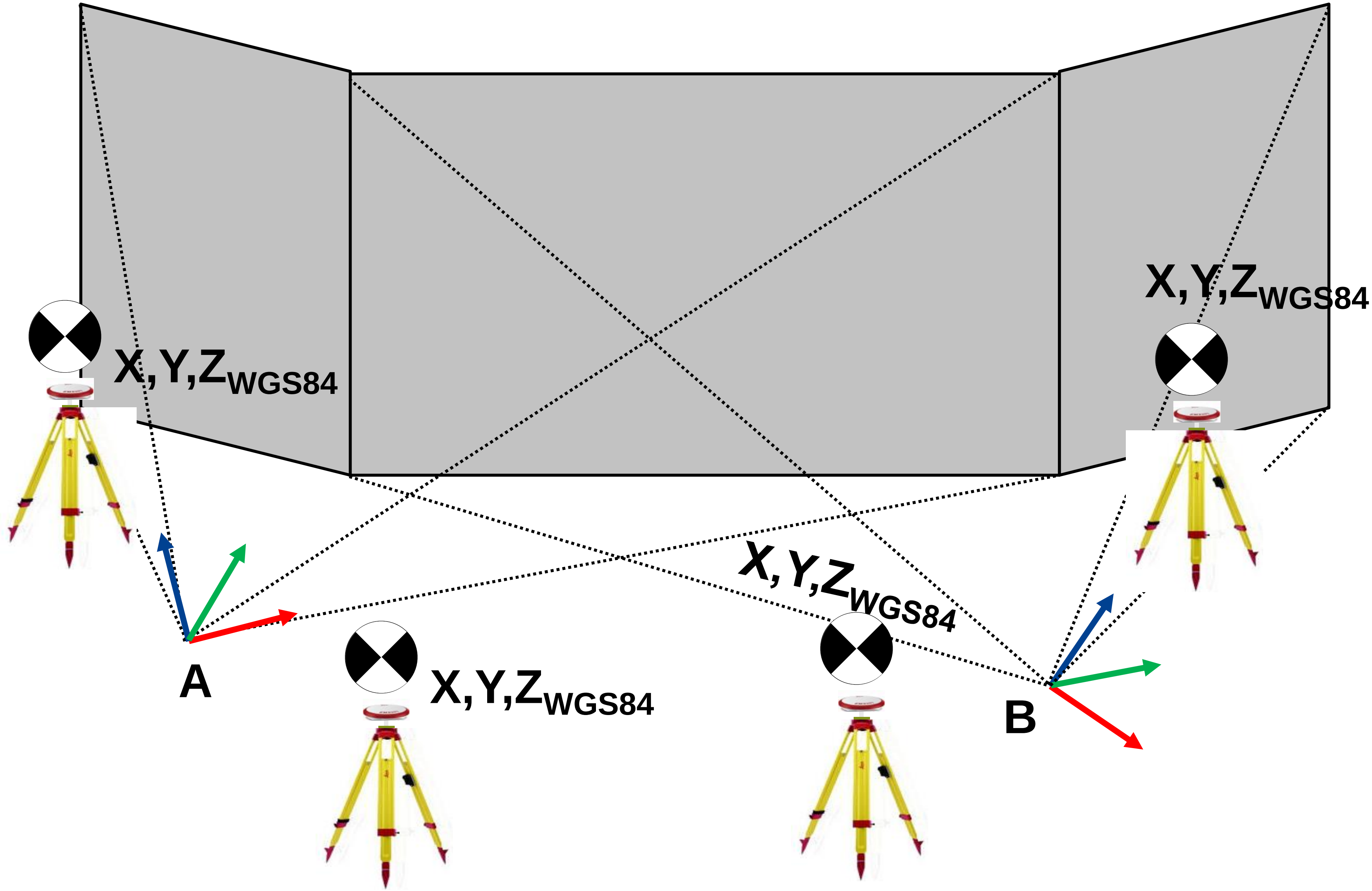
3D Laser scanner



Registration of laser scans



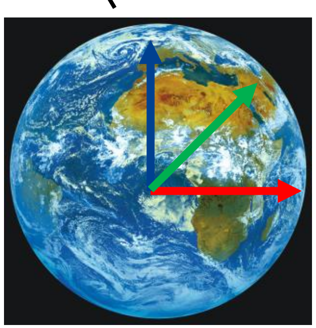
Geo-referencing of laser scans

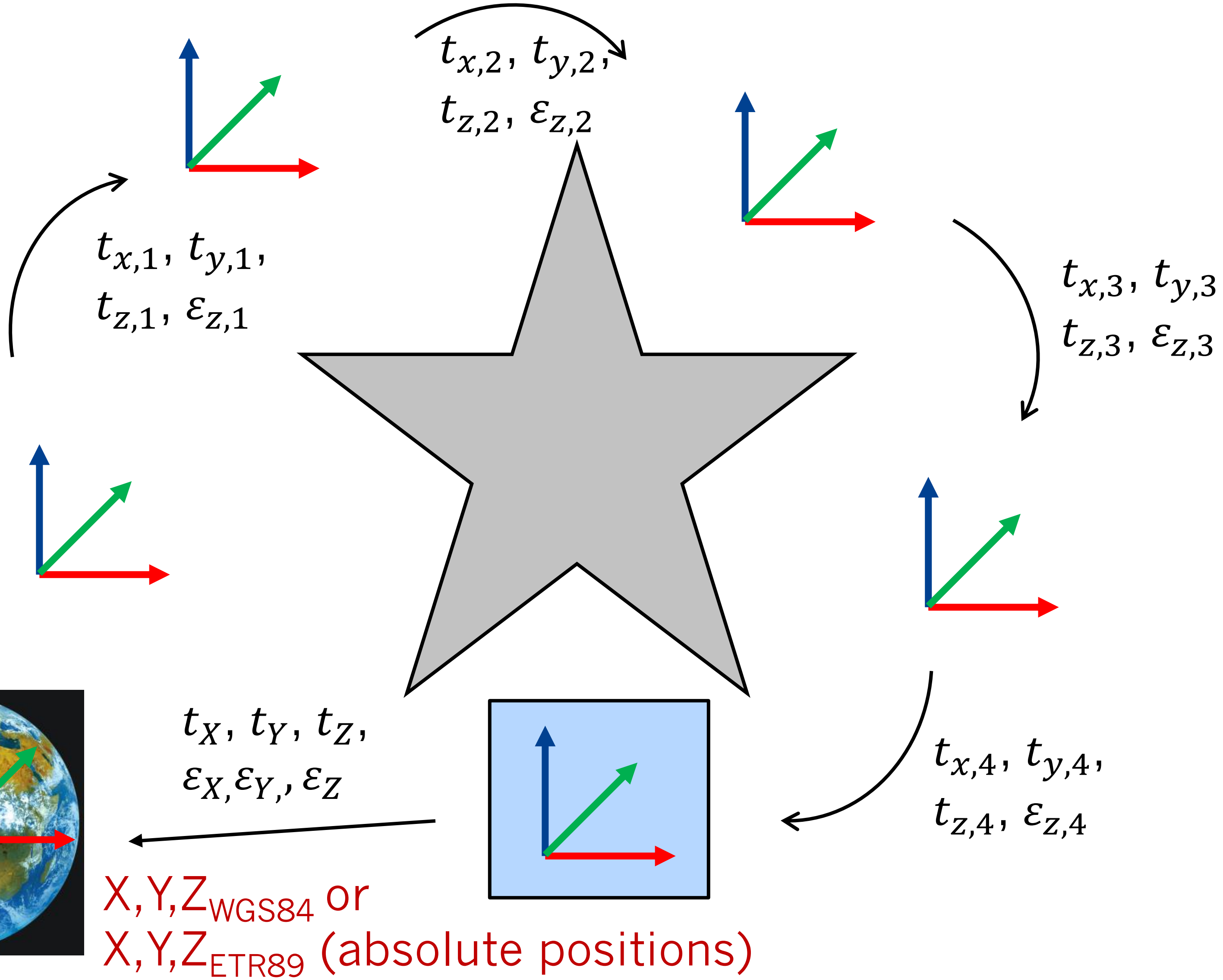


Geo-referencing of laser scans

Process each single laser scan point similarly to GNSS coordinates;

Start from Step 1 (WGS84) or Step 2 (ETRS89)





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