

Lecture Coordinate Systems, WiSe 2020/2021 Homework assignment

Please submit all tasks as one PDF file via eCampus.

Task 1

On eCampus, you find a zip file `buildings.zip` containing the footprints of buildings in Bonn as they are recorded by OpenStreetMap (OSM). The coordinates of these footprints are given as spherical coordinates φ , λ with respect to a sphere with radius $r = 6\,378\,137$ m.

- (a) Create a new file containing the footprints projected with the Mercator projection for a sphere with radius r (EPSG:3785). Use the field calculator to compute the area of the footprints.

Give the area of the footprints with the OSM id **418 636** (Kanzlerbungalow, former residence of the German chancellor), **1 934 665** (Poppelsdorf Palace), **2 881 966** (Electoral Palace, main building of the university) and **3 633 958** (Beethovenhalle, concert hall).

Note: Saving a layer in a new file, you get the opportunity to select the projection (CRS, Coordinate Reference System).

- (b) Now, compute the real area of the footprints by repeating the steps of task (a) with the cylindrical equal area projection (ESRI: 54034).

Again, give the area for the footprints with the OSM id **418 636**, **1 934 665**, **2 881 966** and **3 633 958**.

- (c) As a next step, join the attribute tables of both created layers on the OSM id. Then, use the field calculator to compute the following ratio

$$\frac{\text{area}_{\text{Mercator}}}{\text{area}_{\text{CEA}}}$$

Give this ratio for the footprints with the OSM id **418 636**, **1 934 665**, **2 881 966** and **3 633 958**.

- (d) Create a map using the Mercator projection from above. In this map, the footprints are filled according to the ratio. Therefore, choose a color ramp such that footprints are filled with a color the brighter the less the footprint's area ratio.

Explain the observed effect!

Task 2

On slide 22-6 of the lecture, we compared the area of an infinitesimally small rectangle on a sphere, i.e. $\text{area}_{\text{sphere}} = |r^2 \cos \varphi|$ to the area of this rectangle after using a cylindrical projection f , i.e. $\text{area}_f = |f'(\varphi) \cdot r|$. Demanding equality, i.e. $\text{area}_{\text{sphere}} = \text{area}_f$, we deduced the formula for the cylindrical equal area projection.

In this task, we are interested in the ratio $\frac{\text{area}_f}{\text{area}_{\text{sphere}}}$, which is the area distortion of projection f .

- (a) Give the area distortion of the Mercator projection for an infinitesimally small rectangle depending on its latitude φ .
- (b) Give some example values for the formula of task (a). In particular, give the distortion for $\varphi \in \{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ\}$. What happens for $\varphi \rightarrow 90^\circ$?

Furthermore, give the distortion for Bonn with its coordinates 50.733992° N, 7.099814° E and compare it to your results of task 1.