Chapter 3
Coordinate Transformations applied to Geodetic Sensors

Module MGE-01: Coordinate Systems

Dr.-Ing. Christoph Holst
&
Dr.-Ing. Tomislav Medic

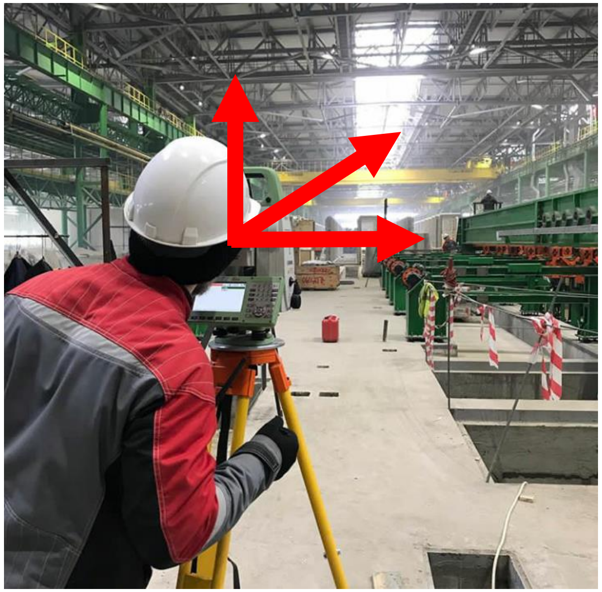
WS 2020/21

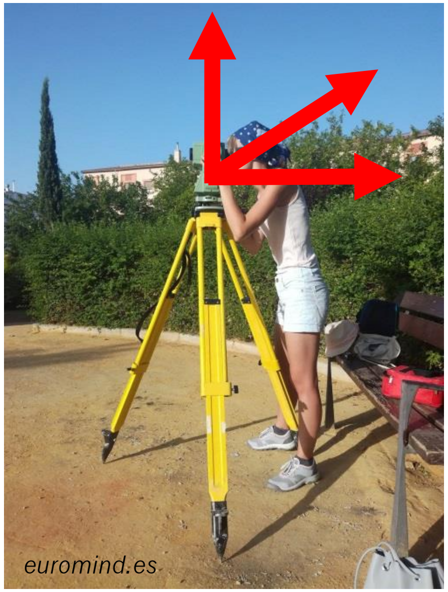
Institute of Geodesy and Geoinformation

Agenda

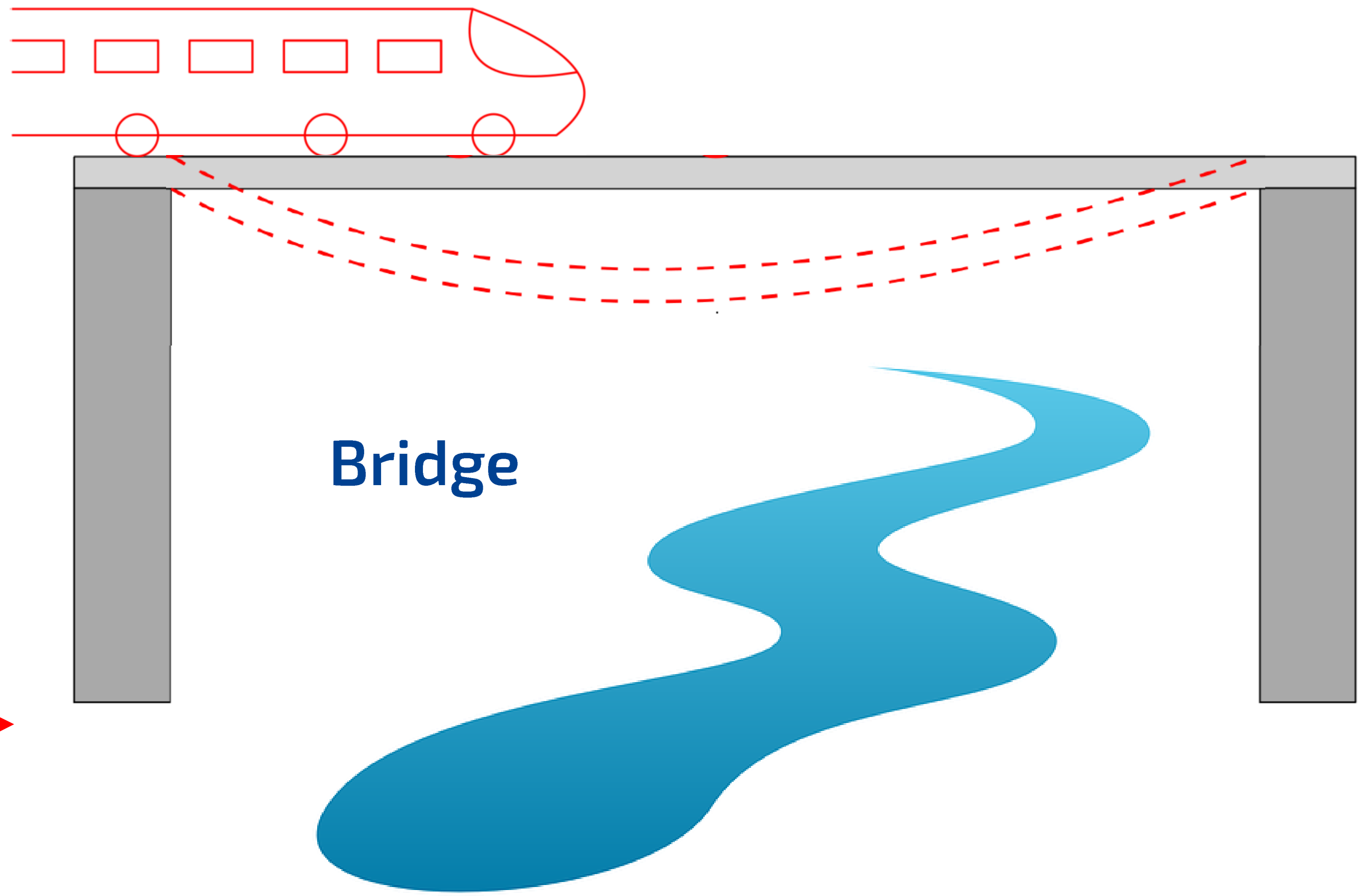
1. Need for coordinate transformations
2. Realization of sensor coordinate systems
3. Local transformations between sensors (Registration)
4. Transformation in global coordinate system (Geo-referencing)

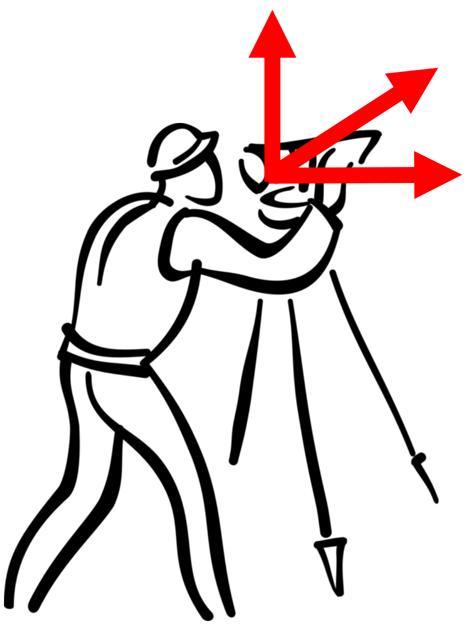
Her Majesty: TOTAL STATION





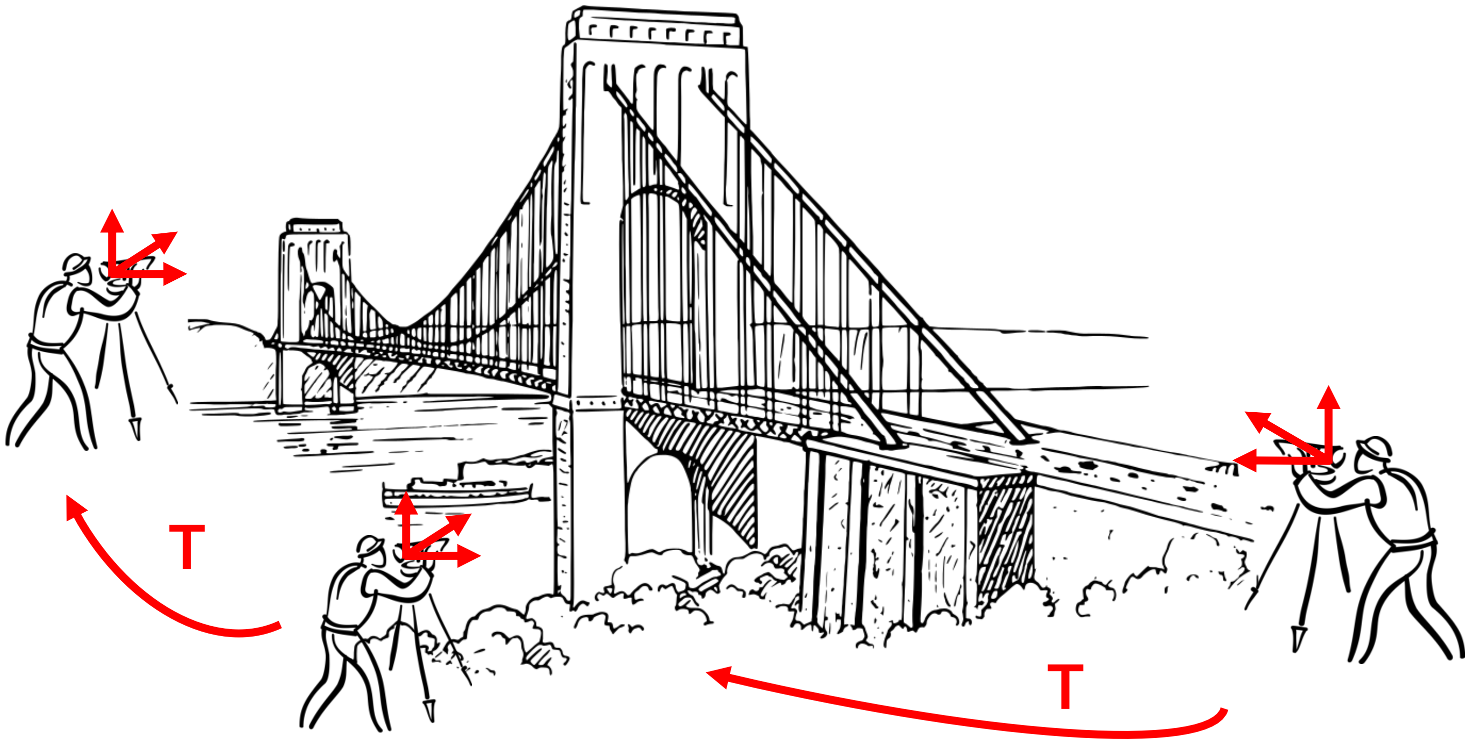
Her Majesty: TOTAL STATION





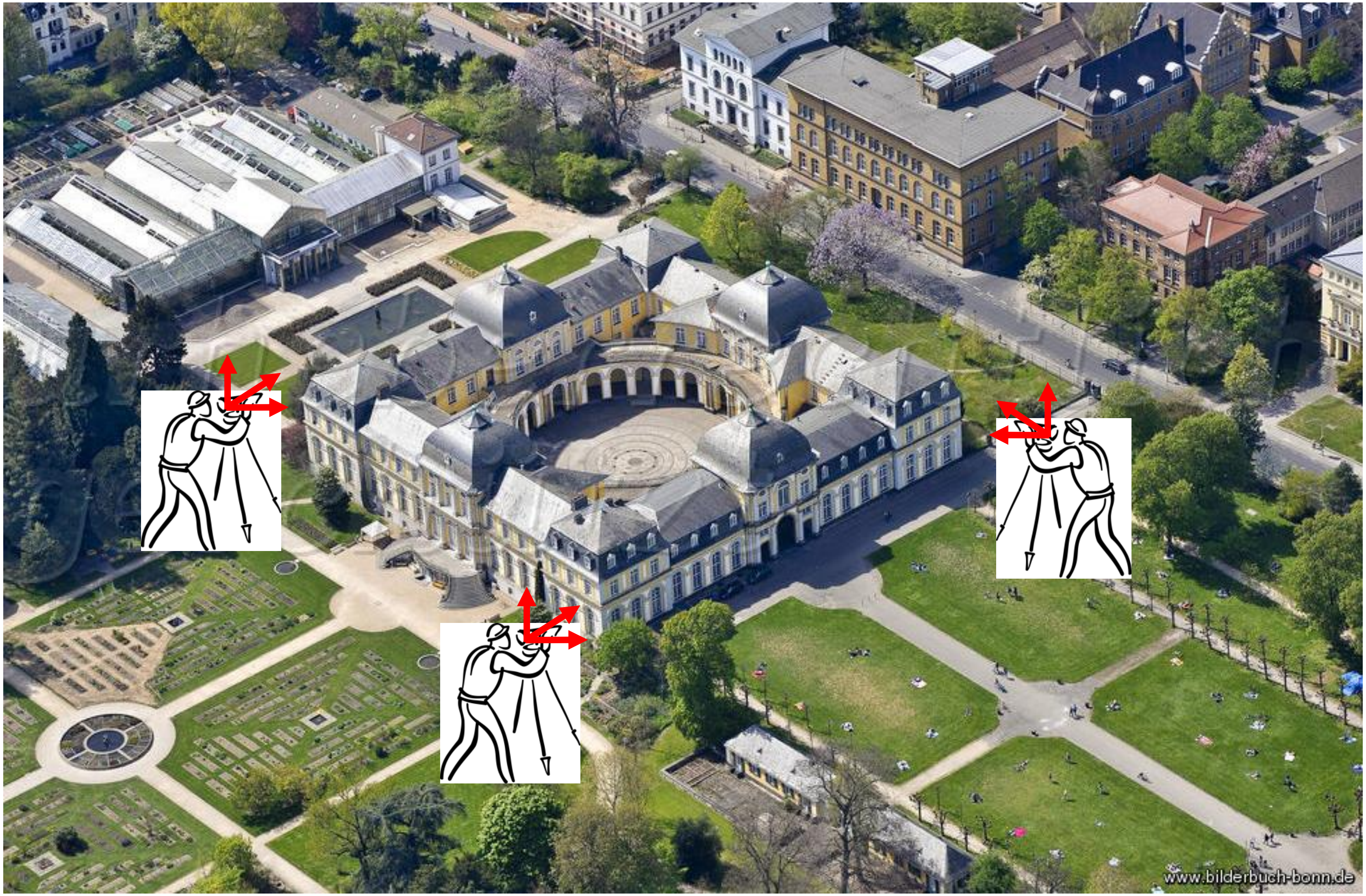
- Simple task – small object
- comparing two epochs (before after)
- Relative to sensor coordinate system

Her Majesty: TOTAL STATION

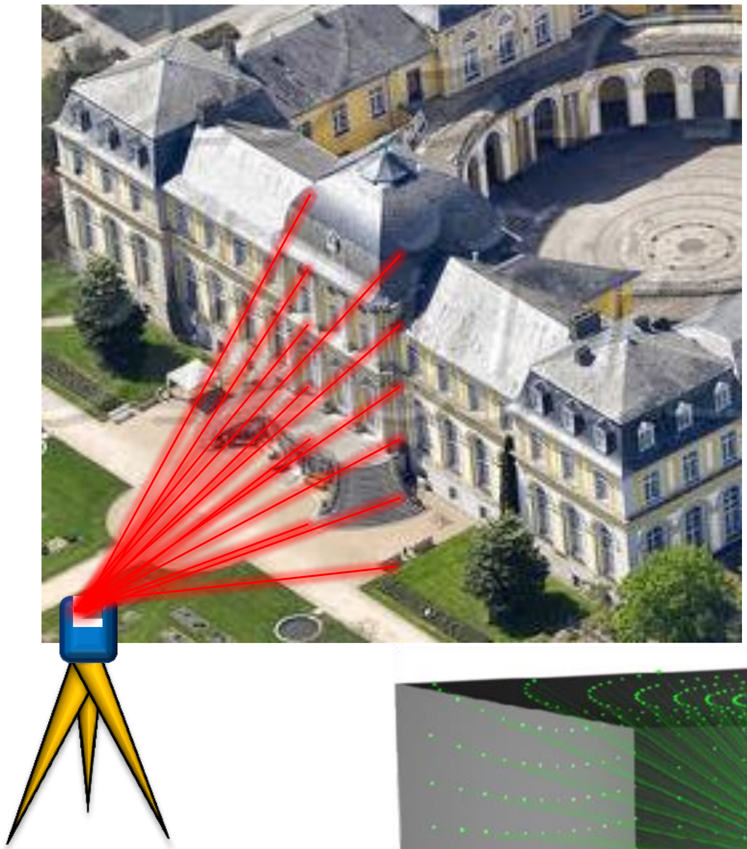


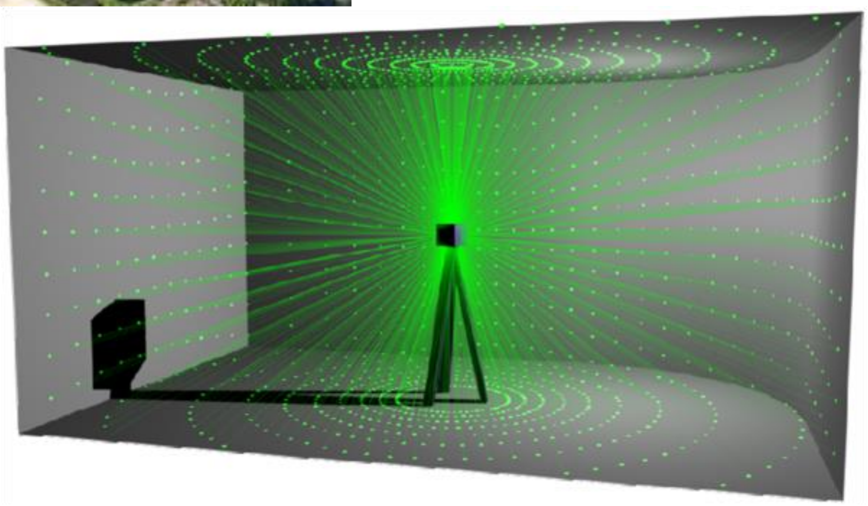
- Large complex object – more stations
- Single coordinate system necessary
- Local transformation between sensors (registration)

Main example: Building 3D model

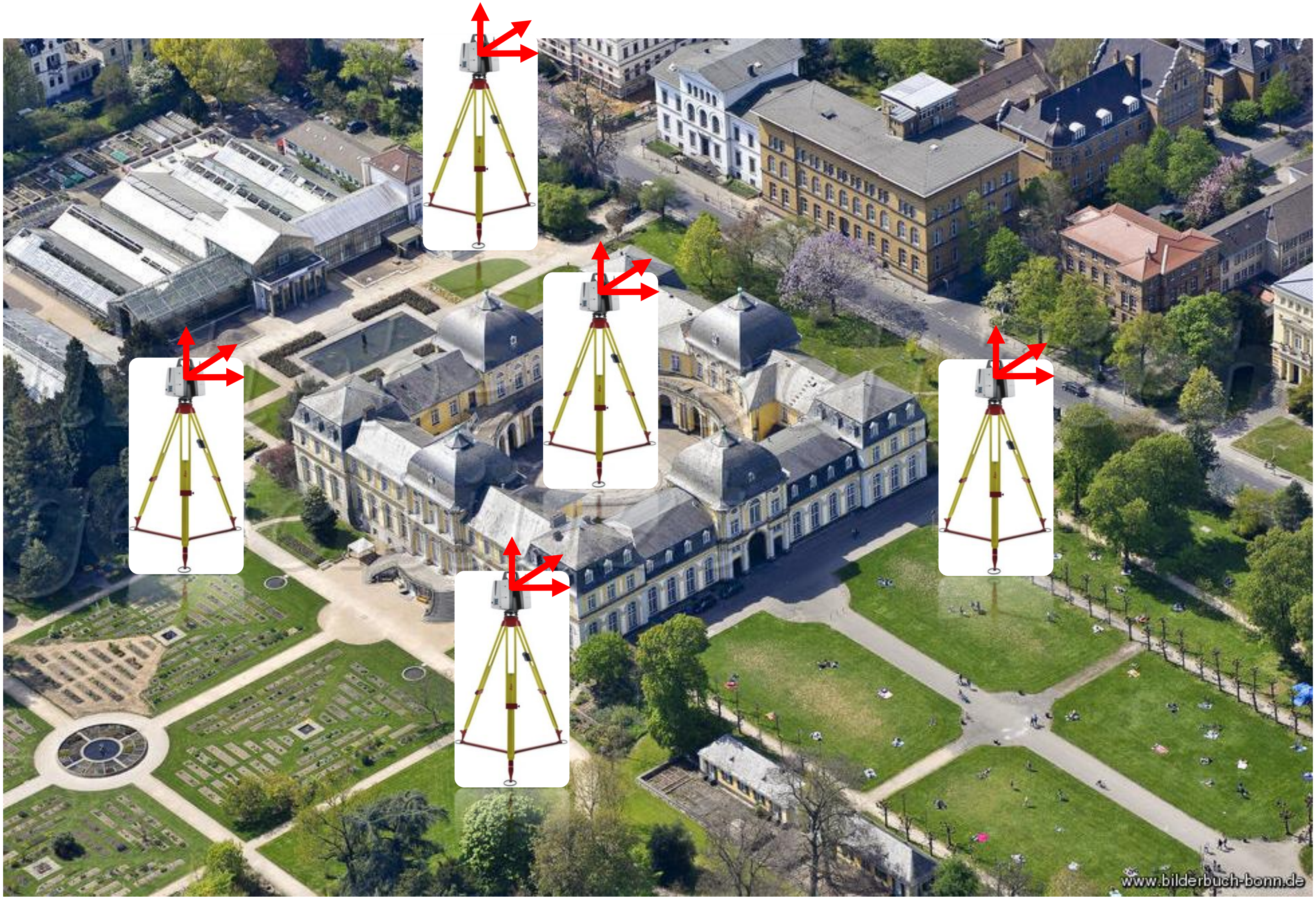


Terrestrial laser scanner (TLS)

