UNIVERSITÄT BONN igg

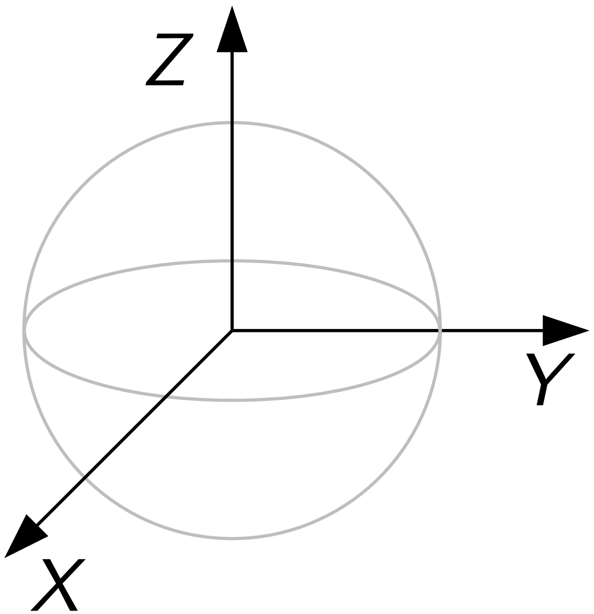
Coordinate Systems

Map Projections

Prof. Dr.-Ing. Jan-Henrik Haunert
Institut für Geodäsie und Geoinformation
Universität Bonn