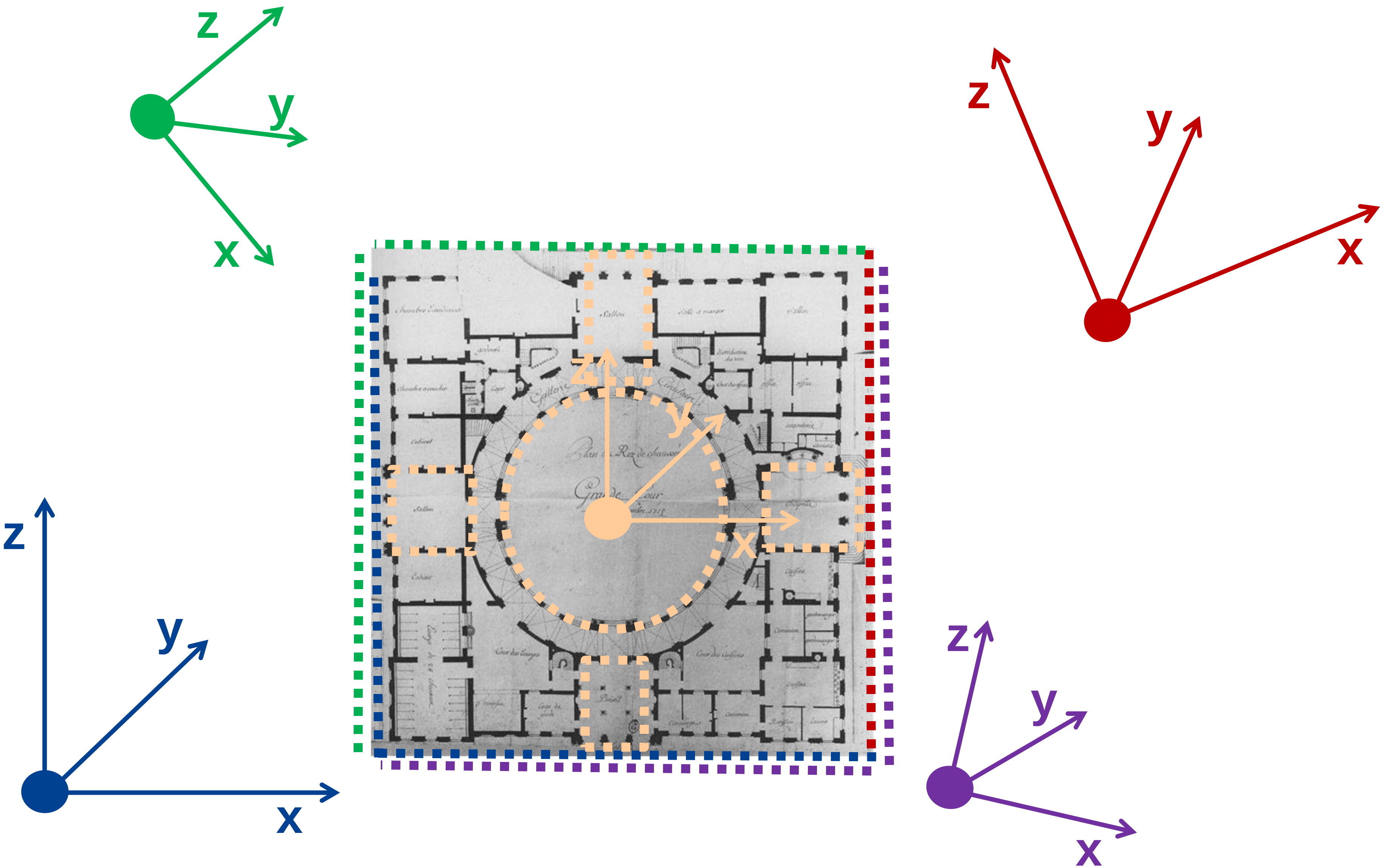




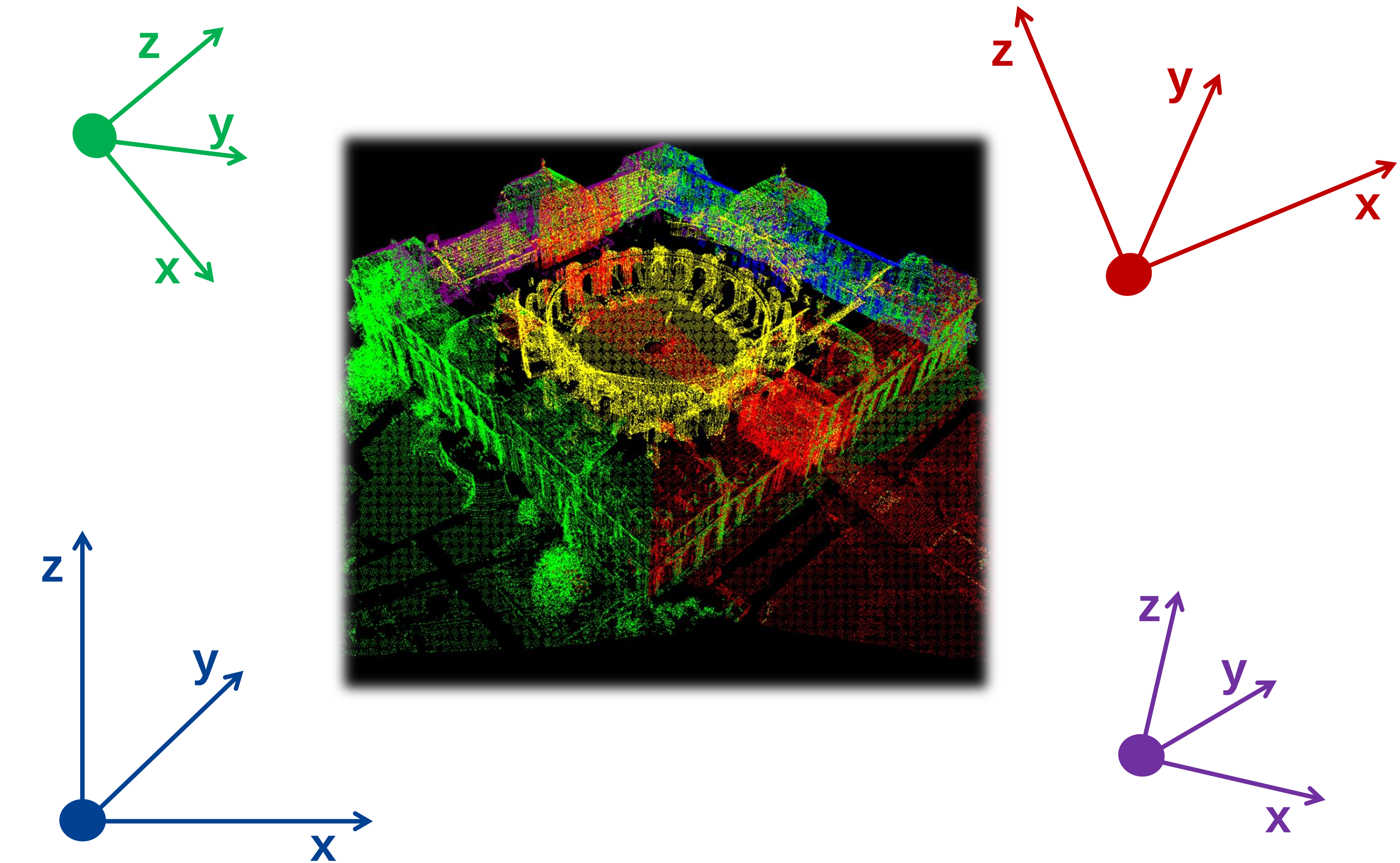
Step1: TLS Registration



Step1: TLS Registration



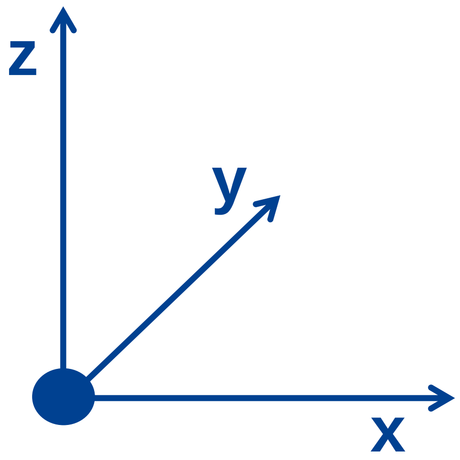
Step1: TLS Registration

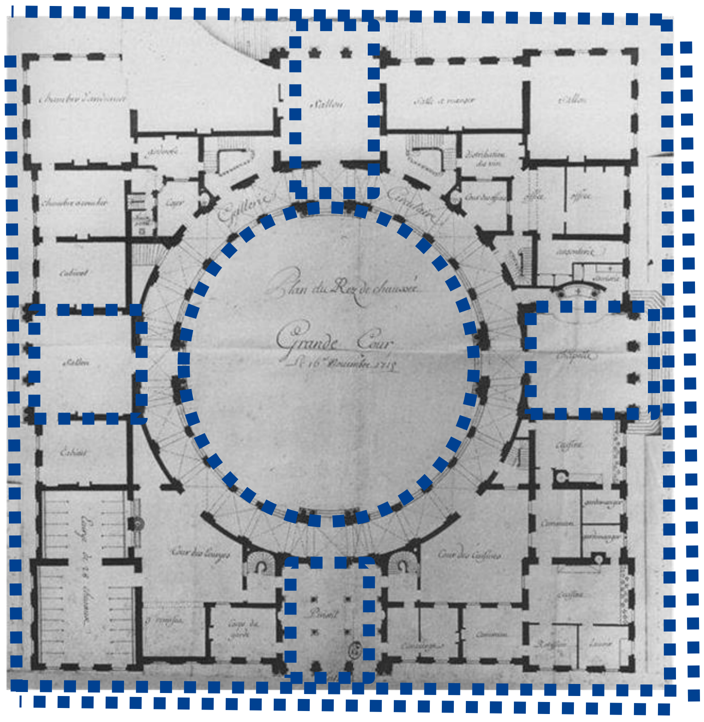


Step1: TLS Registration

- Merging to one single point cloud <=>

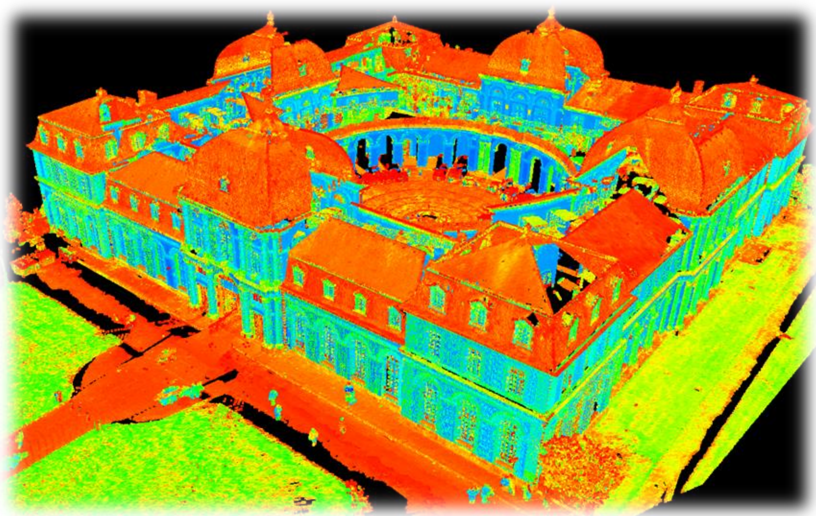
  Coordinate transformations
- **Registration** to local coordinate system x,y,z



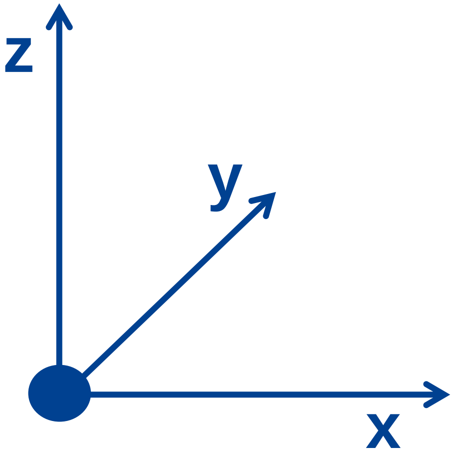


Step1: TLS Registration

Point cloud



3D model

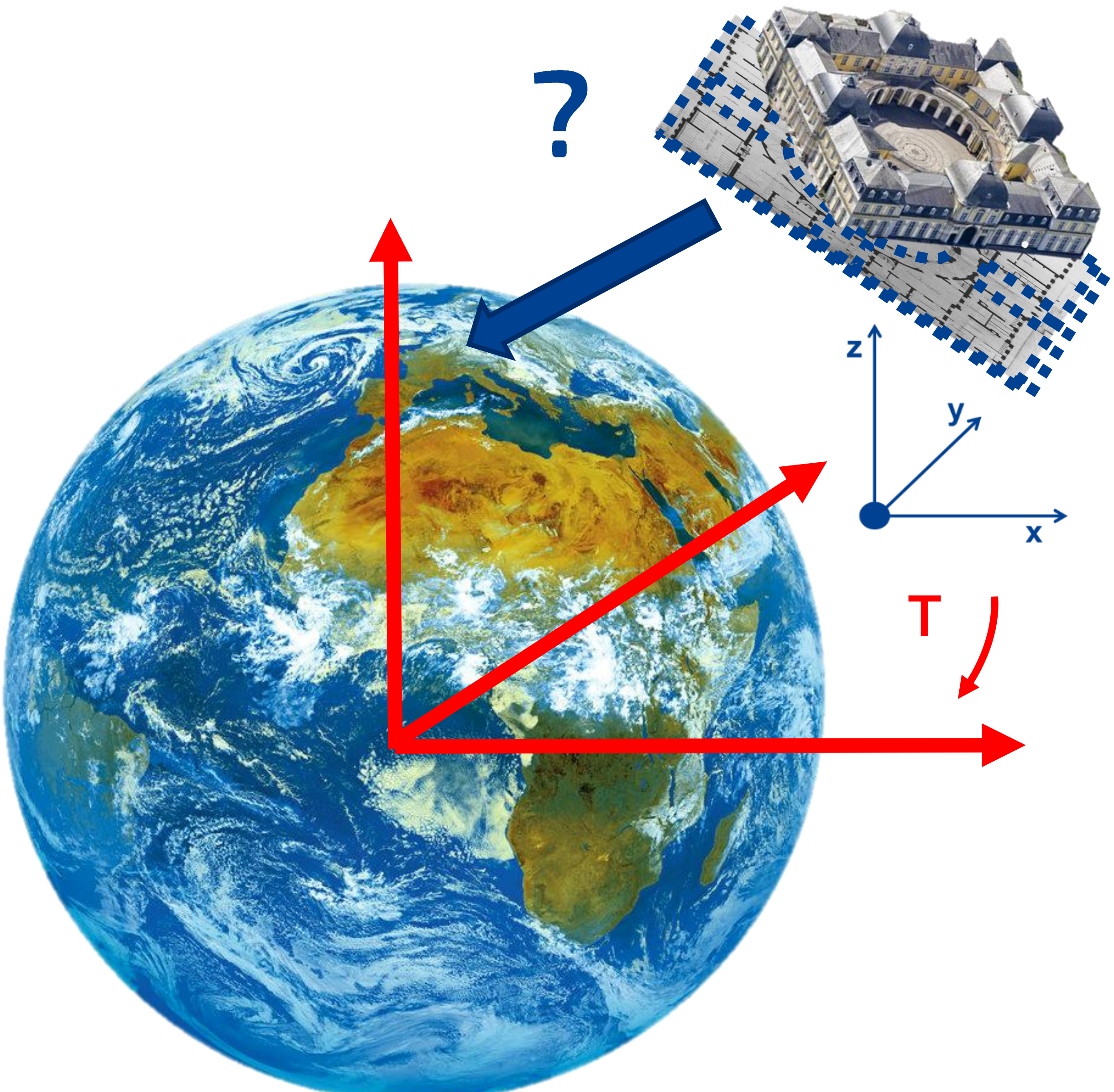


Mathematical approximation

- Arbitrary local coordinate system

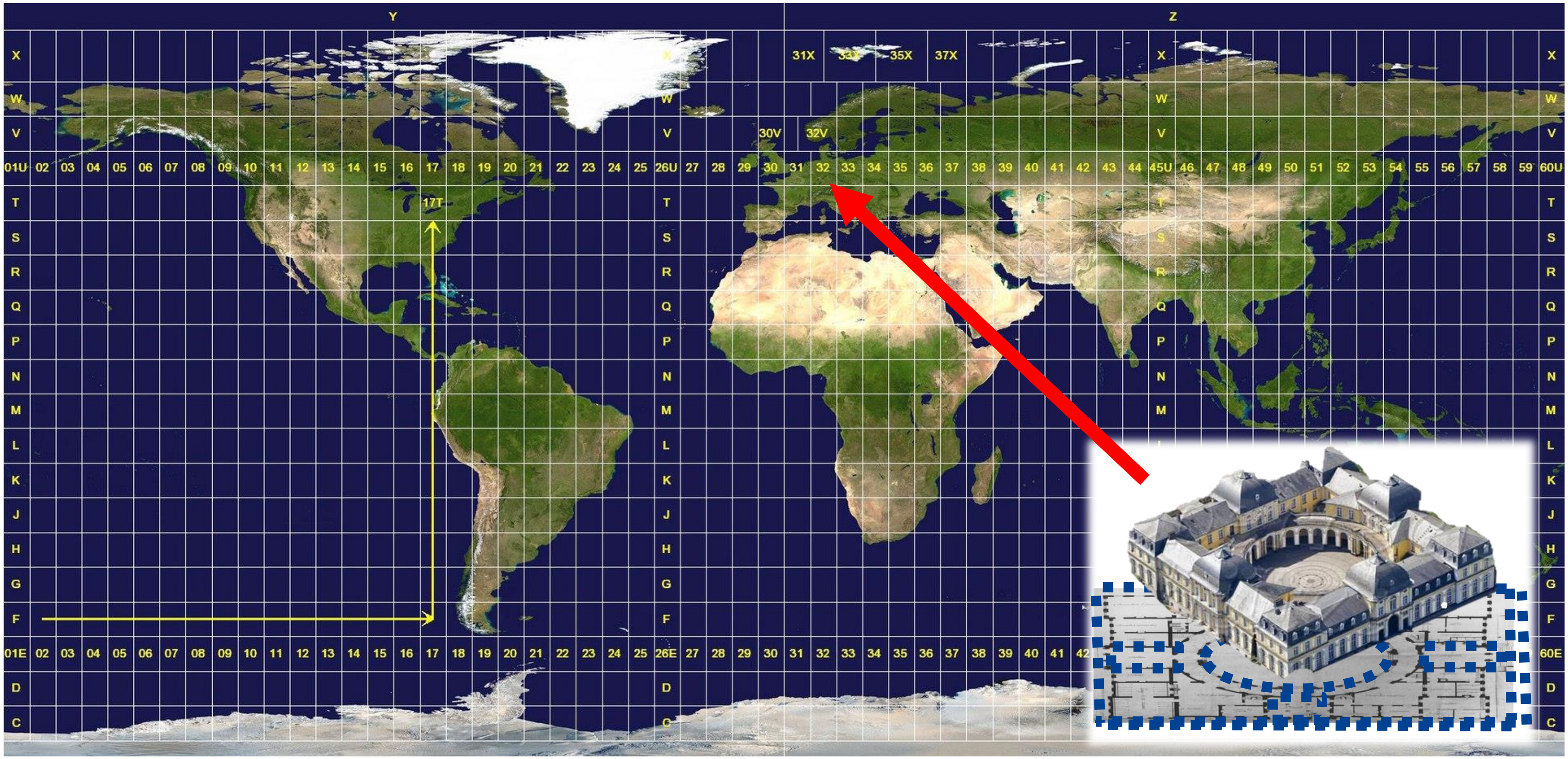
Step 2: Positioning on earth

- Transformation to global, earth centered coordinate system
- E.g., X,Y,Z_{ITRF} or X,Y,Z_{WGS84}
- **Geo-referencing**



Step 3: Giving official coordinates

- Official coordinates are split in position (2D) and height (1D)
- Position is (UTM-)projection due to curved earth



Need for coordinate transformations

- Coordinate system of acquisition often not the one of interest

1. Several different local coordinate systems => Merging in one of them => **Registration**

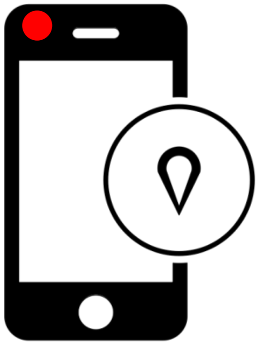
2. Sensor gives local coordinates but geocentric coordinates are needed => Transformation in global coordinate system => **Geo-referencing**

3. Sensor gives 3D geocentric coordinate system but official coordinates (2D+1D) are of interest => **Projection**

Agenda

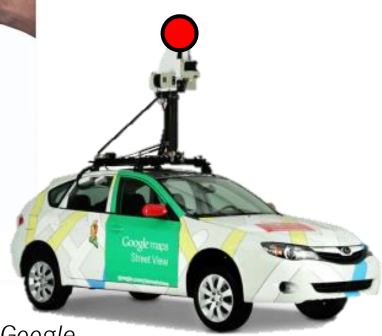
1. Need for coordinate transformations
2. Realization of sensor coordinate systems
3. Local transformations between sensors (Registration)
4. Transformation in global coordinate system (Geo-referencing)

GNSS Receivers

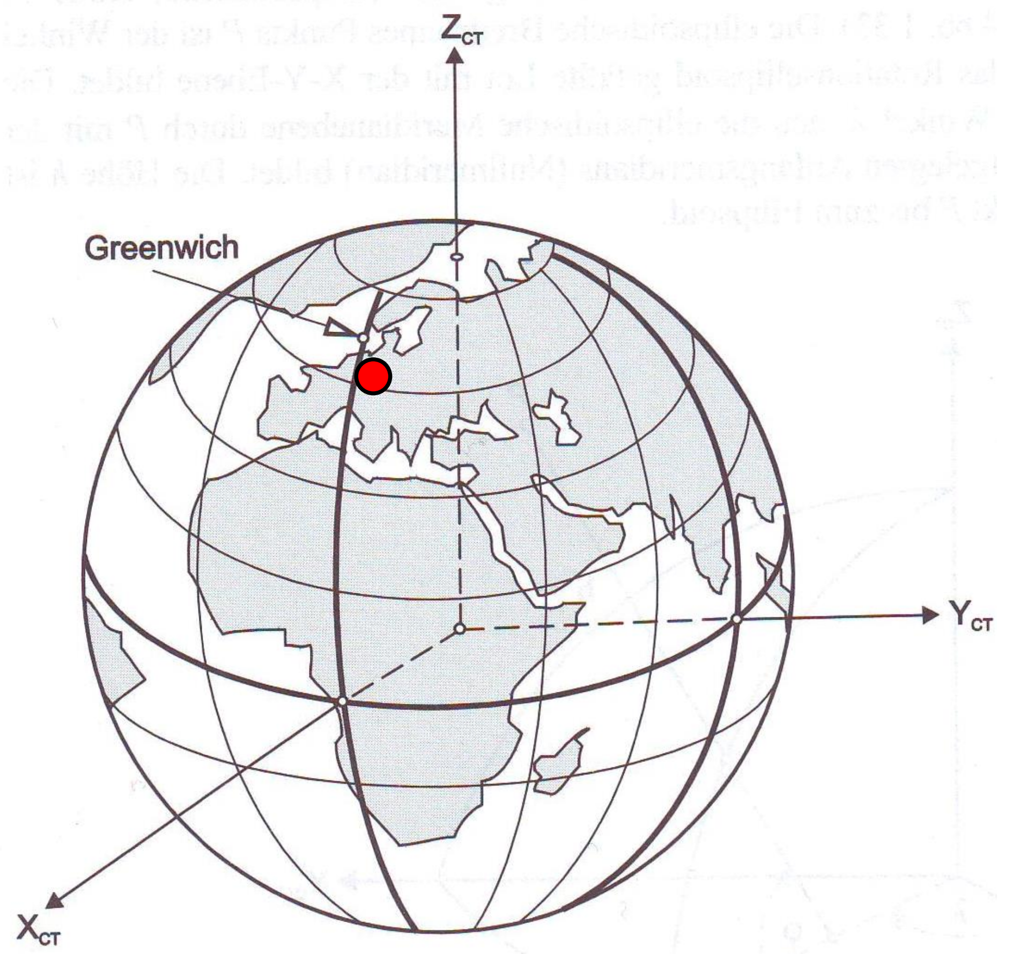


leica-geosystems.com

leica-geosystems.com

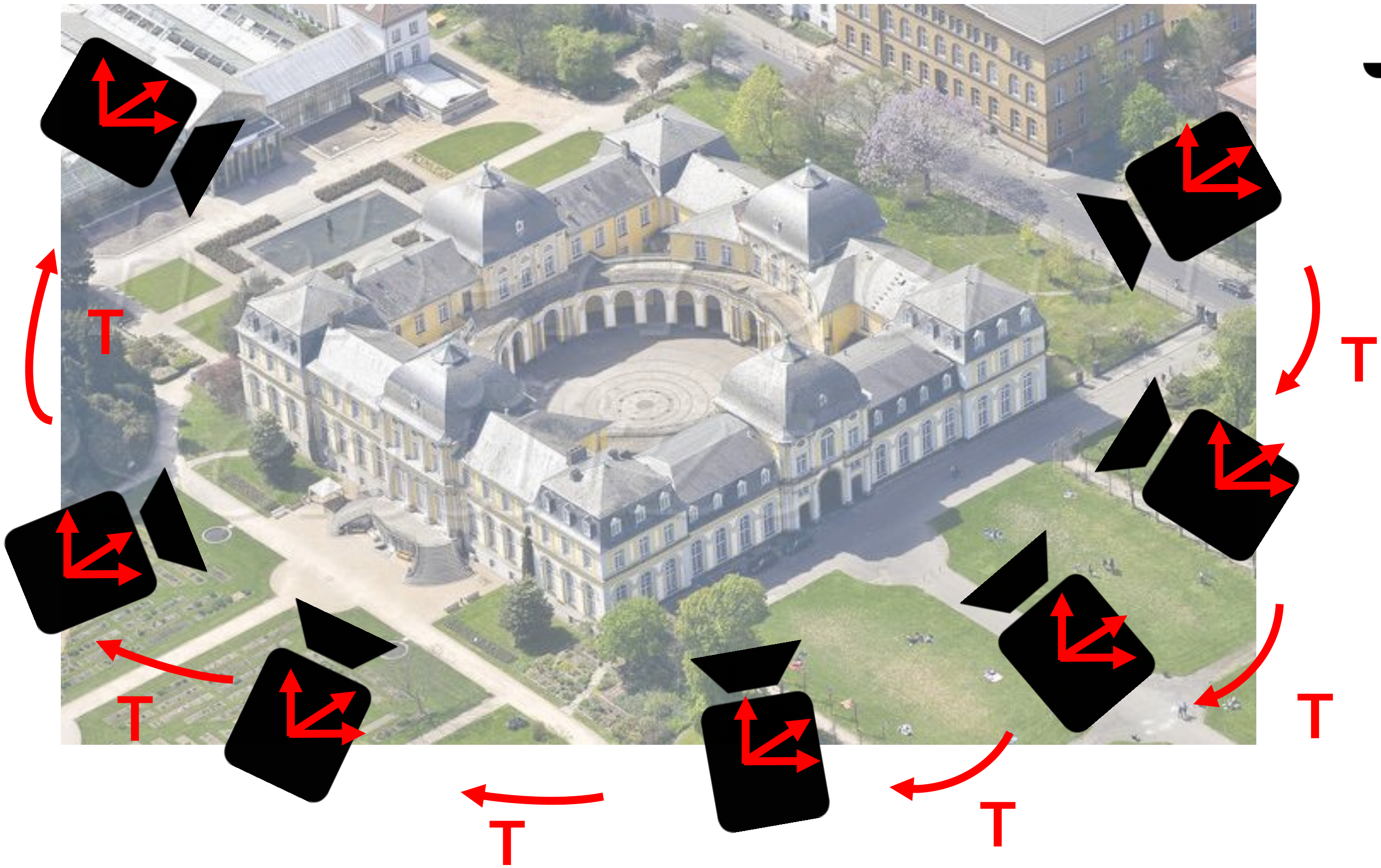


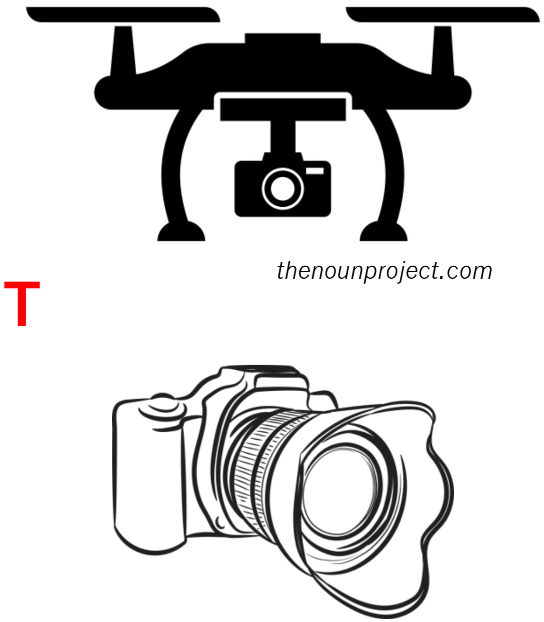
Google



- observables: x,y,z
- Coordinate system: geocentric, ITRF2014 / WGS84

Camera

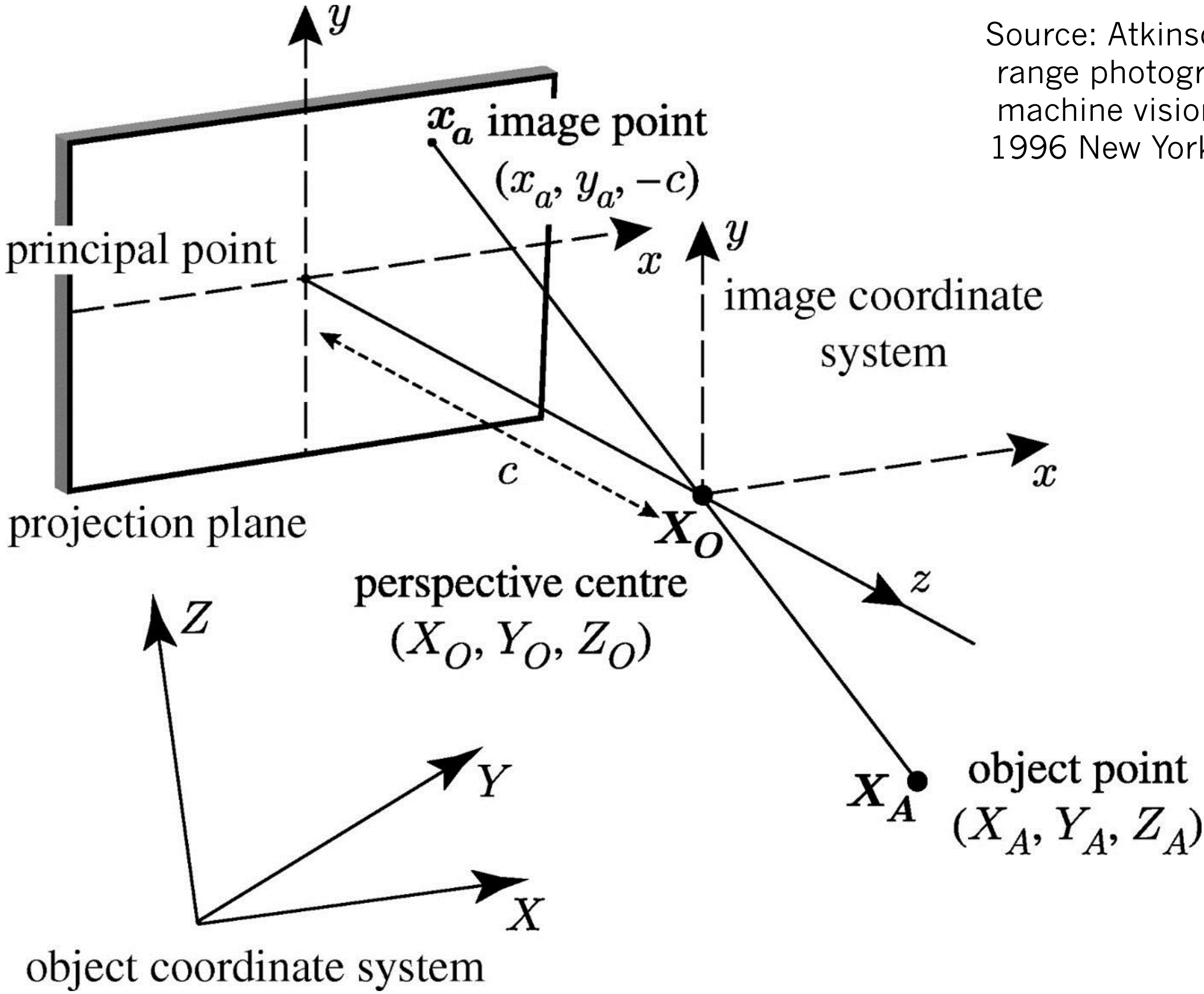




- Photogrammetry – geometry from images
- Structure-from-Motion (SfM) – point clouds

Camera

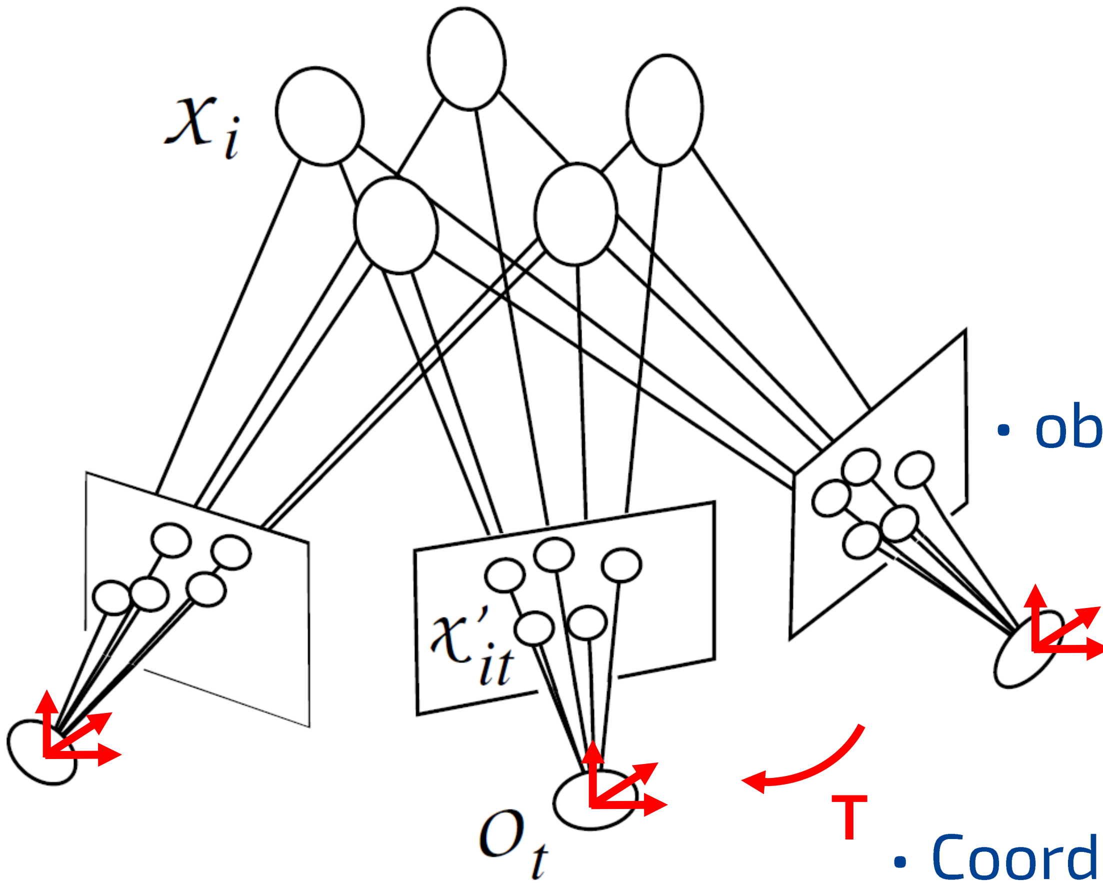
Source: Atkinson K.B: aClose range photogrammetry and machine vision. In Reinhold 1996 New York, NY:Reinhold



- observables: x,y
- Coordinate system: local, image coordinate system

Camera

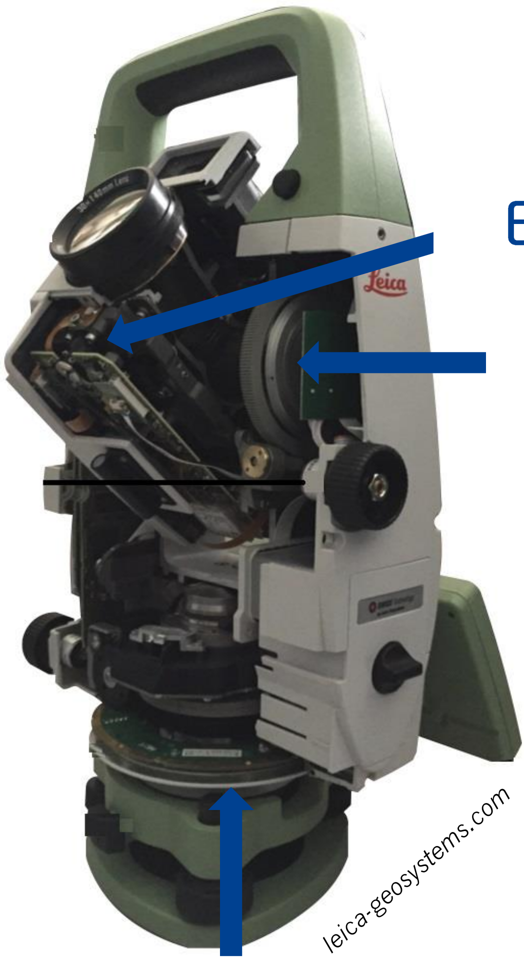
Source: Förstner, Wolfgang, and Bernhard P. Wrobel. Photogrammetric computer vision. Springer International Publishing Switzerland, 2016.



- observations
- sensor coordinate systems
- Coordinate transformation necessary for 3D reconstruction

(Bundle adjustment)

Total station



EDM

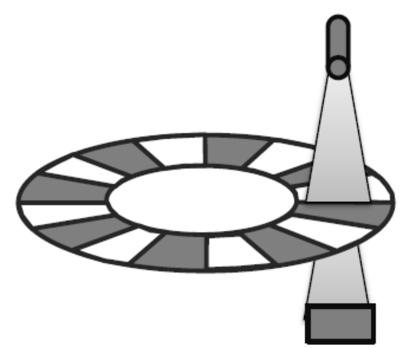
Vertical encoder: θ

Horizontal encoder: φ

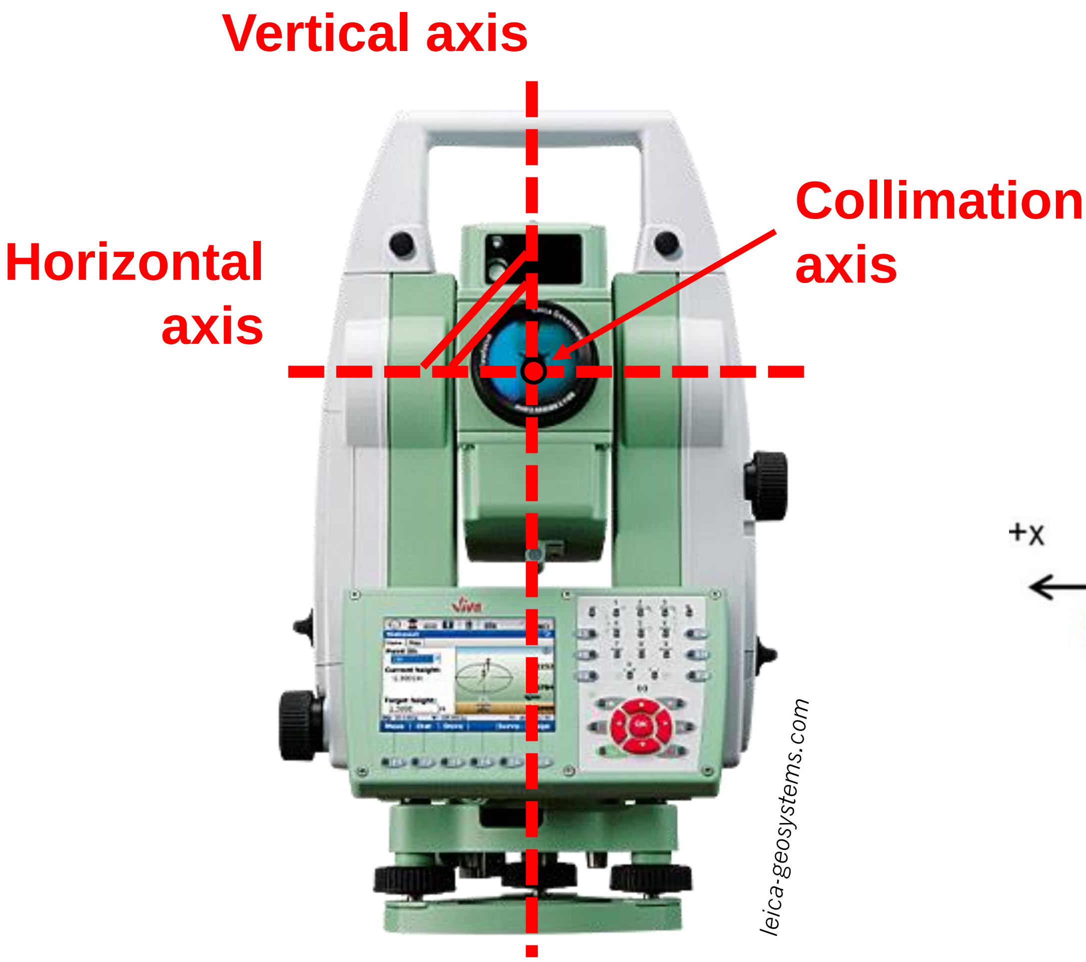
Observables: θ, φ, d:

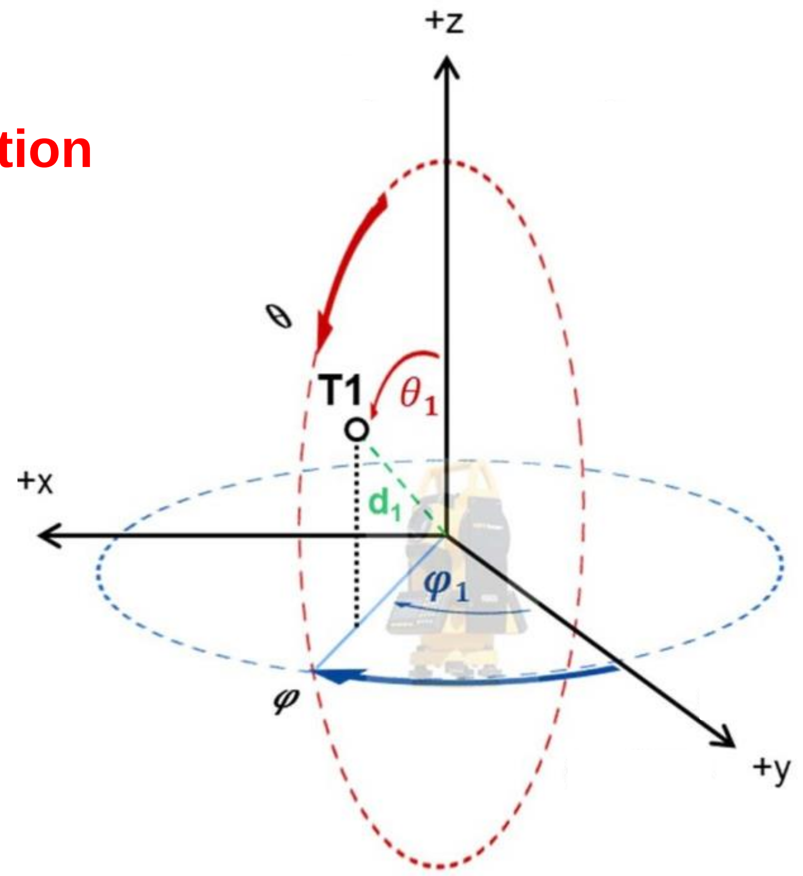
- EDM (laser) - d

- Angular encoders - θ, φ



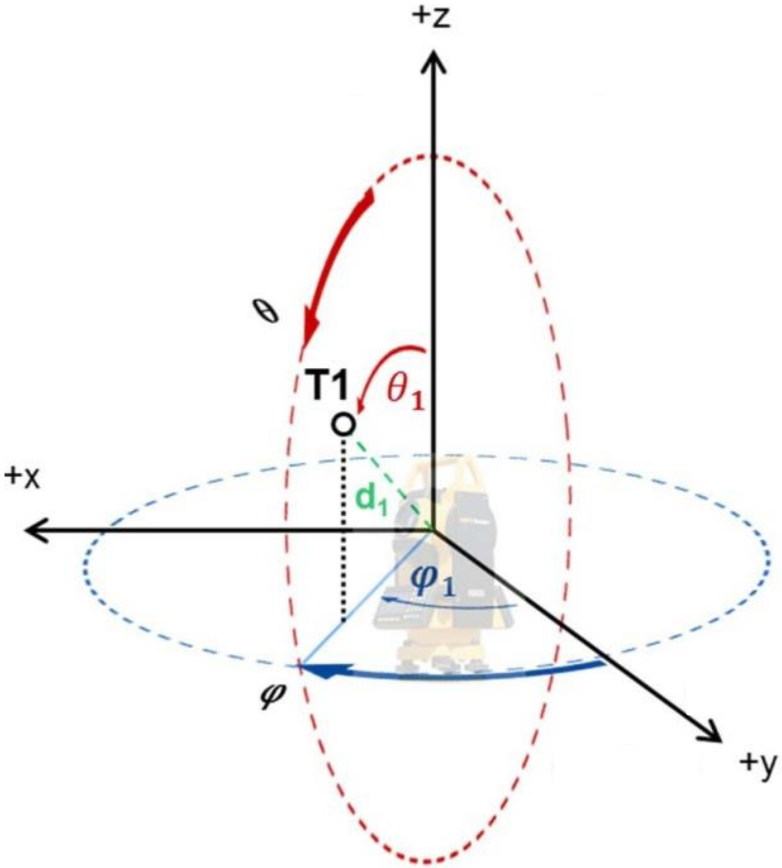
Total station





- Coordinate system: local, intersection of vertical, horizontal and collimation axis