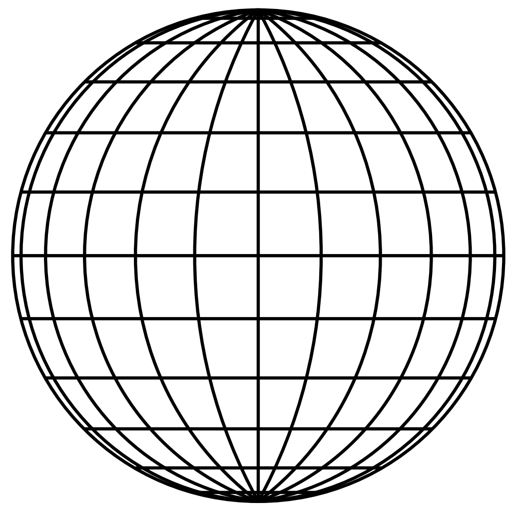
Coordinate Systems for Geoinformation

Earth-centered Cartesian coordinates

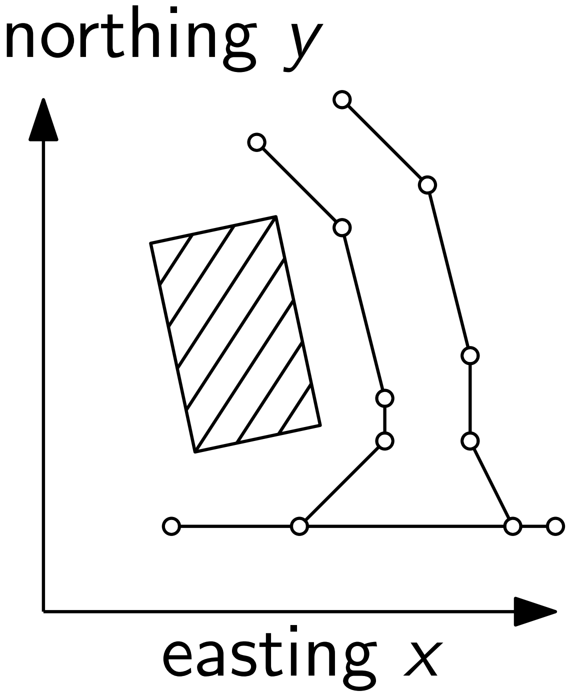


spherical/ellipsoidal coordinates

latitude φ, longitude λ

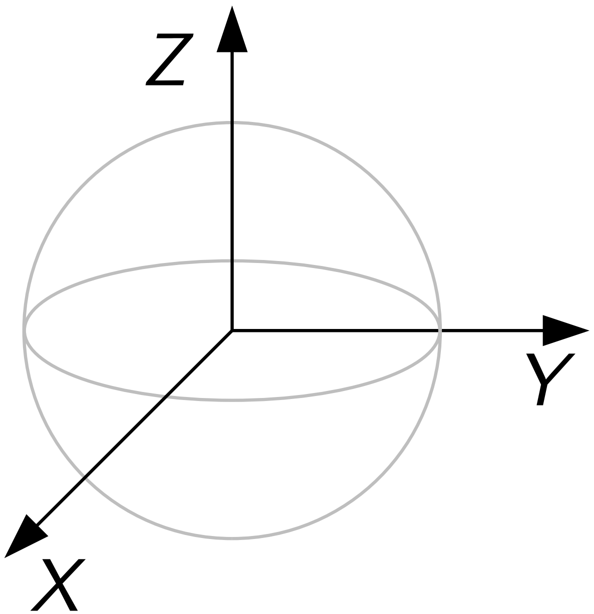


projected coordinates



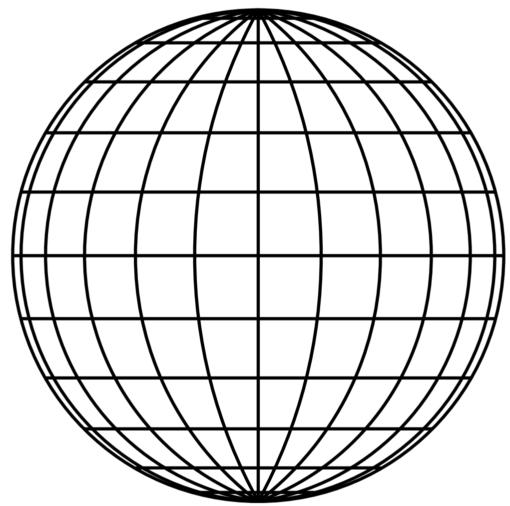
Coordinate Systems for Geoinformation

Earth-centered Cartesian coordinates

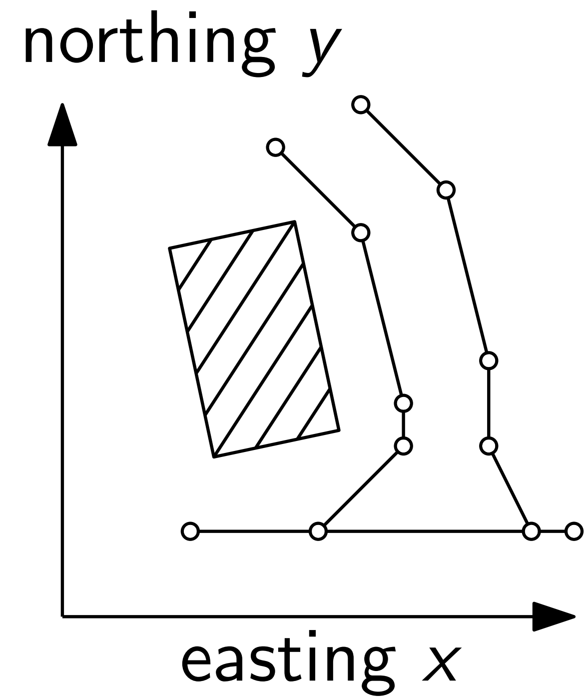


spherical/ellipsoidal coordinates

latitude φ, longitude λ



projected coordinates

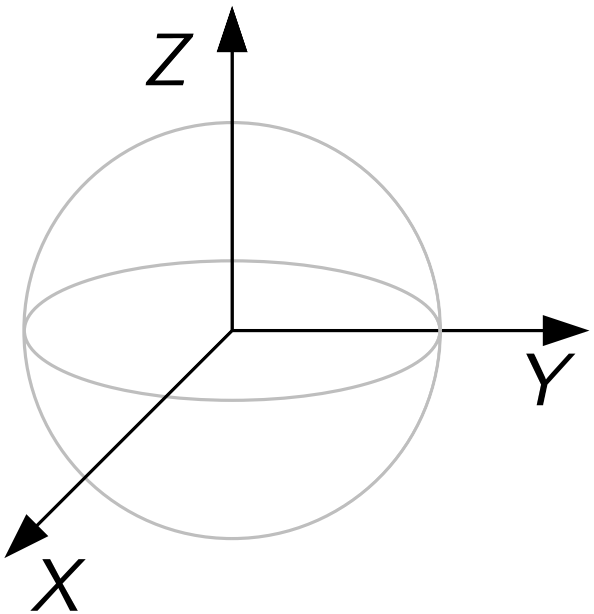


$$\begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix} = \begin{pmatrix} r\cos\varphi_p\cos\lambda_p \\ r\cos\varphi_p\sin\lambda_p \\ r\sin\varphi_p \end{pmatrix}$$