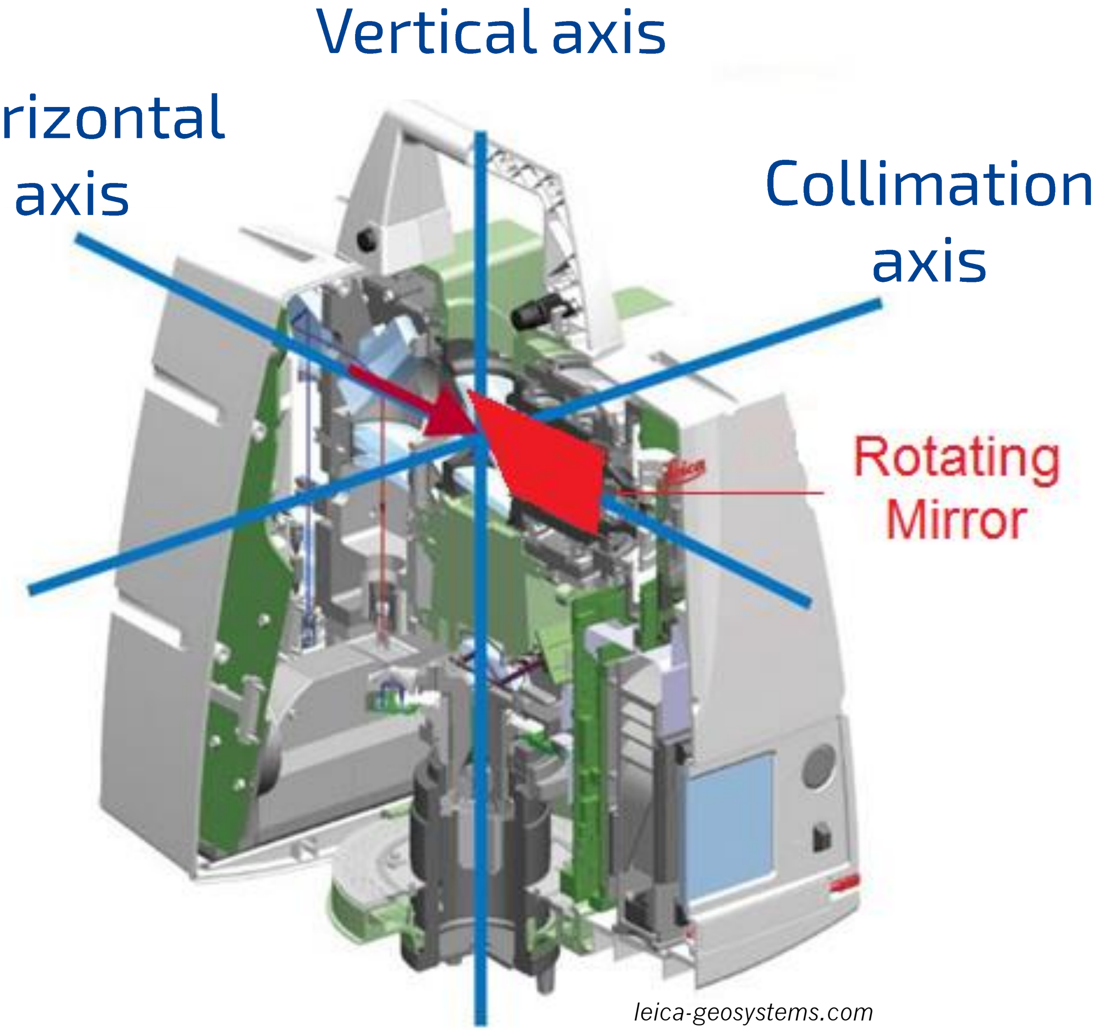
Total station



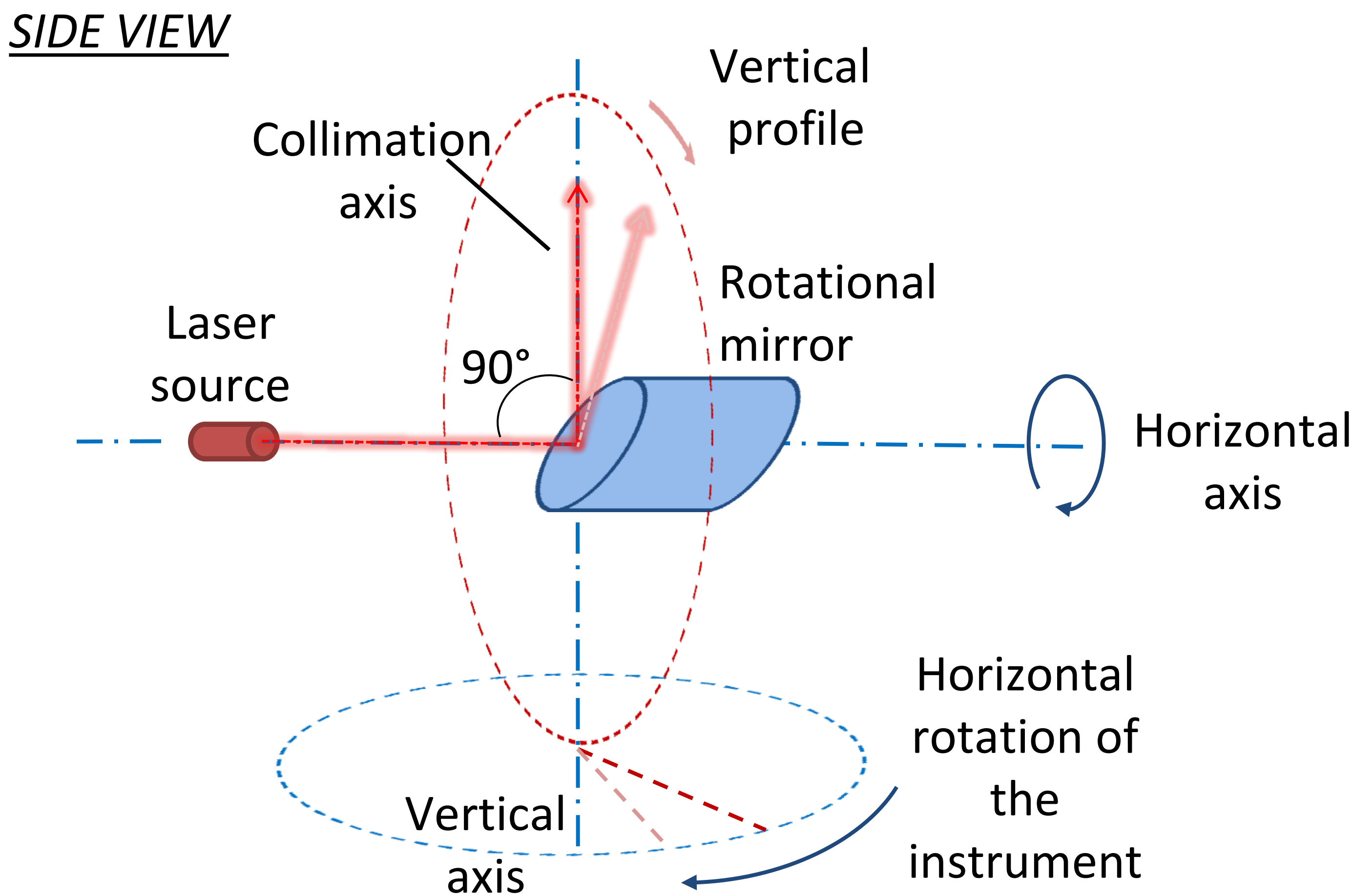
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \cdot \sin\theta \cdot \sin\varphi \\ d \cdot \sin\theta \cdot \cos\varphi \\ d \cdot \cos\theta \end{bmatrix}$$

- Spherical / polar to Cartesian coordinates $\theta,\ \varphi,\ d \rightarrow$ x, y, z

3D Terrestrial Laser Scanner

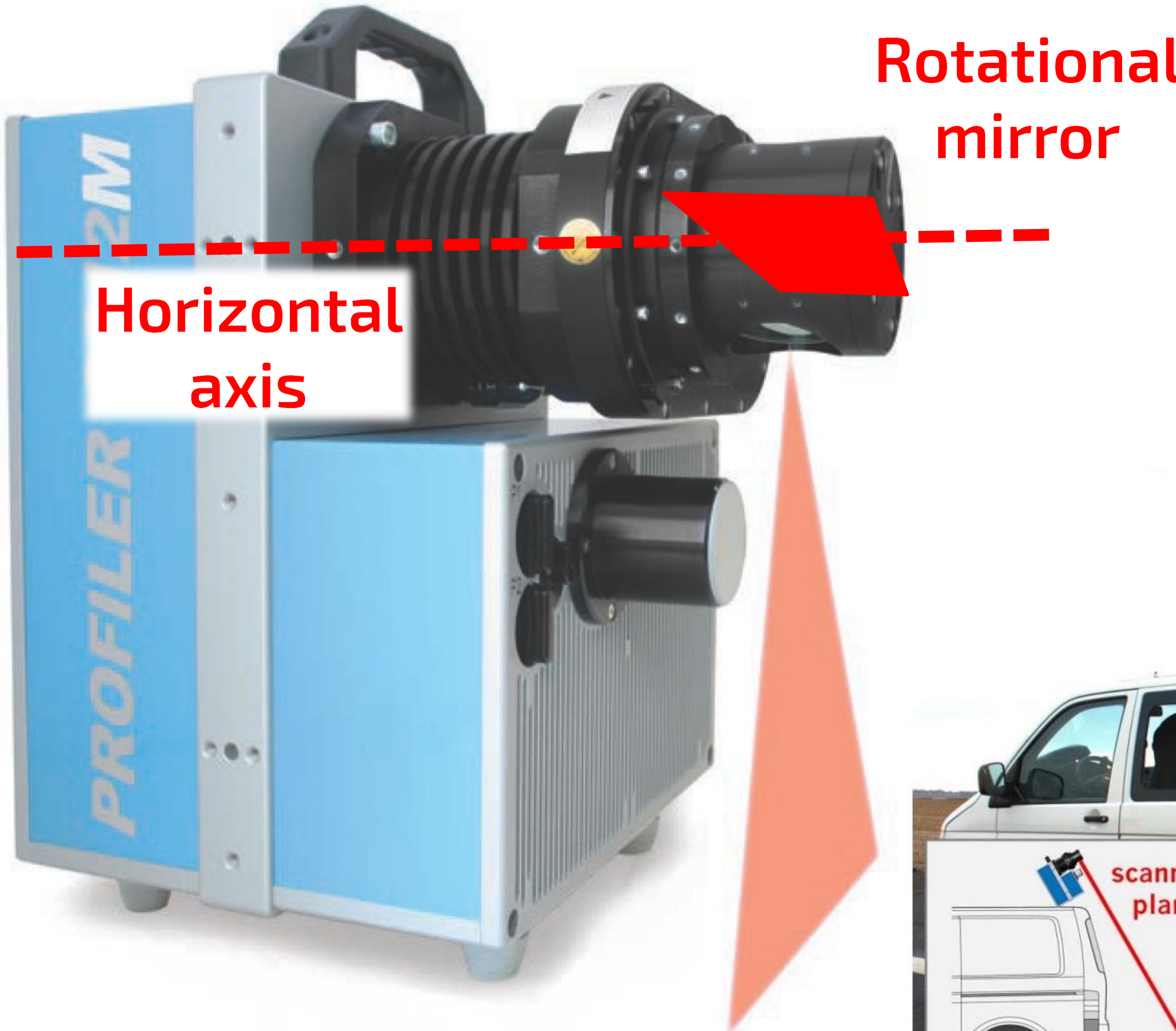


3D Terrestrial Laser Scanner



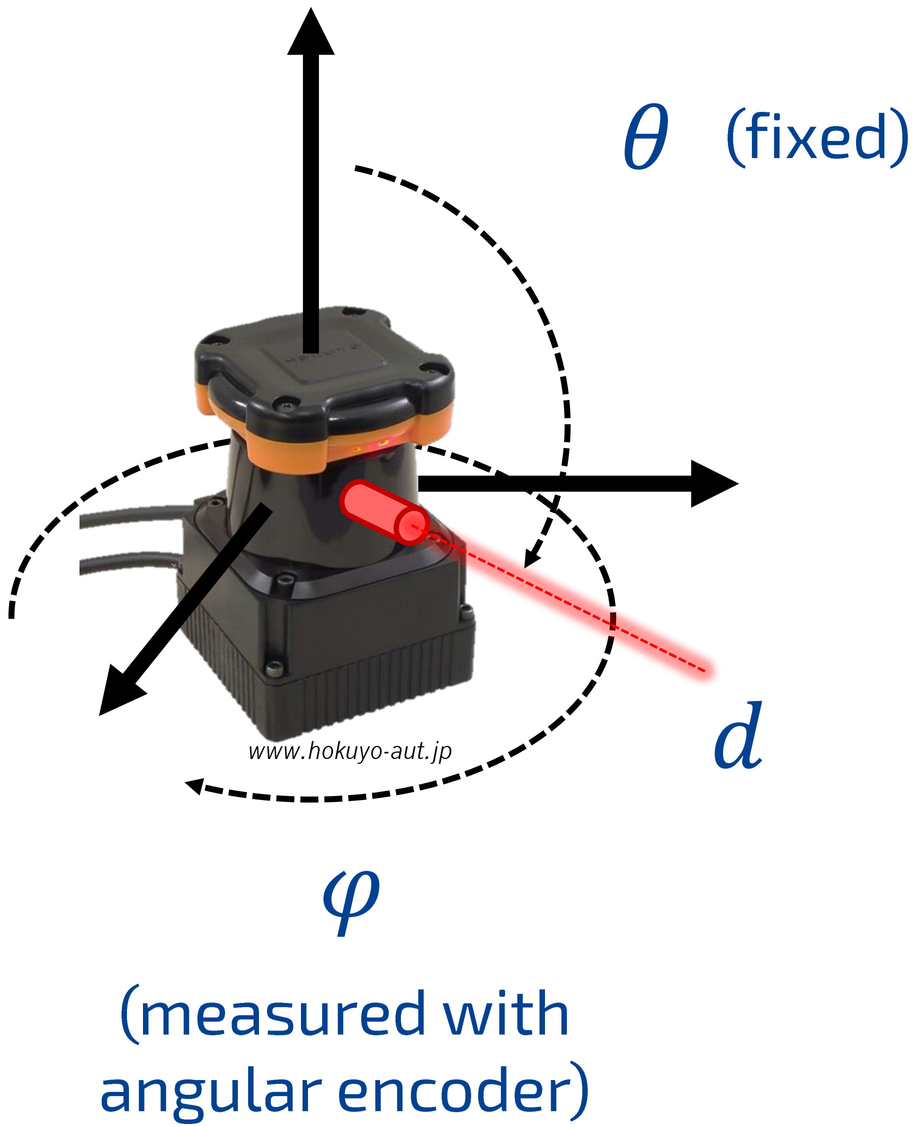
- observables: $\theta, \varphi, d \rightarrow x, y, z$
- Coordinate system: local, intersection of vertical, horizontal and collimation axis