for spherical coordinates

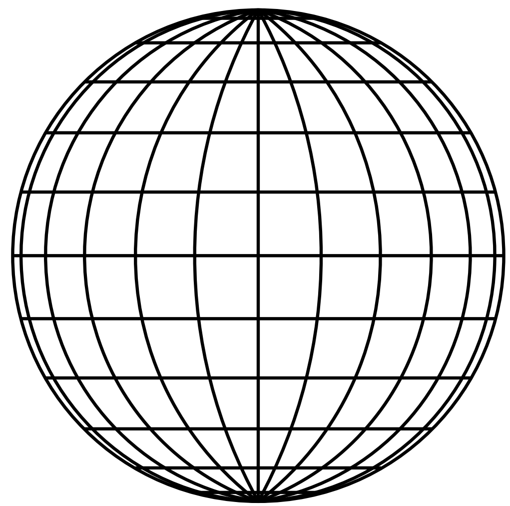
Coordinate Systems for Geoinformation

Earth-centered Cartesian coordinates

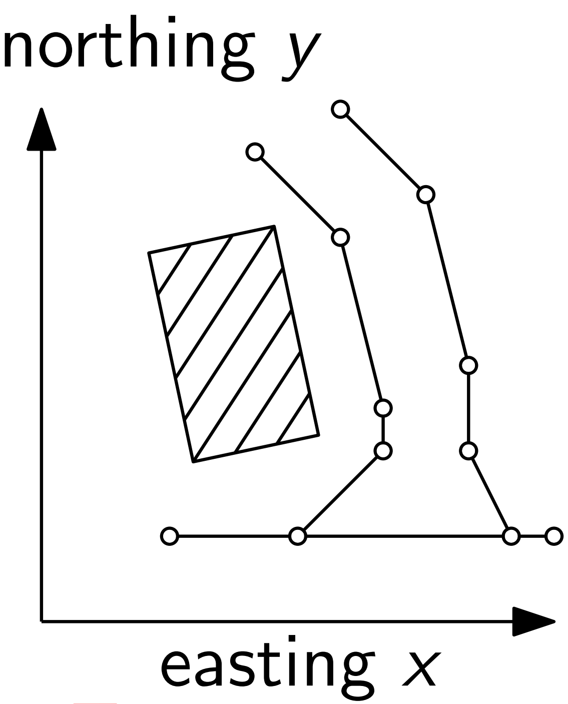


spherical/ellipsoidal coordinates

latitude φ, longitude λ



projected coordinates



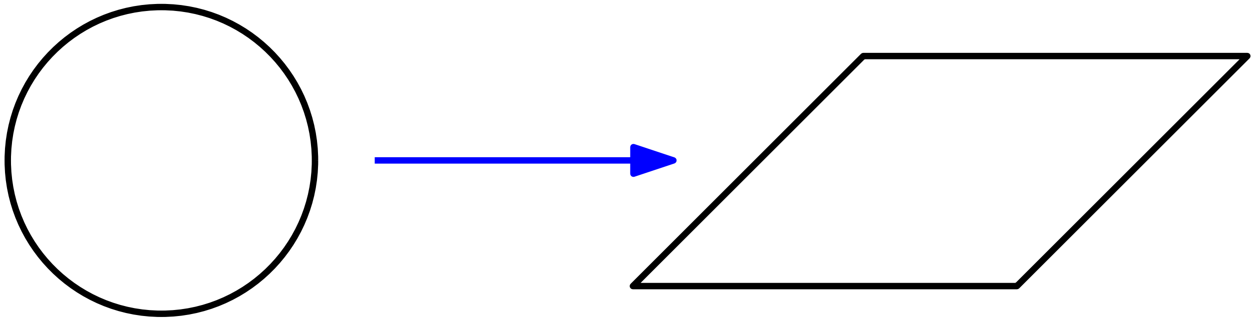
TO DO!

$$\begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix} = \begin{pmatrix} r \cos \varphi_p \cos \lambda_p \\ r \cos \varphi_p \sin \lambda_p \\ r \sin \varphi_p \end{pmatrix}$$

for spherical coordinates

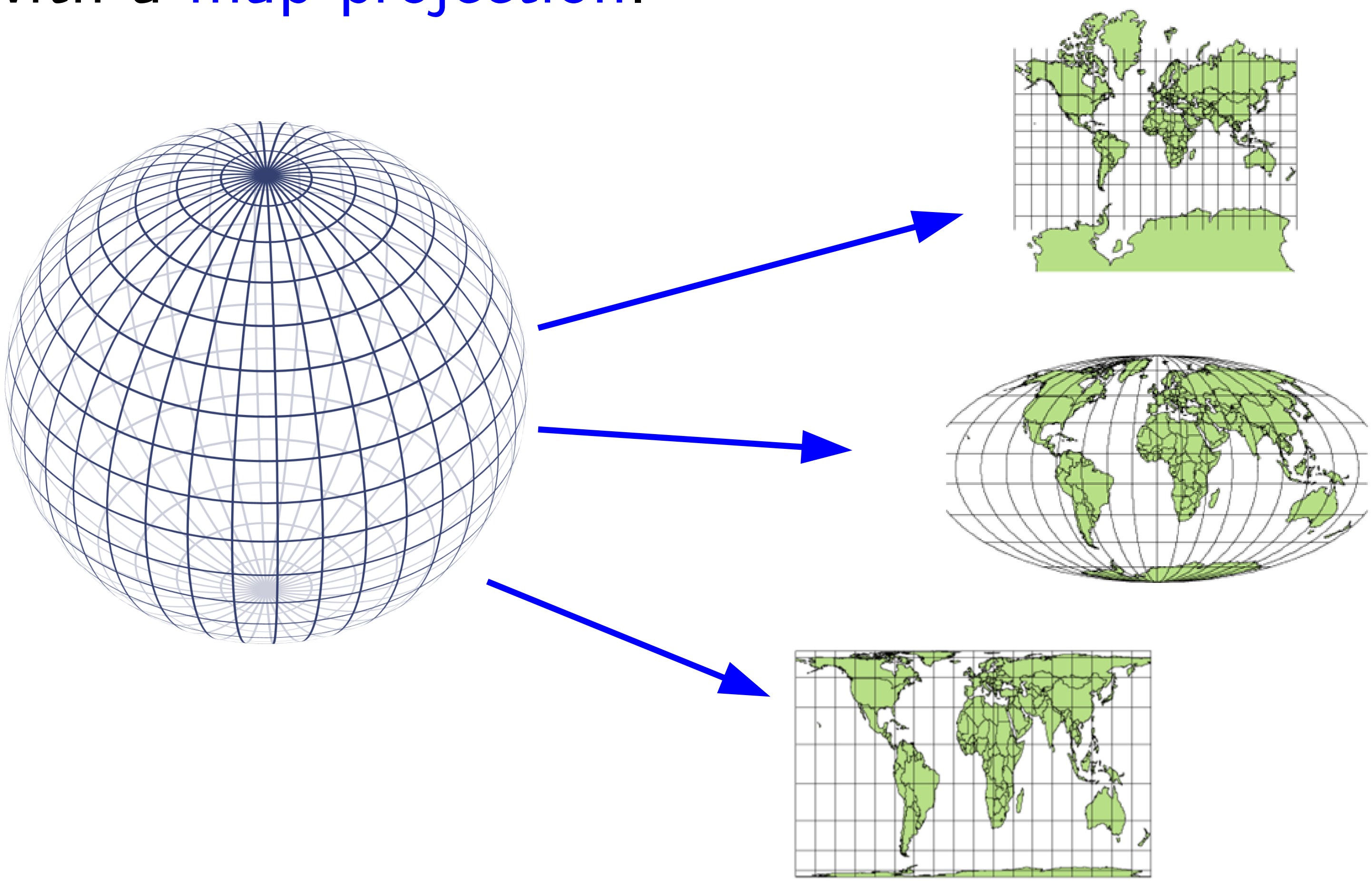
Relationship $(\varphi, \lambda) \leftrightarrow (x, y)$

- Point on reference surface (e.g. sphere, ellipsoid) is mapped to plane with a map projection.