2D Laser Scanner

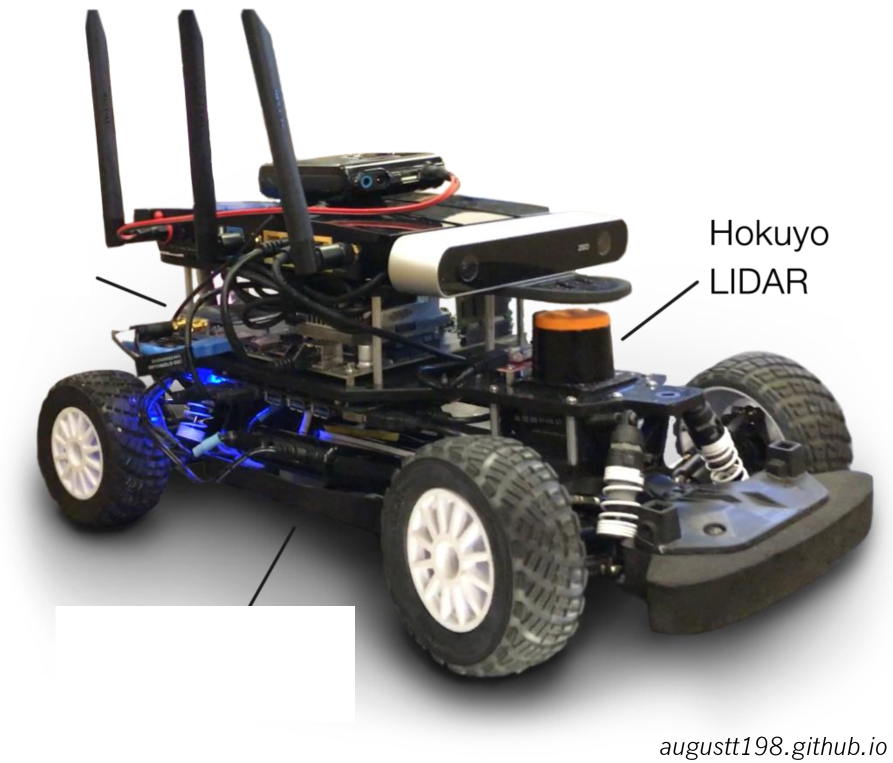


- Mobile mapping

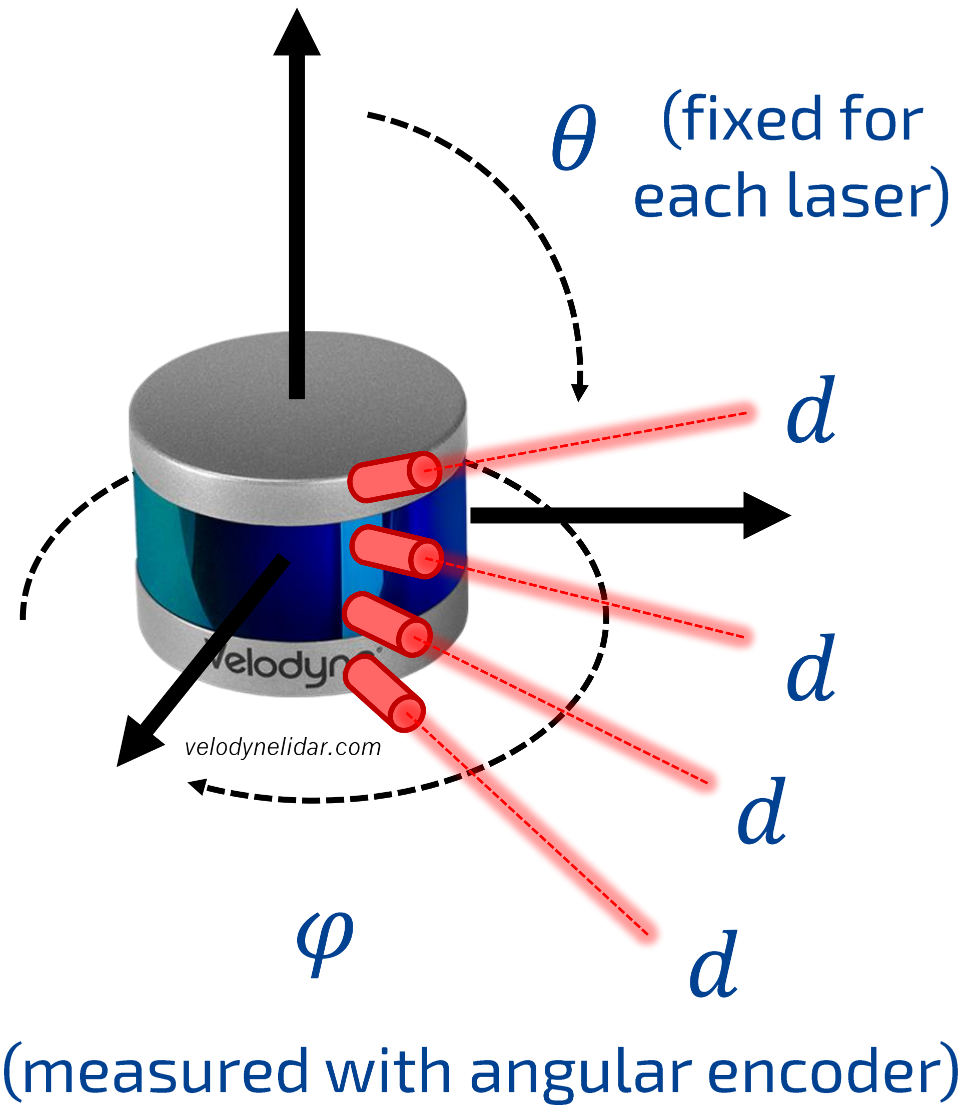
2D Laser Scanner / 2D LIDAR



- Robotics



3D LIDAR



- Autonomous driving

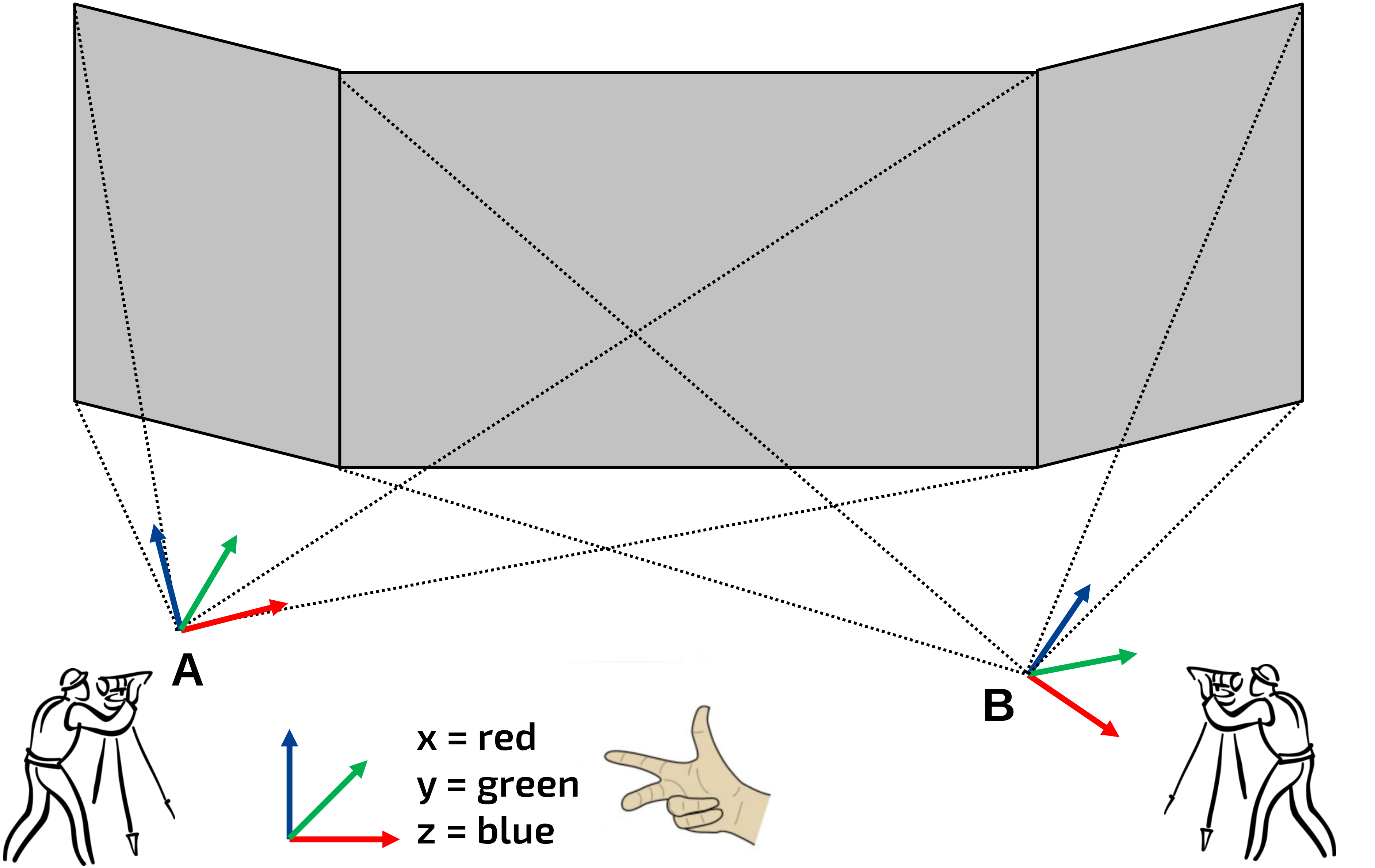
velodynelidar.com

bizjournals.com

Agenda

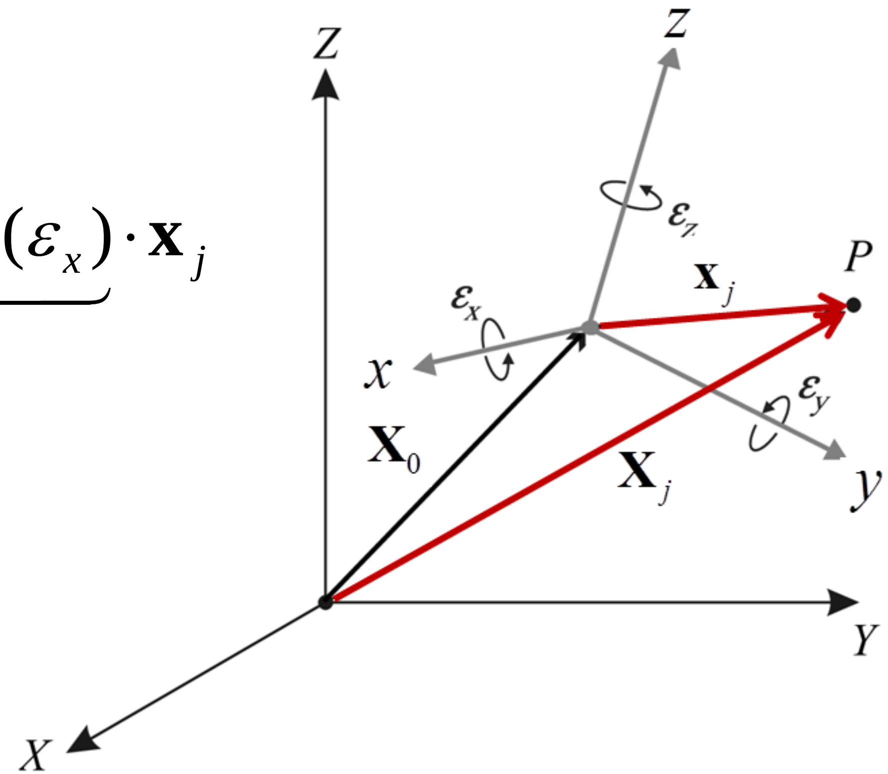
1. Need for coordinate transformations
2. Realization of sensor coordinate systems
3. Local transformations between sensors (Registration)
4. Transformation in global coordinate system (Geo-referencing)

Local transformations



Similarity transform in 3D

$$\mathbf{X}_j = \mathbf{X}_0 + \mu \cdot \underbrace{\mathbf{R}_3(\varepsilon_z) \cdot \mathbf{R}_2(\varepsilon_y) \cdot \mathbf{R}_1(\varepsilon_x)}_{\mathbf{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z)} \cdot \mathbf{x}_j$$



- Euler angles: $\varepsilon_x, \varepsilon_y, \varepsilon_z$
- Scale: μ
- Translation: $\mathbf{X}_0 = [X_0 \quad Y_0 \quad Z_0]^T$

- Coordinates in starting system: $\mathbf{x}_j = [x \quad y \quad z]_j^T$
- Coordinates in target system: $\mathbf{X}_j = [X \quad Y \quad Z]_j^T$

Representation of angles

- Euler angles: $\varepsilon_x, \varepsilon_y, \varepsilon_z$

$$\boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x) = \boldsymbol{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z) = \boldsymbol{R}$$

- Alternative: Axis-angle representations: $\theta, \boldsymbol{r} = [r_1 \quad r_2 \quad r_3]^T$

$$\boldsymbol{R} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}^2$$

- *Quaternions (not handled here)*

Similarity transform in 3D

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mu \underbrace{\boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)}_{\boldsymbol{R}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

7 DoF / 7 parameters:

- Scale change (1DoF)
- Rotation (3DoF)
- Translation (3DoF)

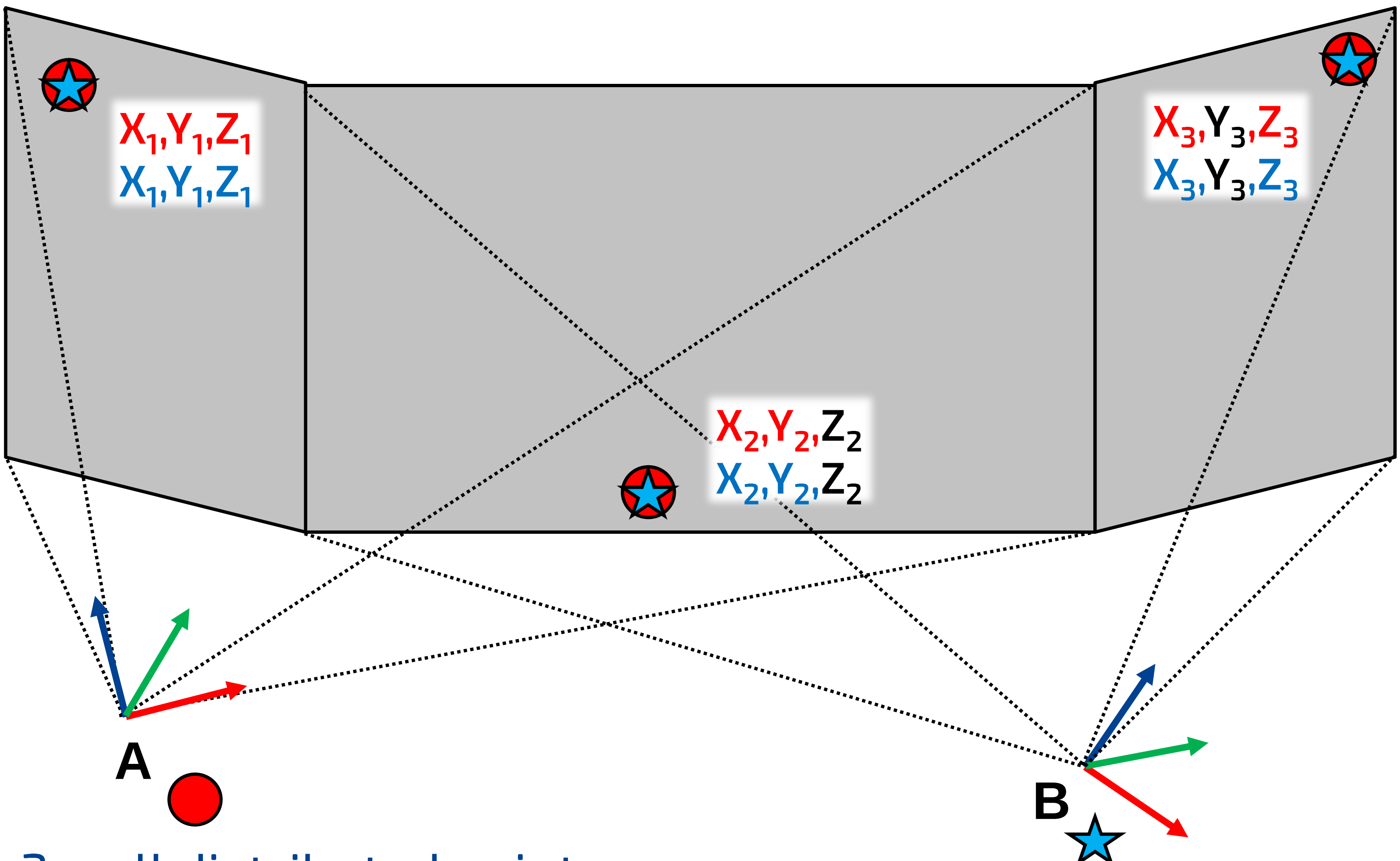
Determination of parameters

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mu \underbrace{\boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)}_{\boldsymbol{R}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

How to determine these 7 parameters?

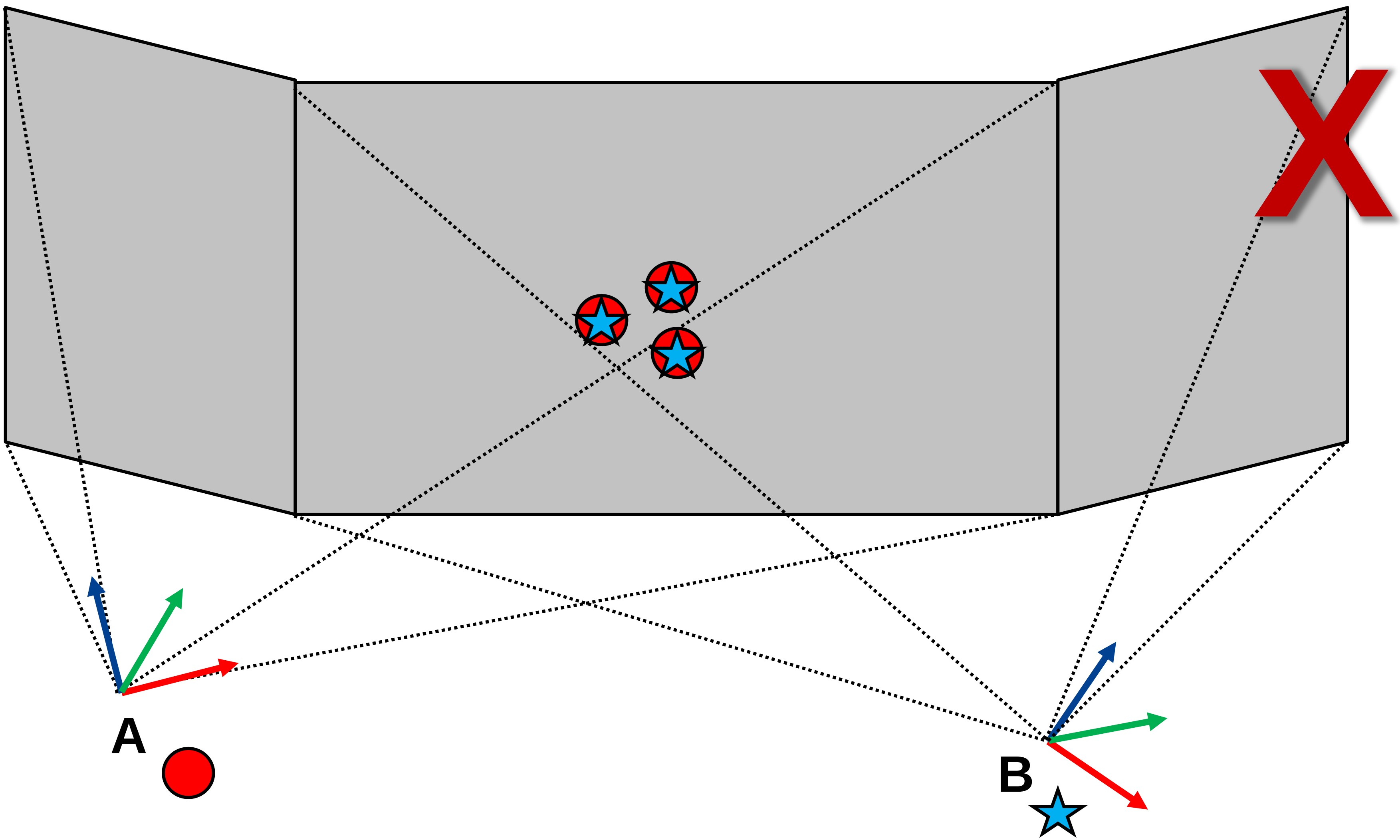
- 7 or more observations (coordinates) in each system
  7 = analytical solution
  >7 = adjustment (MGE-03 Statistics & Adjustment theory)
- 3 well distributed points (otherwise ill posed problem)
- known correspondences !

Determination of parameters

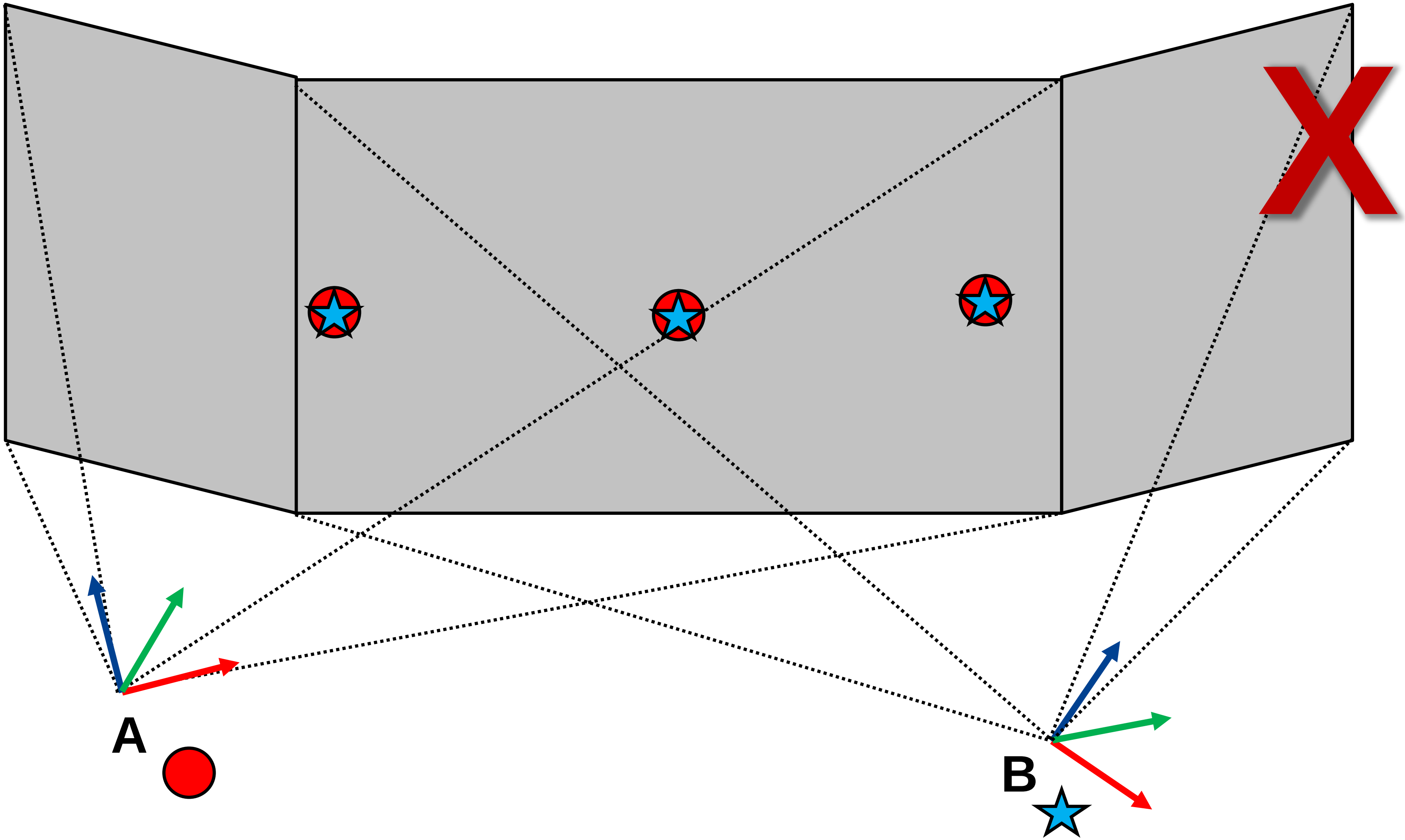


- 3 well distributed points
- 7 or more observations (coordinates) in each system