Relationship $(\varphi, \lambda) \leftrightarrow (x, y)$

- Point on reference surface (e.g. sphere, ellipsoid) is mapped to plane with a map projection.



- Different map projections cause different distortions.
- Best choice of map projection depends on application.

Outline

- Plate carrée projection (= most basic cylindrical projection)
- What is a cylindrical projection?
- Mercator projection (= most important cylindrical projection)
- cylindrical equal-area projection

Plate Carrée Projection

Idea:

- Map areas between lines of constant latitude/longitude to squares of constant size (assuming $\Delta\varphi = \Delta\lambda$).

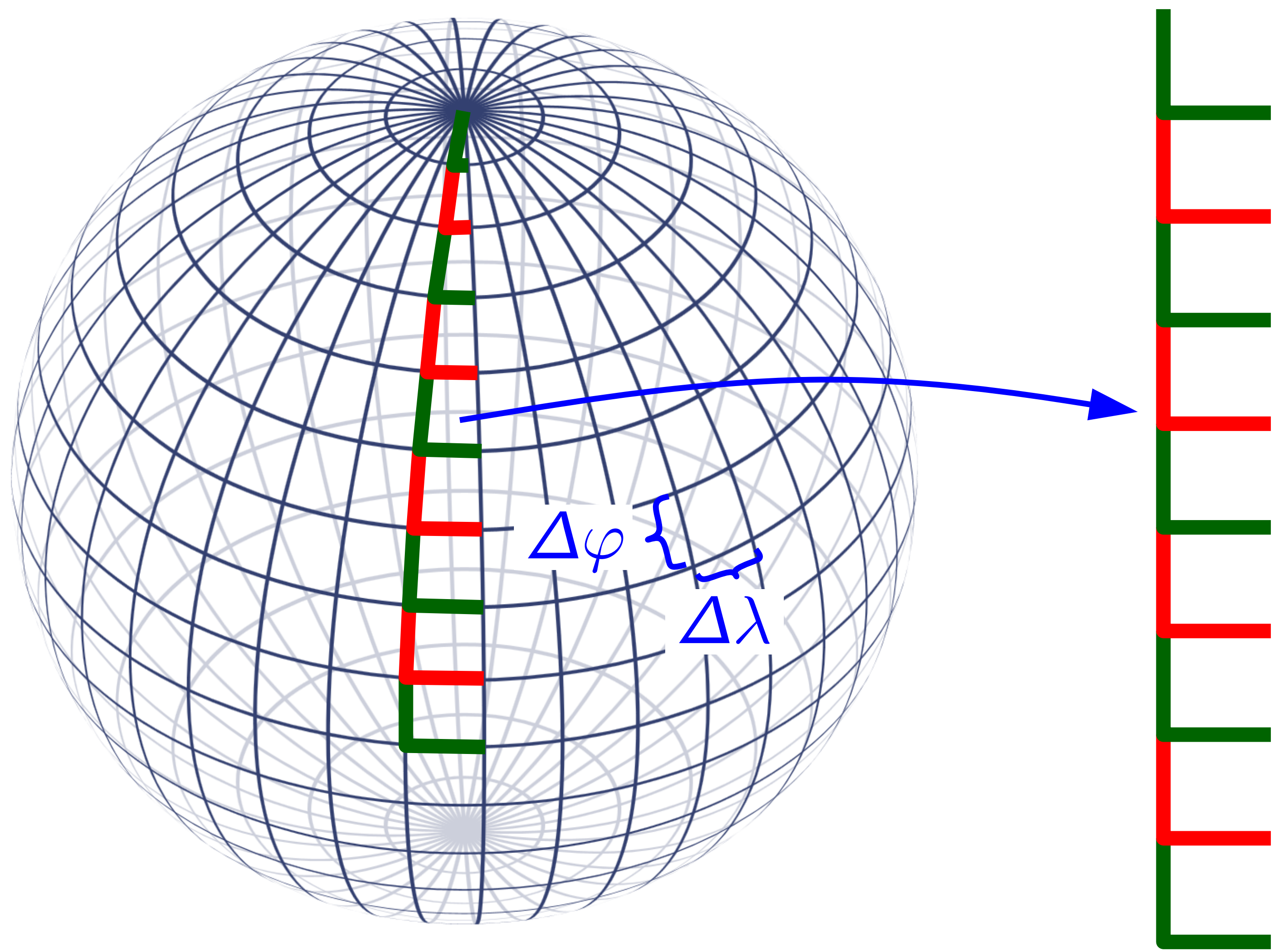


Plate Carrée Projection

Idea:

- Map areas between lines of constant latitude/longitude to squares of constant size (assuming $\Delta\varphi = \Delta\lambda$).
- Preserve lengths of equator and meridians.