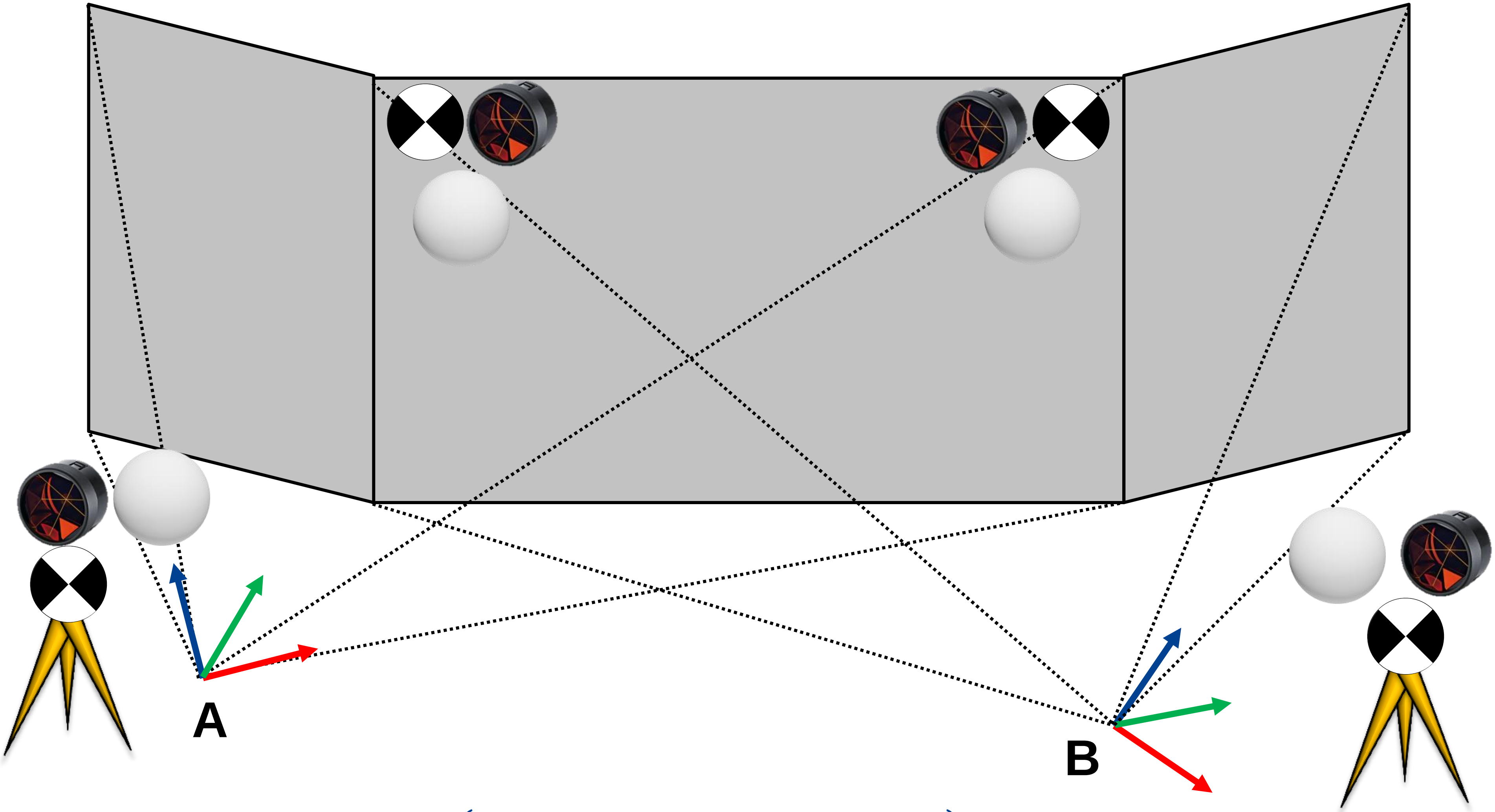
Determination of parameters



Determination of parameters

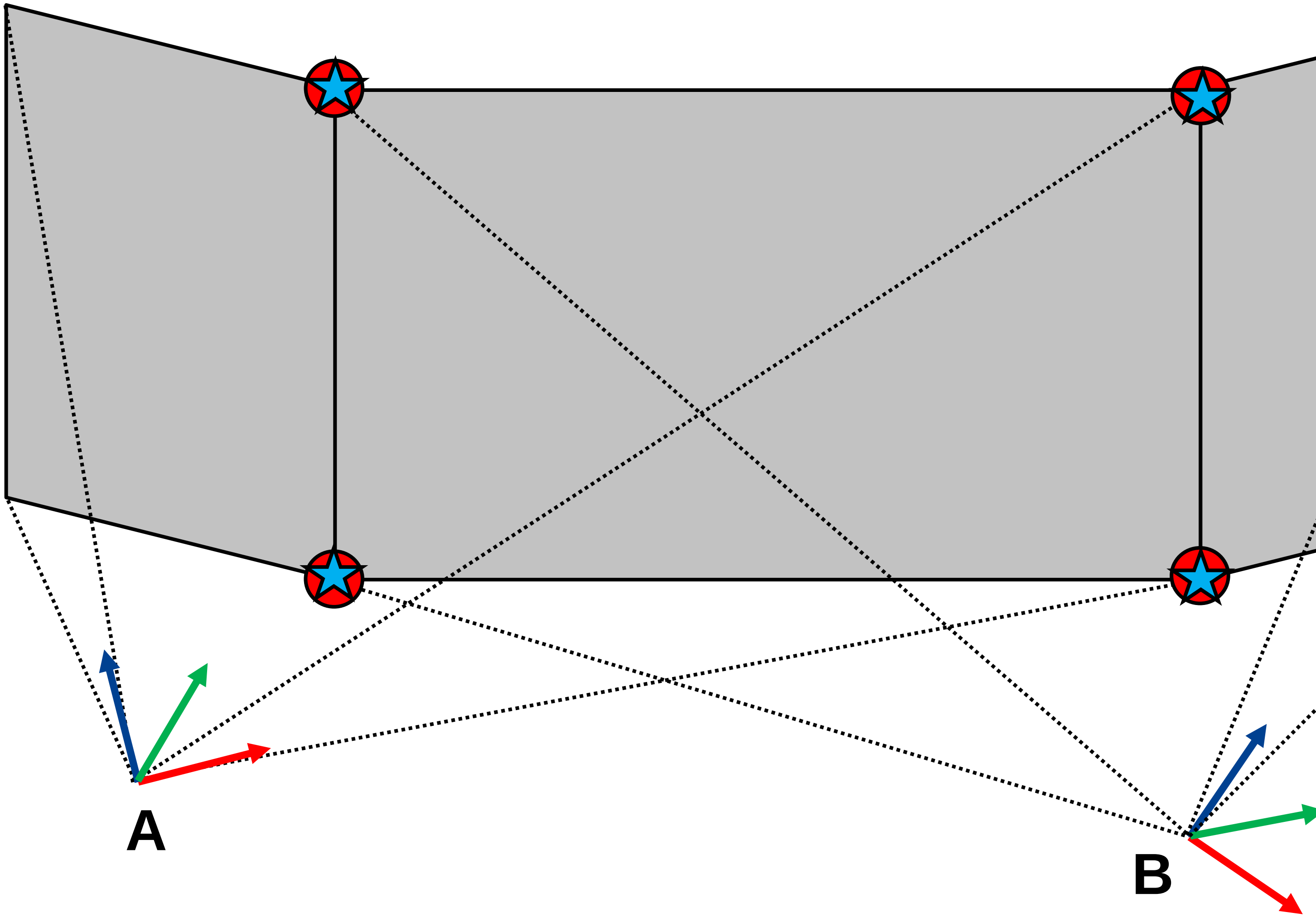


Using artificial targets



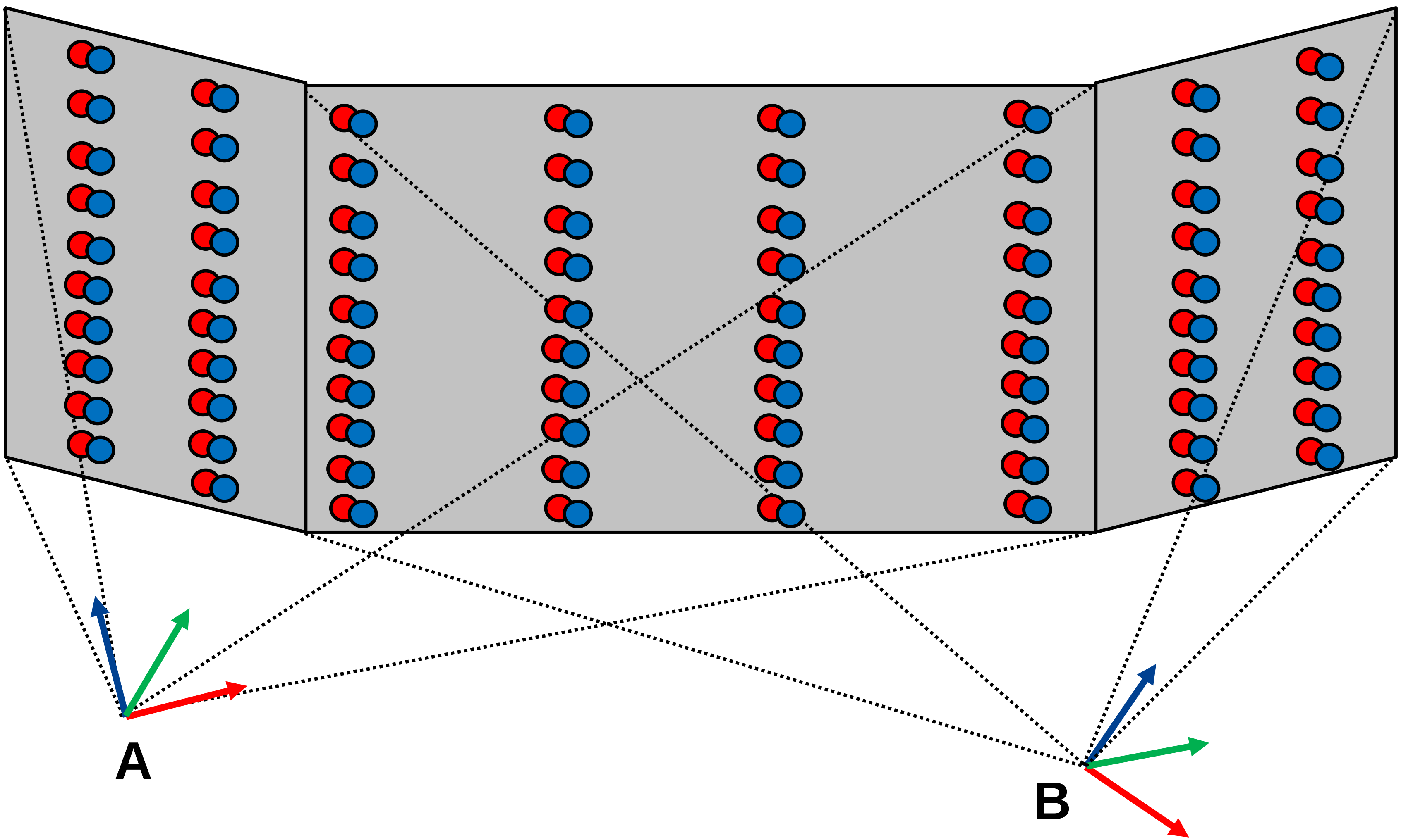
- 1. Artificial targets (e.g. prism, target, sphere)
- aimed (e.g. total station) or automatically detected (TLS)

Using measured object



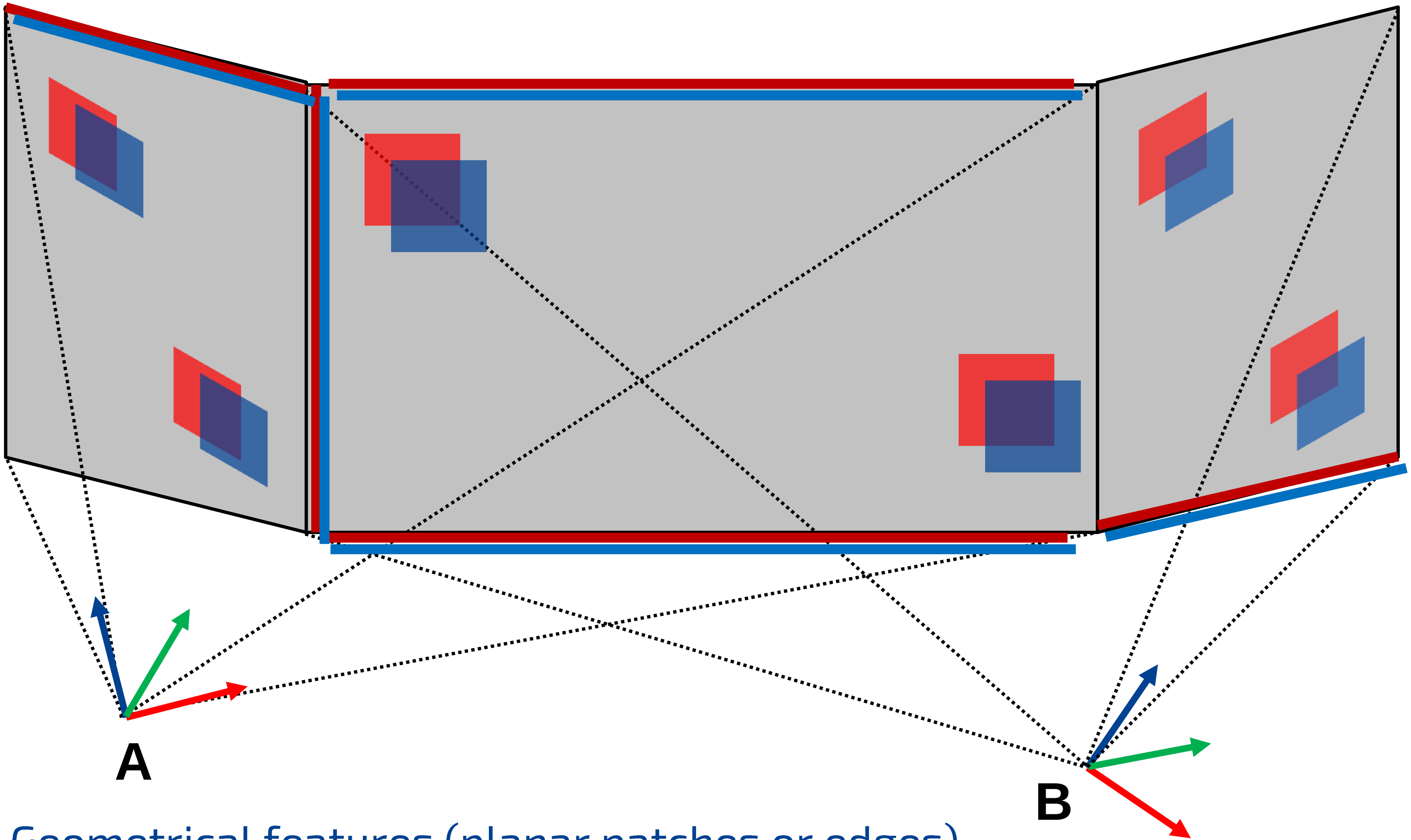
- 2. Well-defined keypoints (e.g. corners)
- aimed (e.g. total station) or automatically detected (TLS)

Using measured object



- 2. Point clouds
- Most often solved by ICP

Using measured object



- Geometrical features (planar patches or edges)
- Varying in all 3 dimensions
- (modified transformation equation)

Determination of parameters

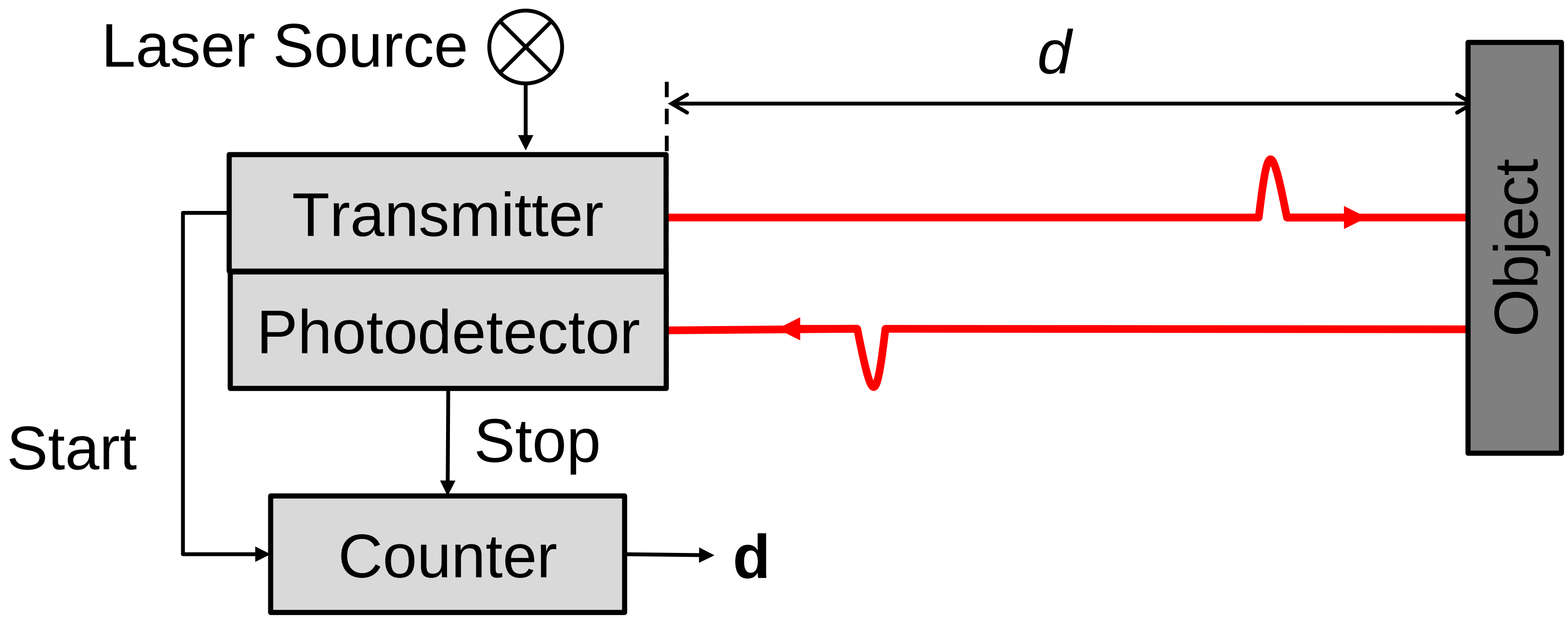
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mu \underbrace{\boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)}_{\boldsymbol{R}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Alternative:

- Directly observing transformation parameters with additional hardware/sensors
- Most often used for partial solution (some parameters) – reducing DoF
- Reducing number of parameters increases efficiency and accuracy of result
- (Complete solutions not covered here)

Reducing DoF: Scale μ

EDM (laser): Time-of-Flight (ToF)

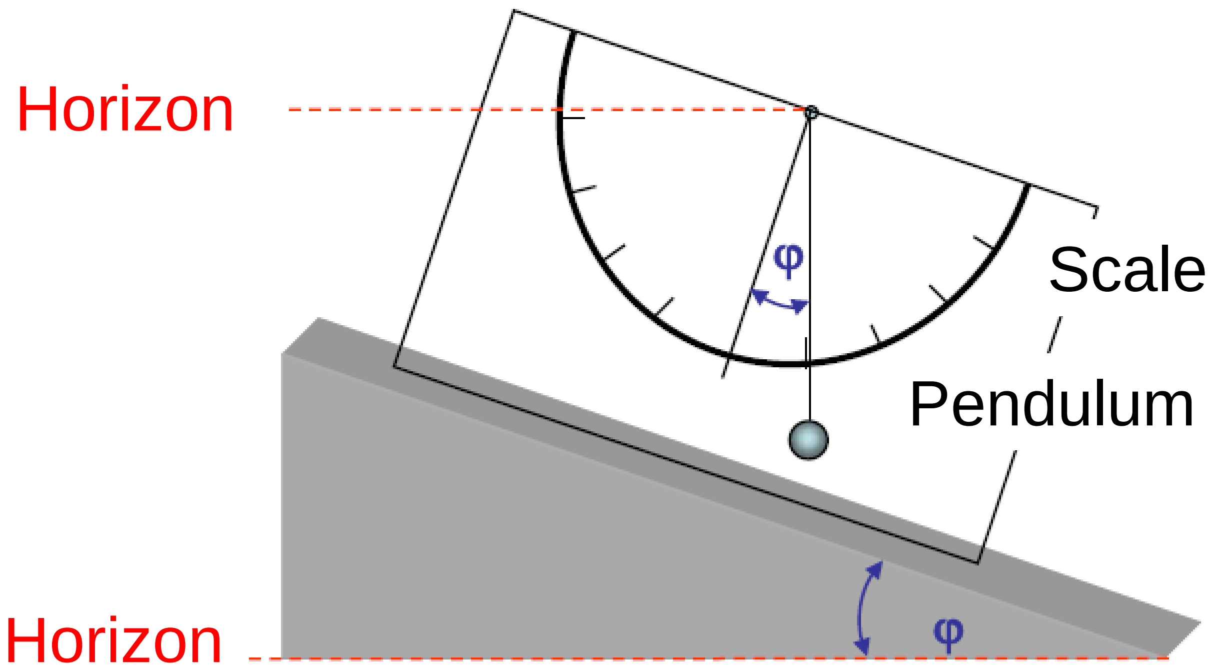


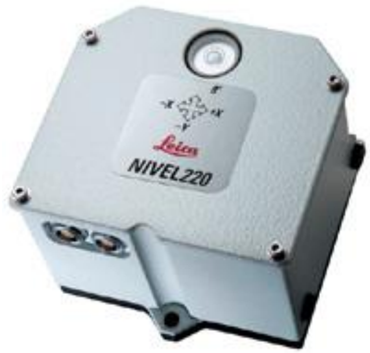
$$d = \frac{(t_{Stop} - t_{Start})}{2} \cdot c$$

- c: Speed of light in vacuum
- Distance is given in meter => Scale fixed
- Propagation delay of current atmosphere needs to be known

Reducing DoF: Rotations $\varepsilon_x, \varepsilon_y$

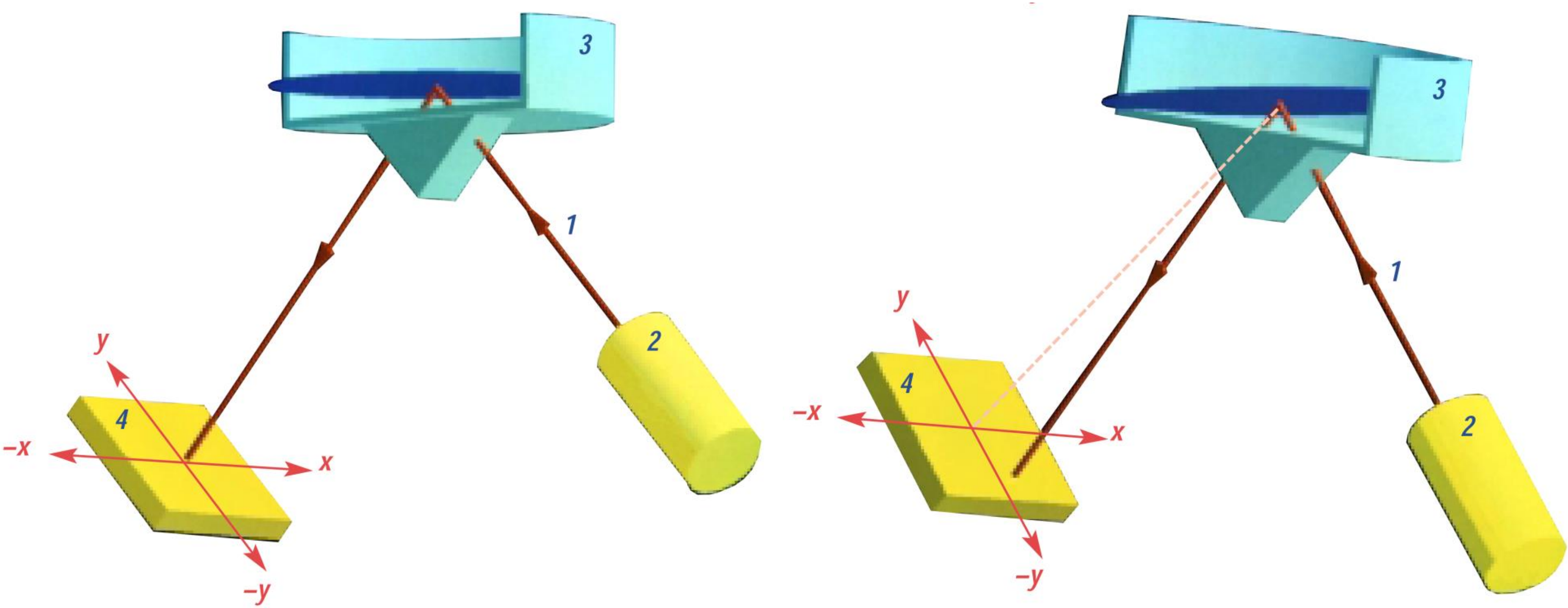
- $\varepsilon_x, \varepsilon_y$ are Euler angles in x- and y-direction
- If reference system is levelled, $\varepsilon_x, \varepsilon_y$ equal inclinations
- Deviations from horizontal plane => measurable by inclinometer





Reducing DoF: Rotations $\varepsilon_x, \varepsilon_y$

Levelled => $\varepsilon_x = \varepsilon_y = 0$ **Not levelled =>** $\varepsilon_x \neq \varepsilon_y \neq 0$



- 3D sensors often contain compensator
- Compensator = Inclinometer + unit correcting θ, φ
- Requirement for compensator: course levelling of sensor