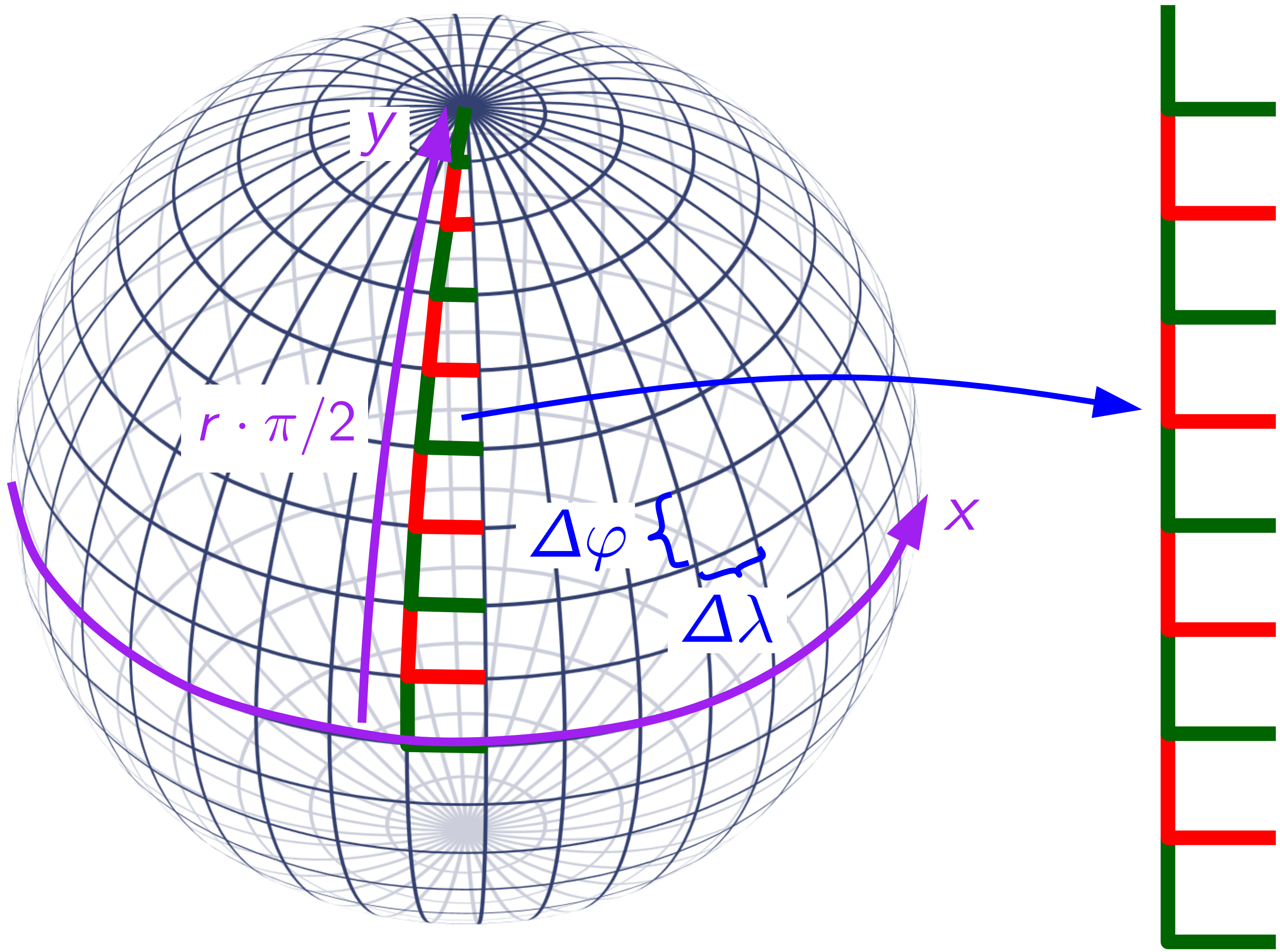


Plate Carrée Projection

- mapping function:

$$\text{easting } x = \lambda_{[\text{deg}]}$$
$$\text{northing } y = \varphi_{[\text{deg}]}$$