=> $\varepsilon_x = \varepsilon_y = 0$ if using only sensors with compensator

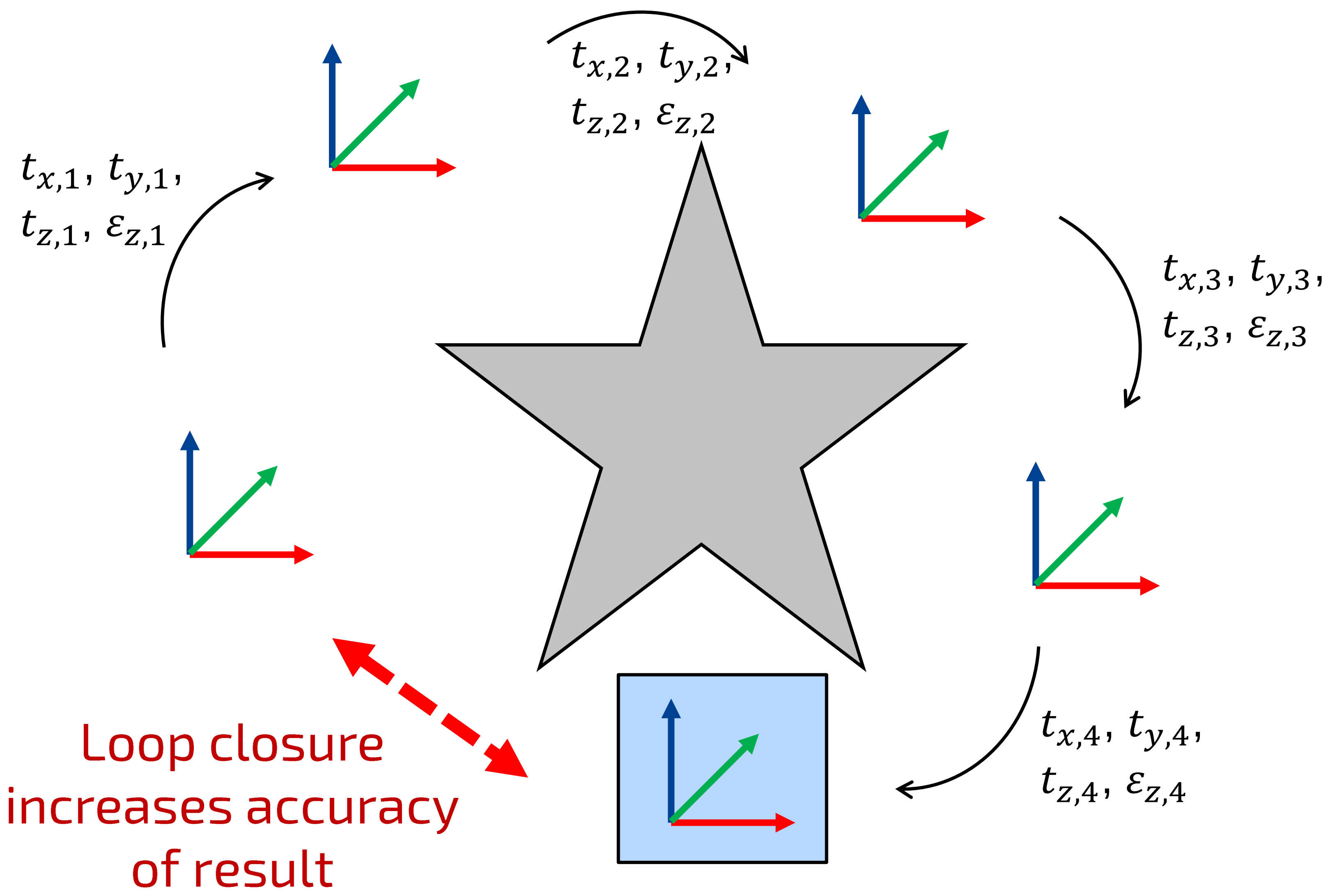
From 7 to 6 or 4 DoF

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \lambda \boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

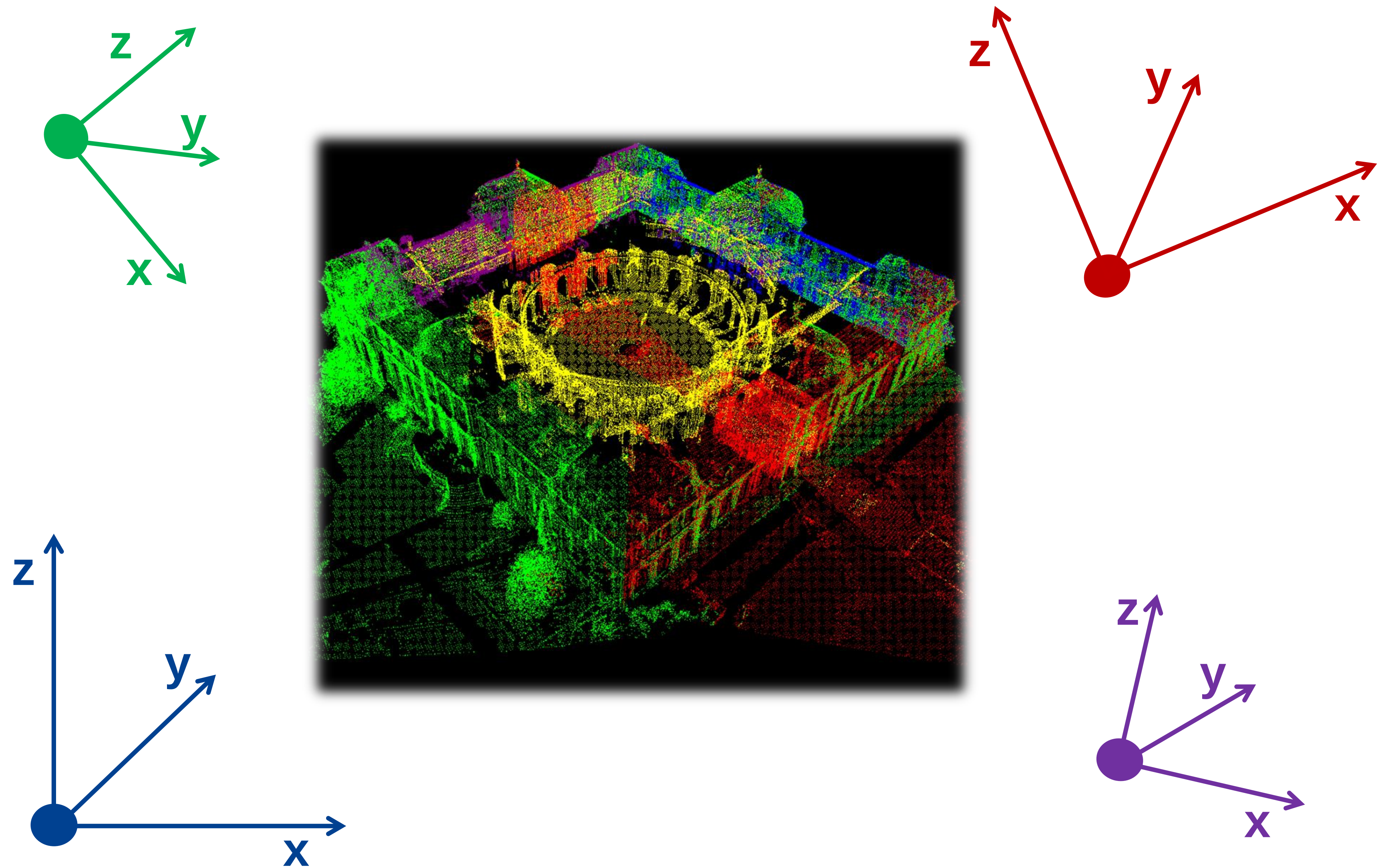
Most times $\lambda = 1$; $\varepsilon_y = 0;\ \varepsilon_x = 0$ Often at static data acquisition

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \boldsymbol{R}_3(\varepsilon_z) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Composition of registrations



Composition of registrations



Determination of parameters

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mu \underbrace{\boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)}_{\boldsymbol{R}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Summary:

- using identical artificial points (targets)
- using measured object (keypoints, point cloud, geometric features)
- using additional hardware
- Combination

leica-geosystems.com

leica-geosystems.com

velodynelidar.com

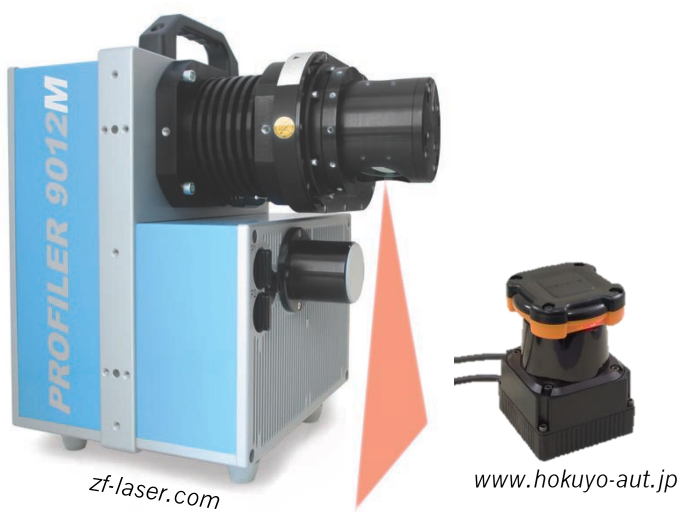
leica-geosystems.com

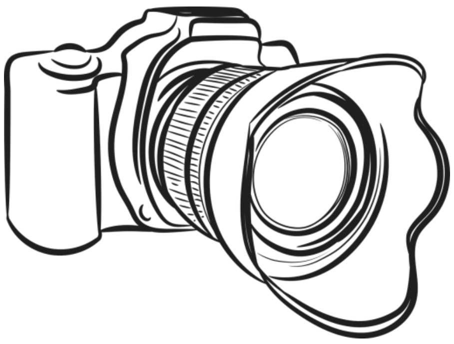
More in the next chapter:

Georeferencing

Can become more complicated

- Relative sensor registration with no 3D observations





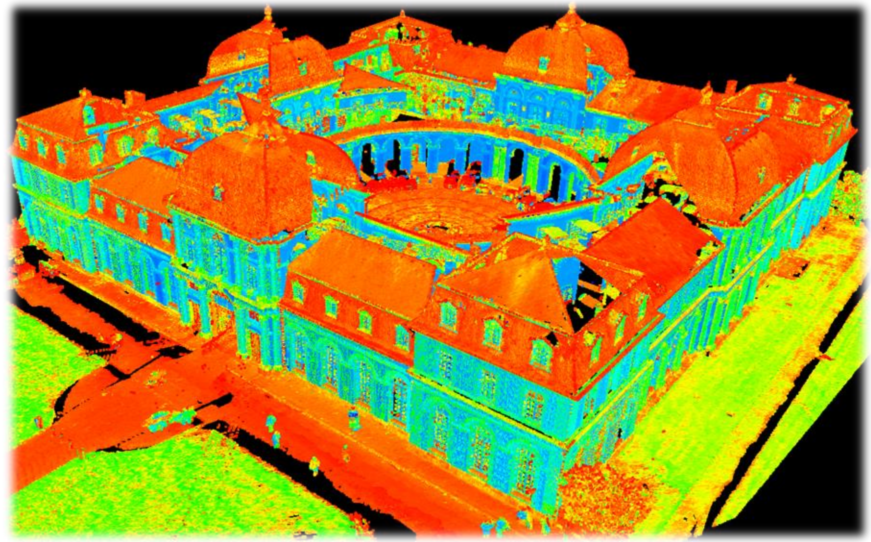
- Observations in 3D (but within 1 plane ~2D)
- no targets, keypoints, planes
- Lines & point clouds

- 2D image coordinates (no 3D space)
- collinearity equations
- Scale μ issue (all 7 DoF)

Other reason for sensor registration

riegl.com

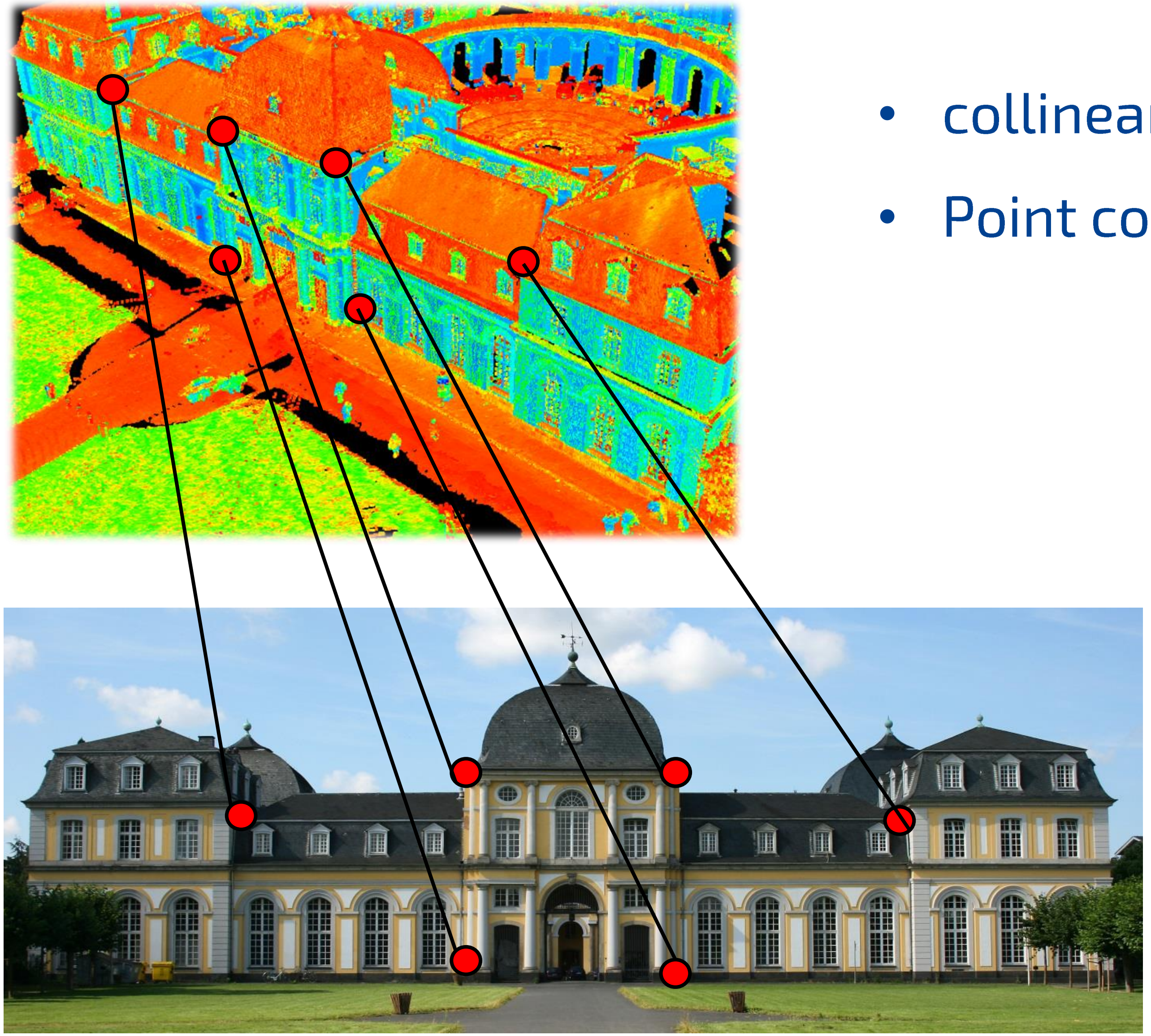
- Coloring the point cloud



zf-laser.com

Z+F MapCam® S

Other reason for sensor registration

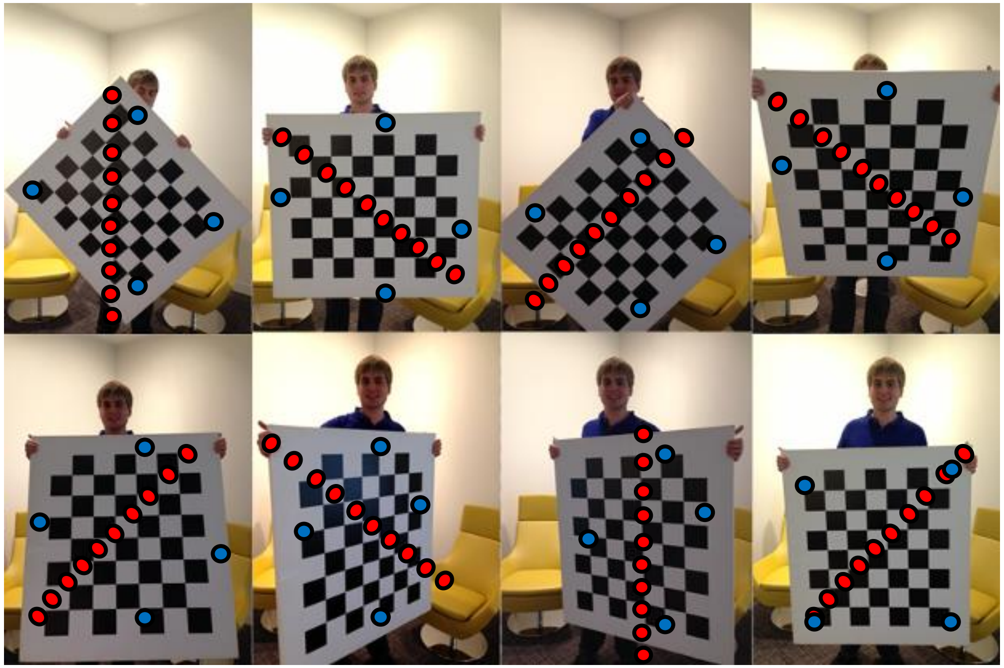


- collinearity equations
- Point correspondences

riegl.com

Other reason for sensor registration

- 2D laser scanner Lines falling on a plane
- Varying in all 3 dimensions
- Measurement correspondence over time



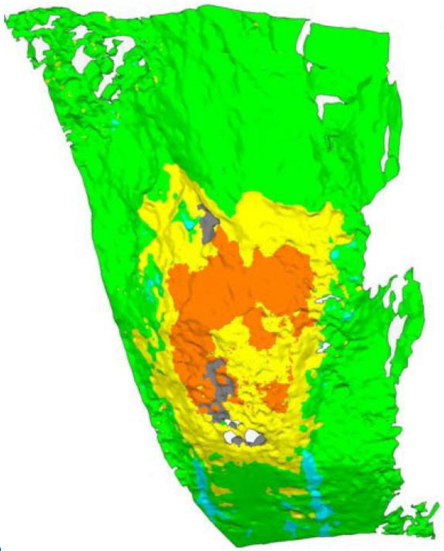
Other reason for sensor registration

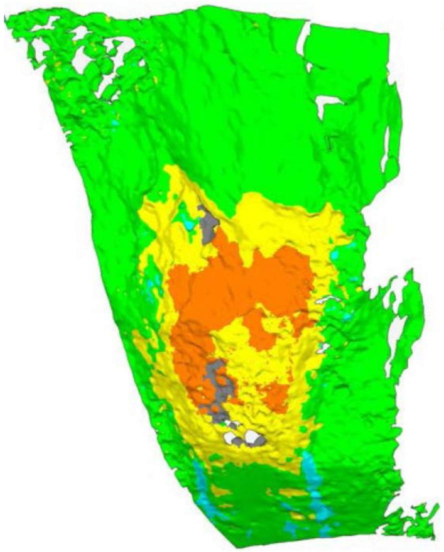
- Change detection & deformation monitoring
- Registration of measurements in different time points

2011

vs.

2012





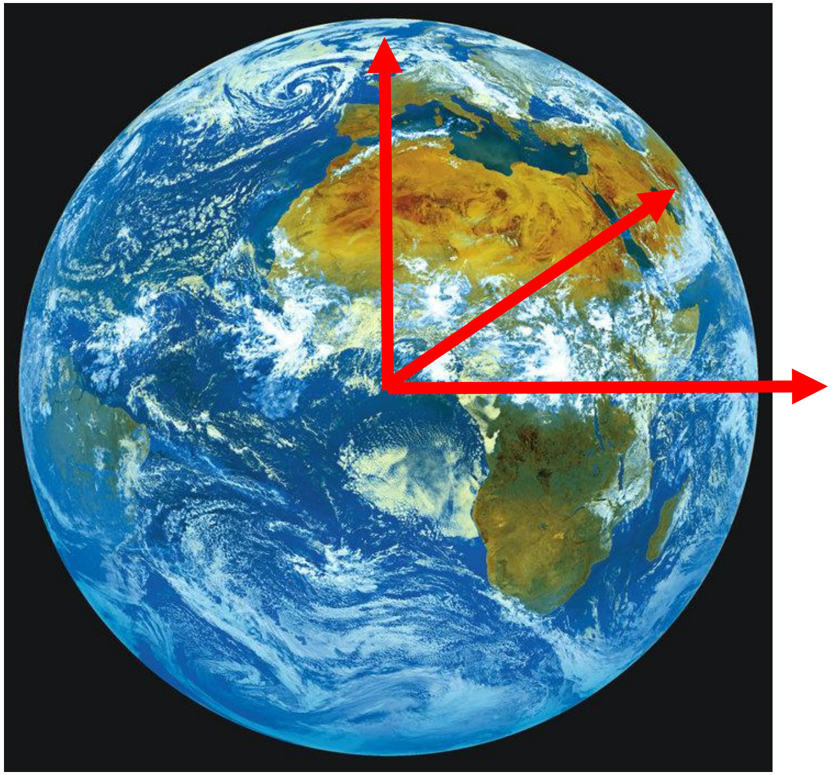
Wujanz (2016)

Agenda

1. Need for coordinate transformations
2. Realization of sensor coordinate systems
3. Local transformations between sensors (Registration)
4. Transformation in global coordinate system (Geo-referencing)

Global coordinate systems

- Different realizations of global coordinate systems
- See next lectures in MGE-01
- E.g., X,Y,Z_{ITRF} or X,Y,Z_{WGS84}
- Center = Center of certain ellipsoid
- X-axis = Greenwich
- Z-axis = rotation axis of earth
- Y-axis = completion of right-handed coordinate system



Similarity transform in 3D

- Same equation
- ~~Scale change (1DoF)~~
- Rotation (3DoF)
- Translation (3DoF)
- 6 DoF / 6 parameters

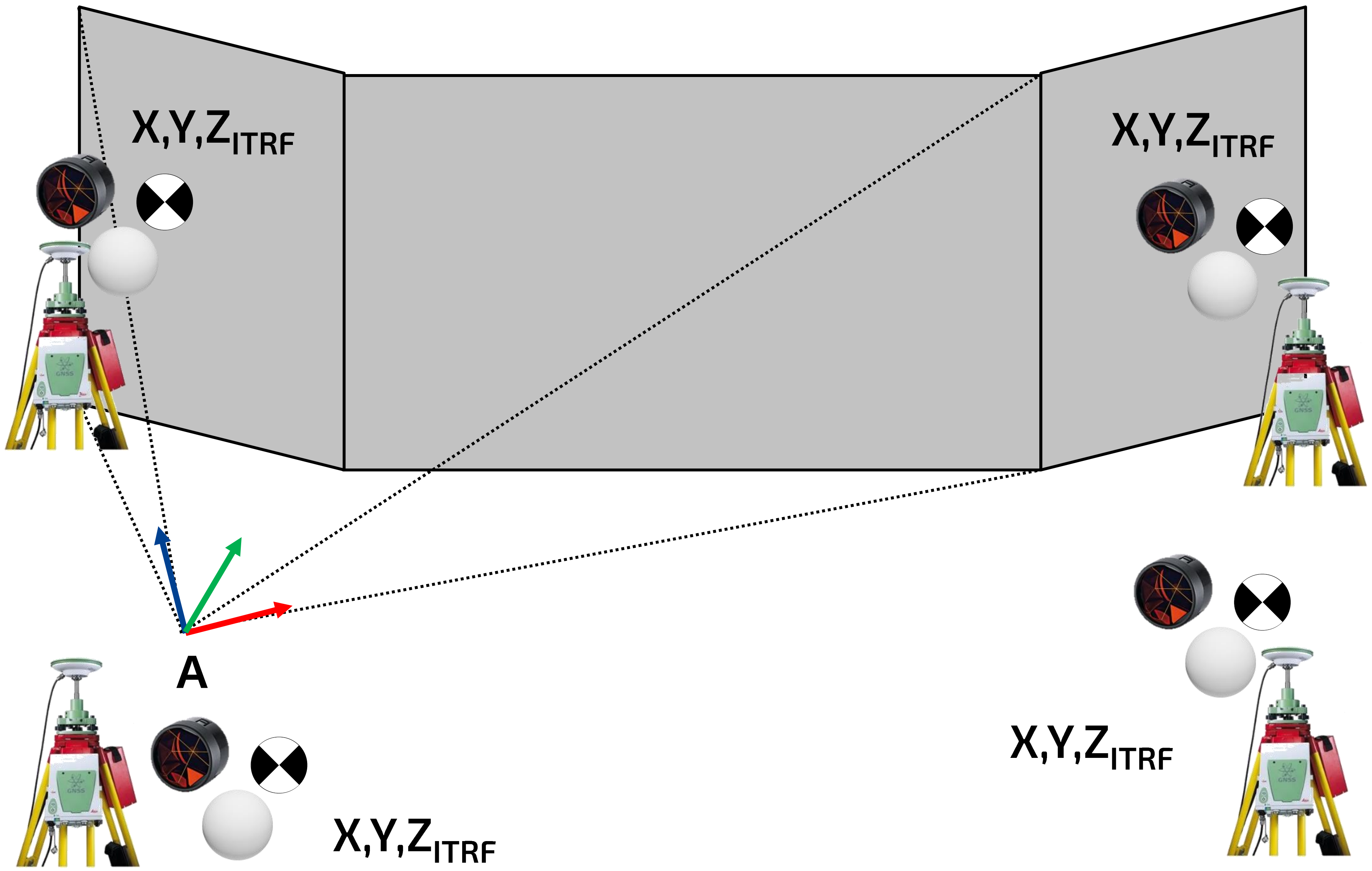
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Determination of parameters

- Task for positioning on earth: transformation to global, earth centered coordinate system
- How to get the 6 parameters?

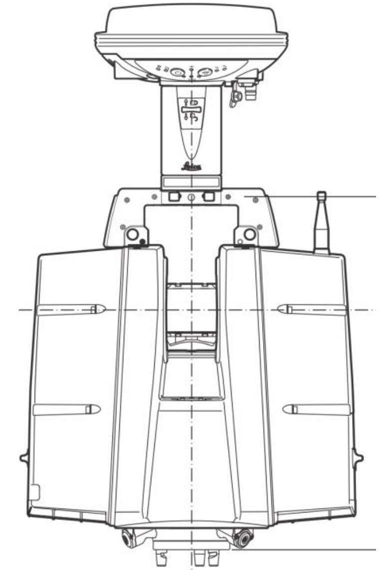
1. using identical, artificial points **that are already given in global coordinate system**
2. Using additional hardware **that acquires data in global coordinate system**

Using artificial targets



Using additional hardware

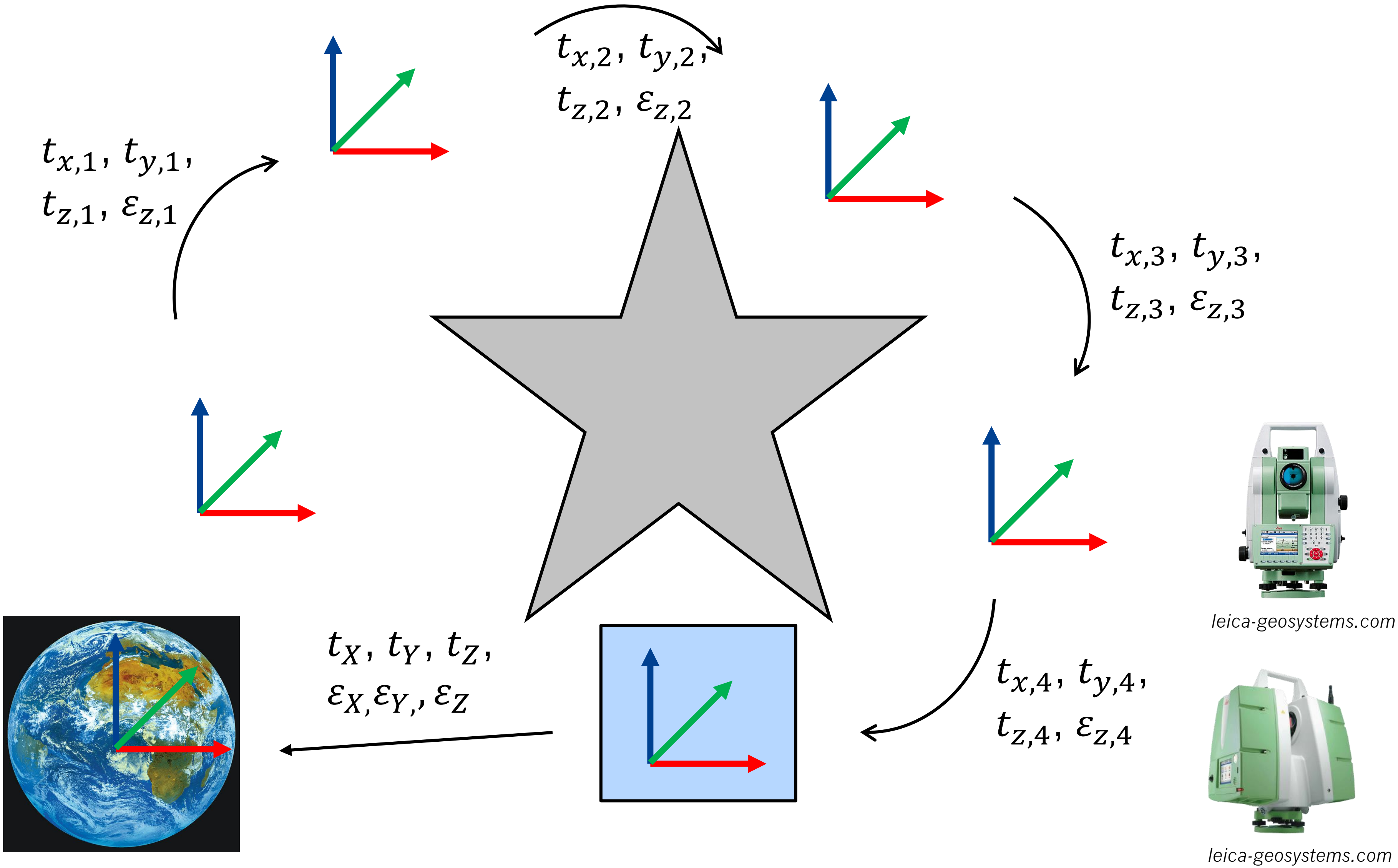
- Using additional hardware that gives global information, e.g.:
- GNSS for t_x, t_y, t_z (most times)
- Two GNSS for ε_z
- Inclinometers for $\varepsilon_x, \varepsilon_y$
- Compass for ε_z



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \boldsymbol{R}_3(\varepsilon_z)\boldsymbol{R}_2(\varepsilon_y)\boldsymbol{R}_1(\varepsilon_x)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

- **...Static or kinematic**

Static geo-referencing



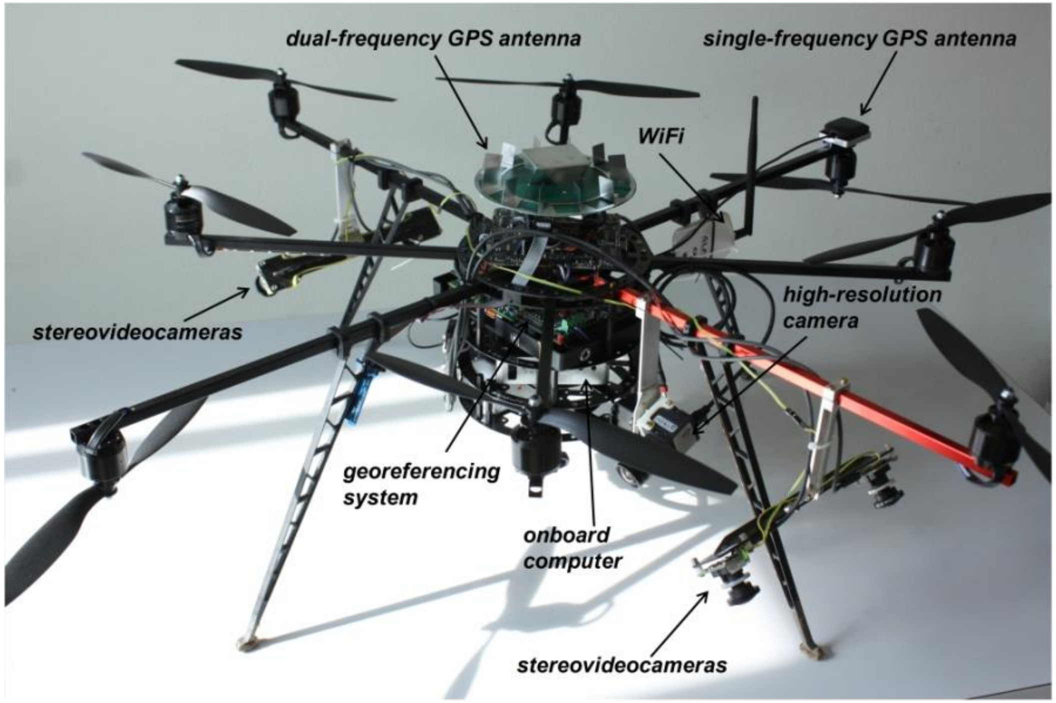
Mobile multi-sensor systems

Google.com

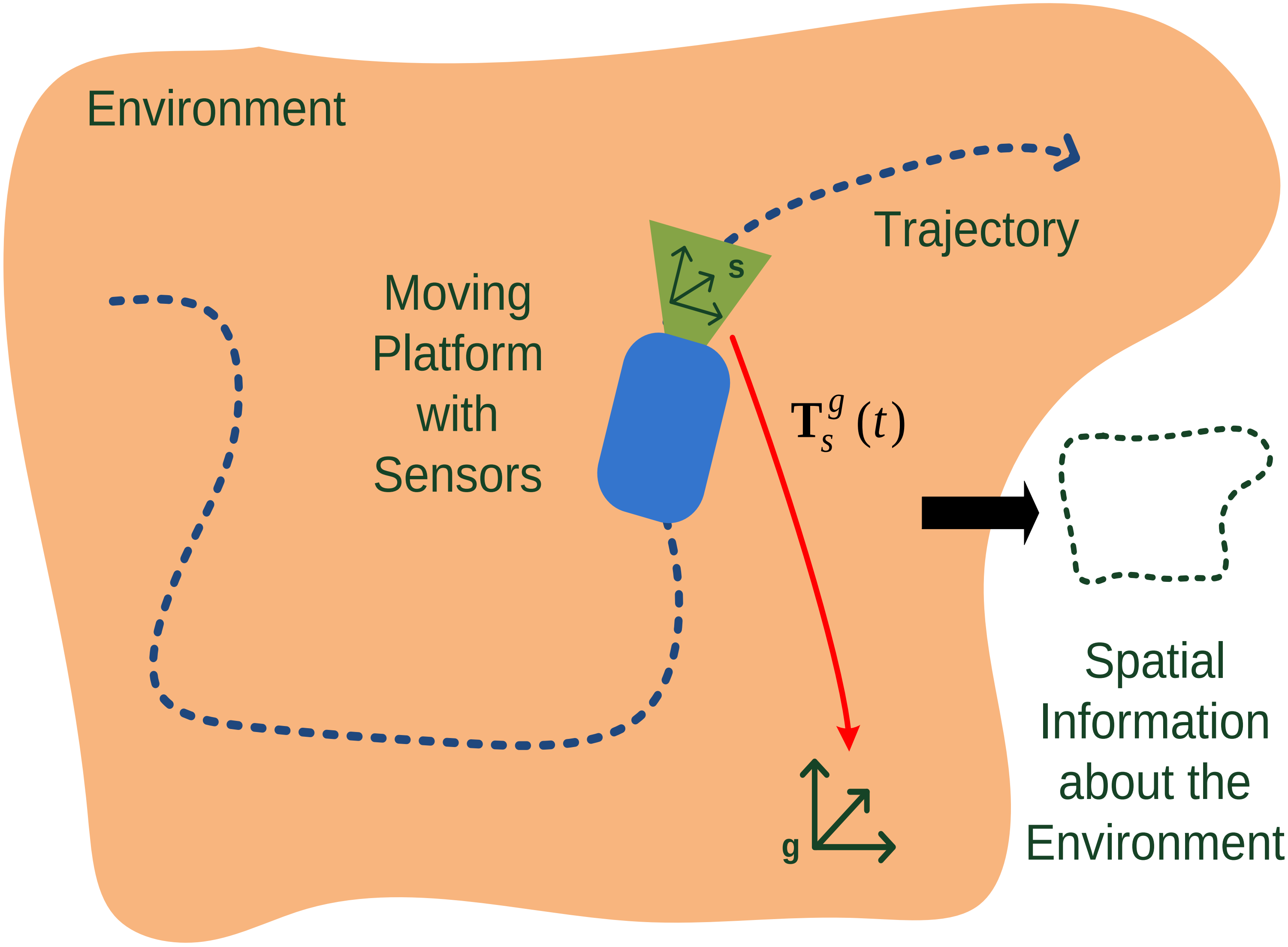
velodynelidar.com

zf-laser.com

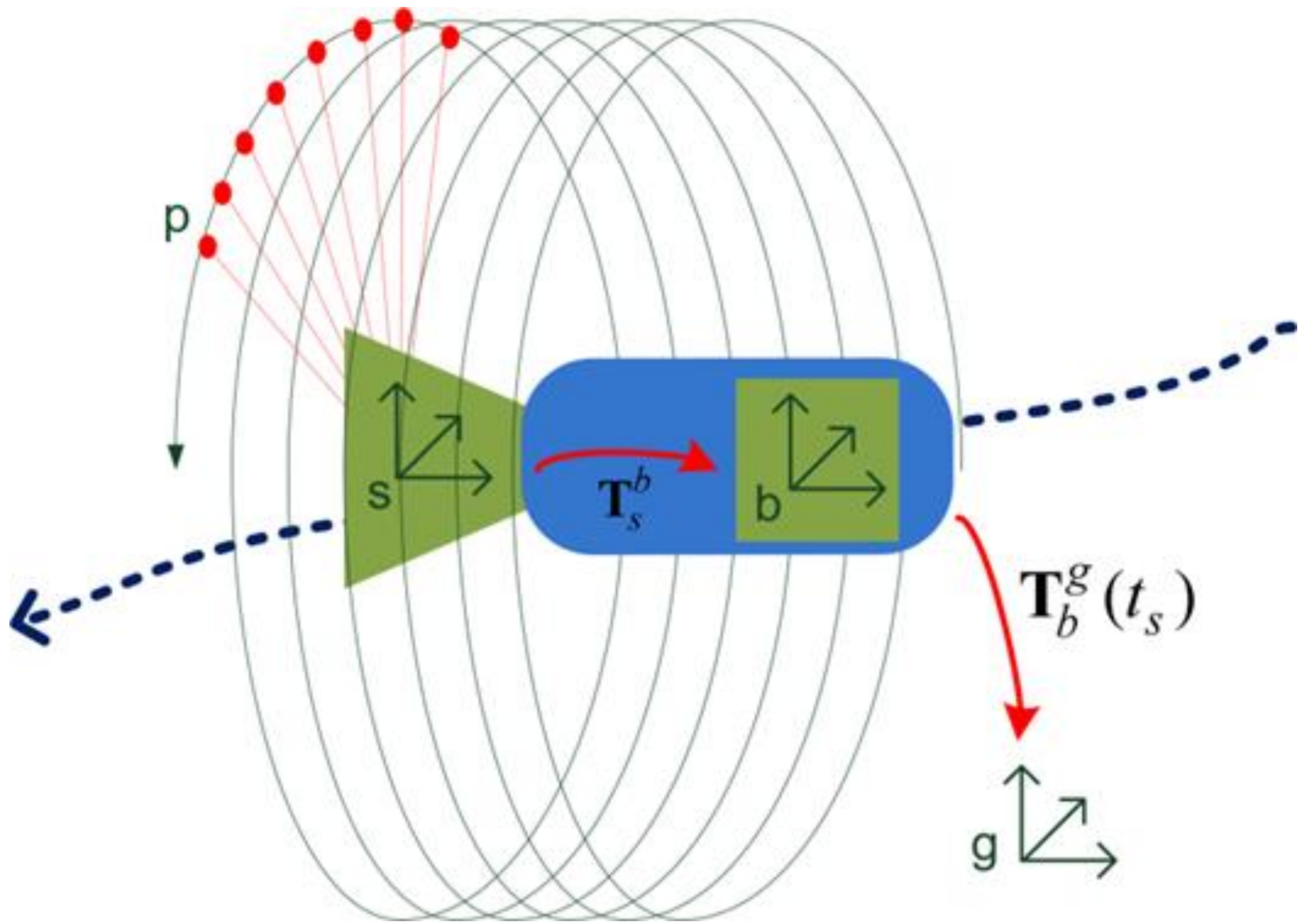
www.hokuyo-aut.jp



Mobile multi-sensor systems

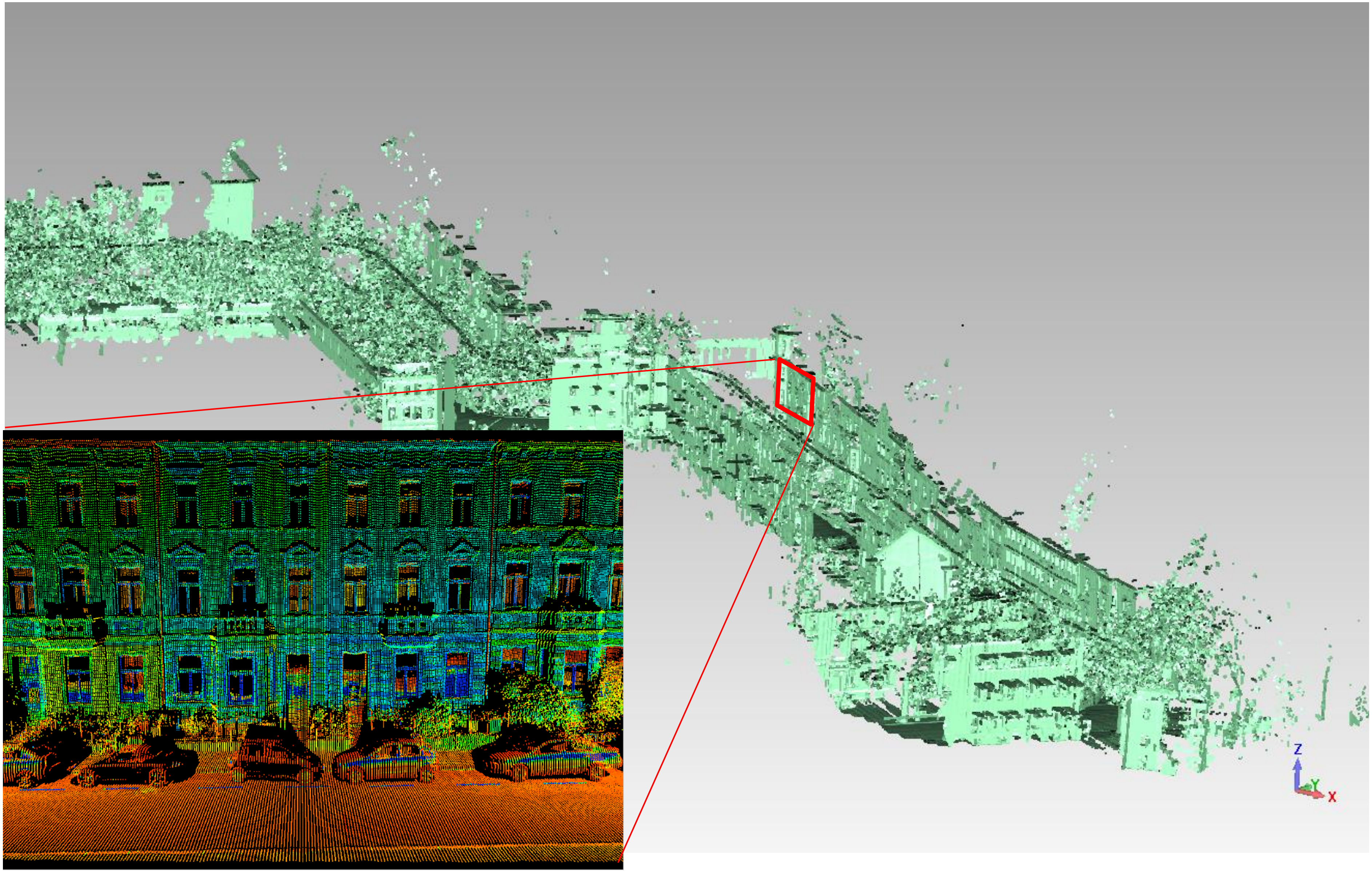


Mobile multi-sensor systems



- **Geo-referencing by:** $\boldsymbol{p}^g = \boldsymbol{T}_b^g(t_s)\boldsymbol{T}_s^b\boldsymbol{p}^s$

3D geo-referenced point cloud



Recapitulation

- Transformation between sensor coordinate systems frequent task in geodesy
- Usually similarity transformation is used => 7 DoF
- **Registration:** Transformation between local coordinate systems
- **Geo-referencing:** Transformation in global coordinate system

References

- Förstner, W., Worbel, P. B.: *Photogrammetric Computer Vision*, Springer: Cham, Switzerland, 2016

- Ogundare, J. O.: *Precision Surveying: The Principles and Geomatics Practice*, Wiley: London, UK, 2015