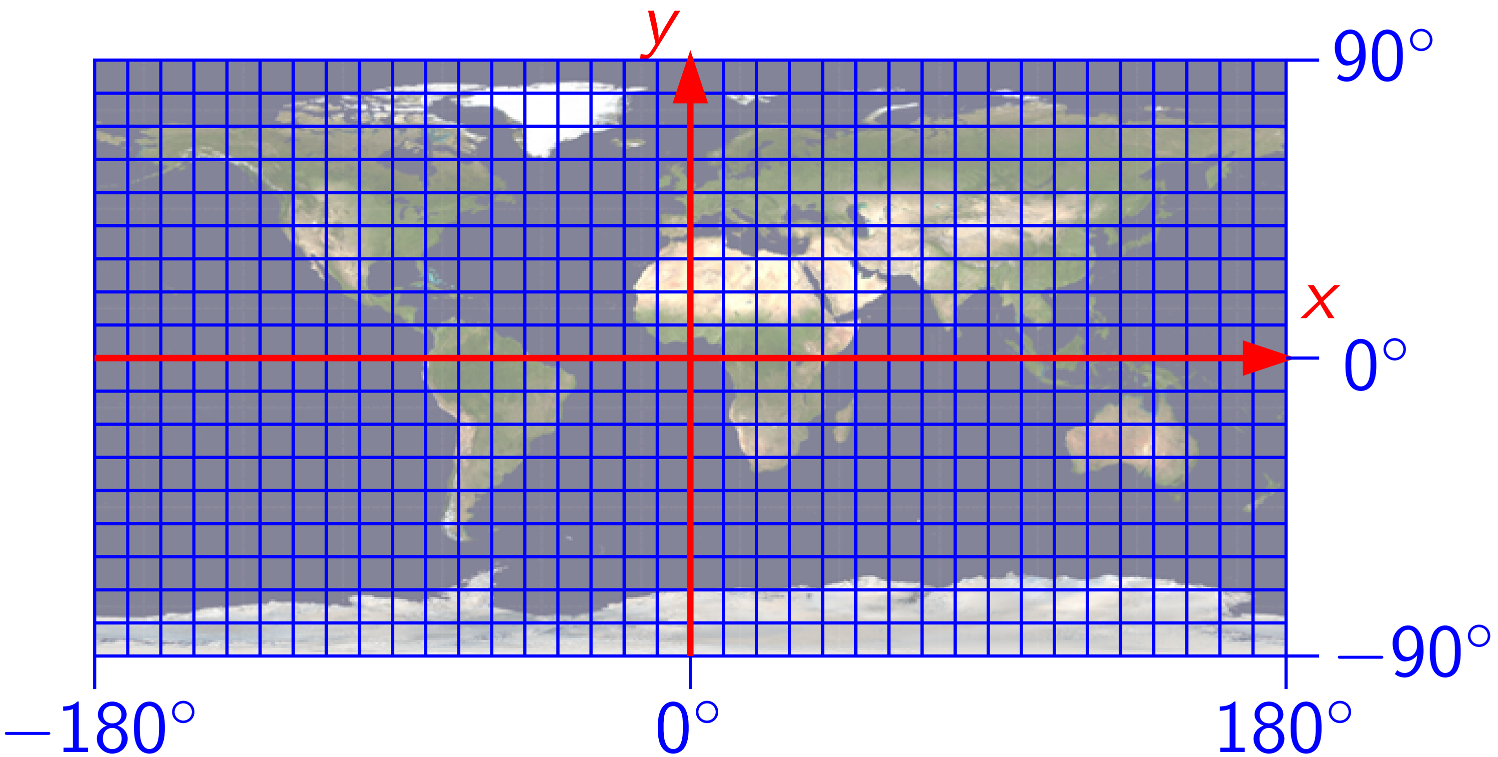


Plate Carrée Projection

- mapping function:

$$\text{easting } x = \lambda_{[\text{deg}]} \cdot \frac{\pi}{180^\circ} \cdot r$$
$$\text{northing } y = \varphi_{[\text{deg}]} \cdot \frac{\pi}{180^\circ} \cdot r$$

r = Earth's radius

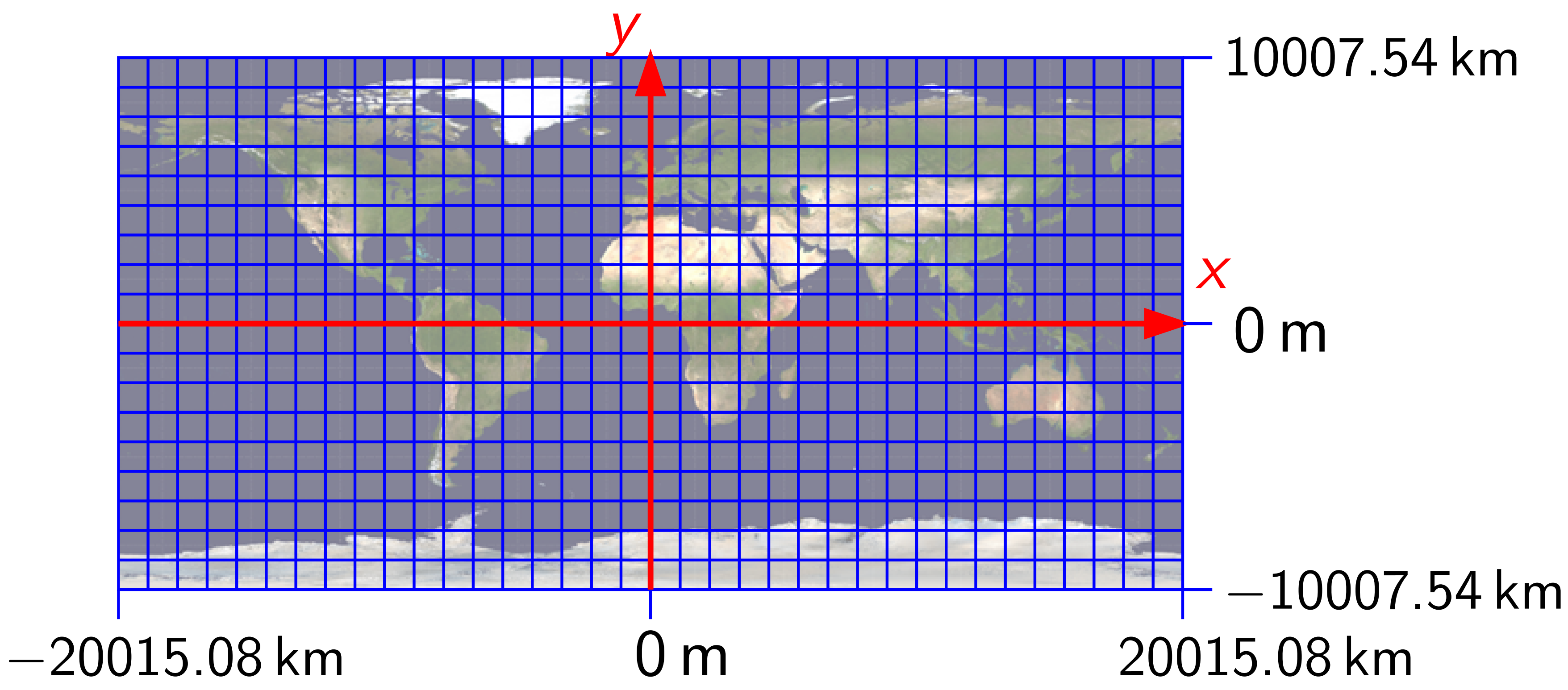


Plate Carrée Projection

Disadvantages:

- shapes get “squeezed”, i.e., aspect ratios change

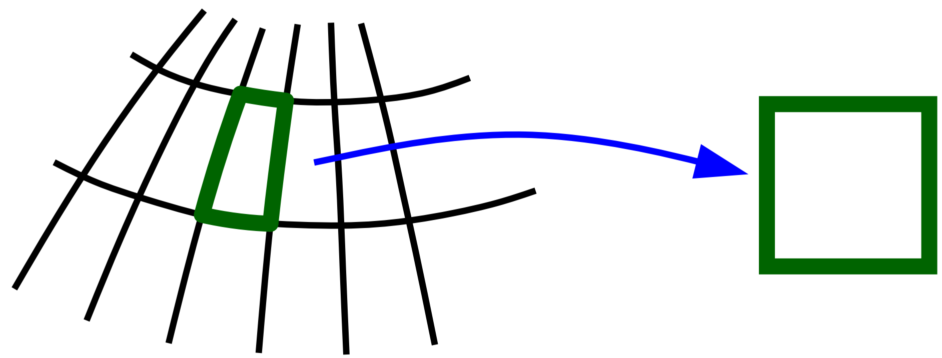
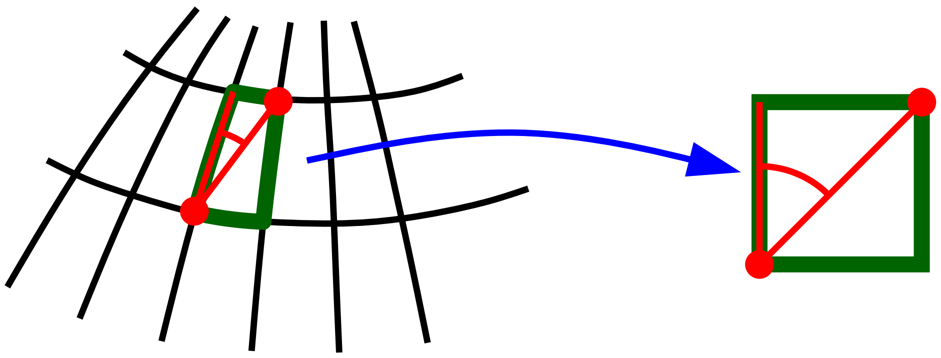


Plate Carrée Projection

Disadvantages:

- shapes get “squeezed”, i.e., aspect ratios change

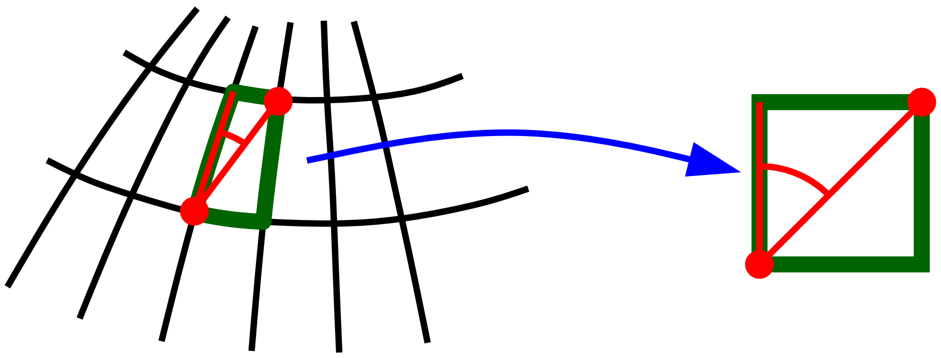


- angles change

Plate Carrée Projection

Disadvantages:

- shapes get "squeezed", i.e., aspect ratios change

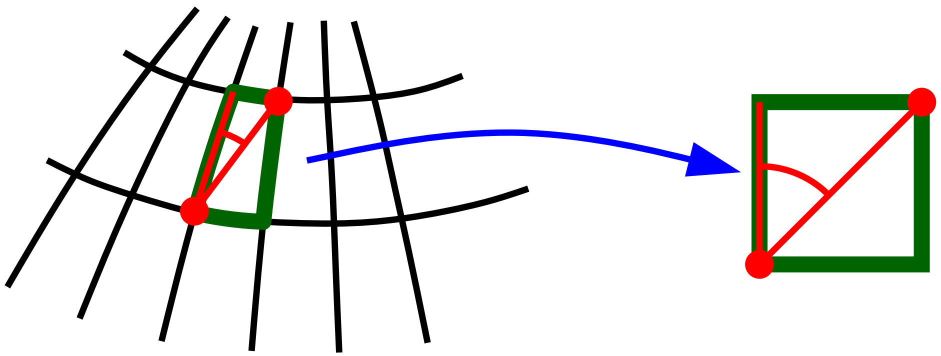


- angles change
- areas change

Plate Carrée Projection

Disadvantages:

- shapes get “squeezed”, i.e., aspect ratios change



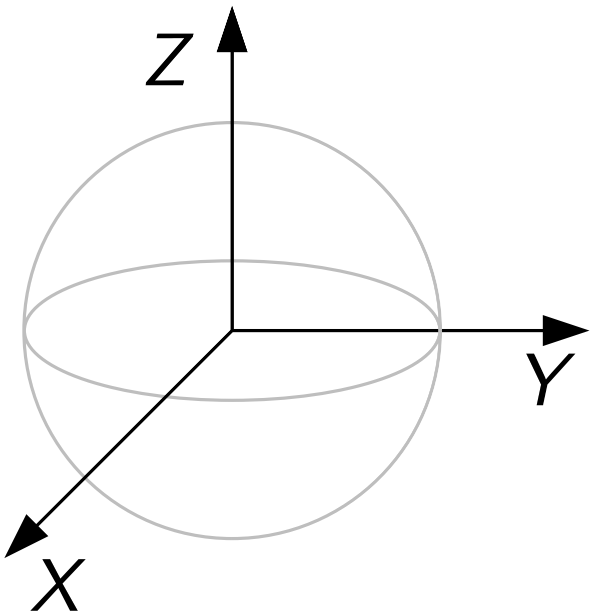
- angles change
- areas change

Advantage:

- lengths of equator and meridians are preserved

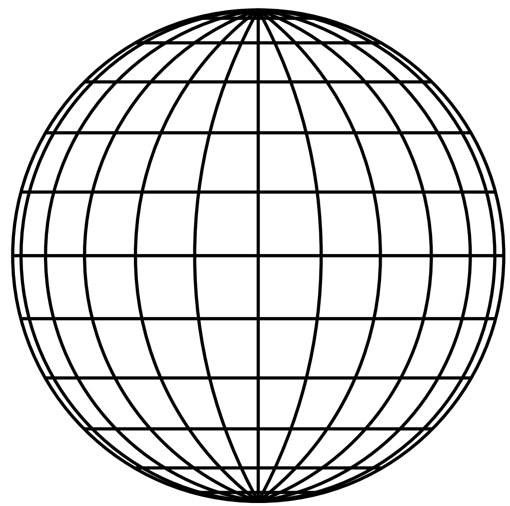
Coordinate Systems for Geoinformation

Earth-centered Cartesian coordinates

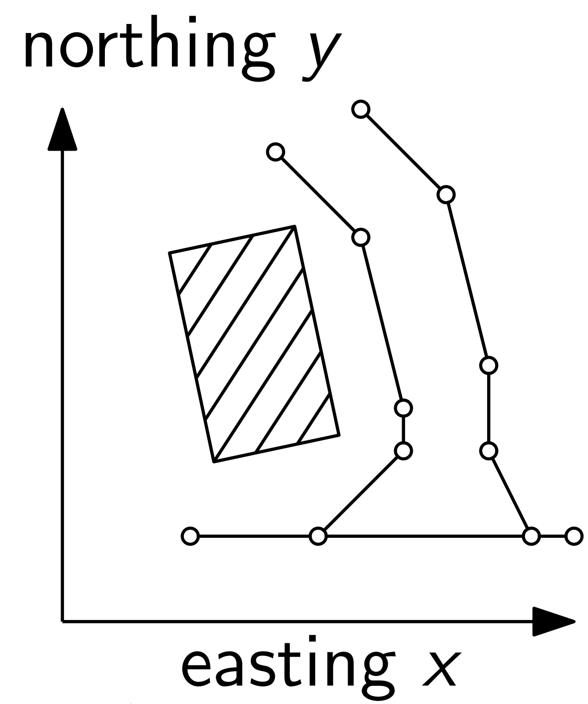


spherical/ellipsoidal coordinates

latitude φ, longitude λ



projected coordinates



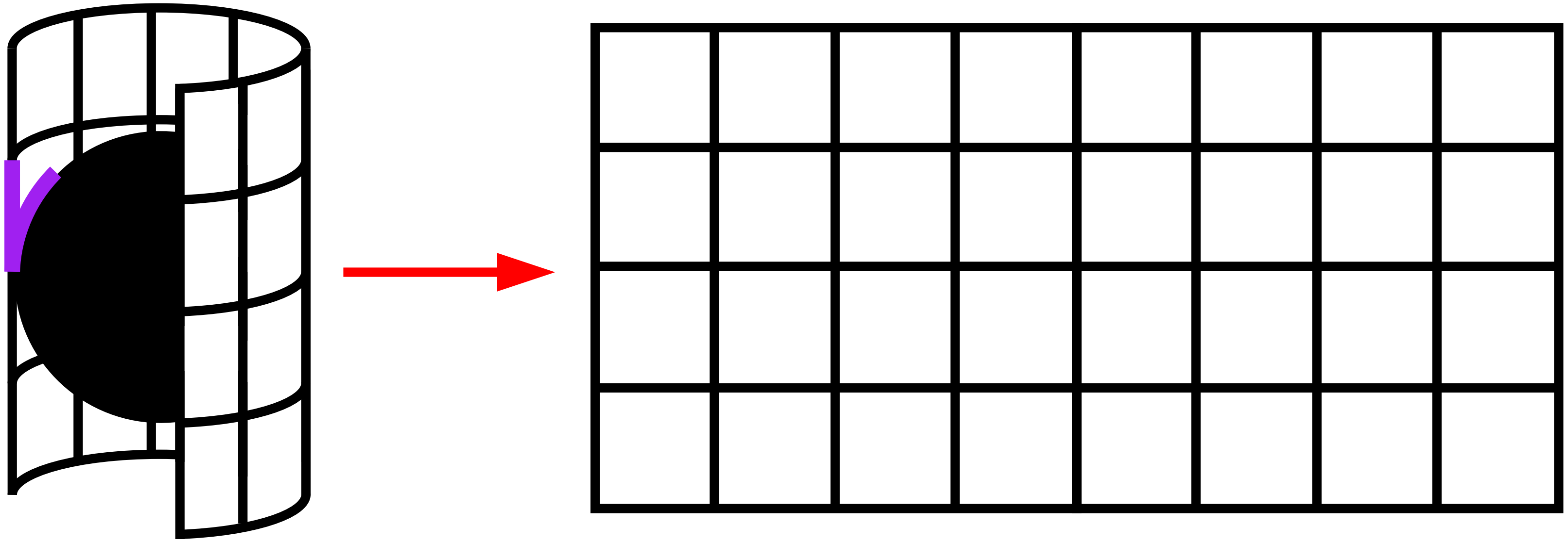
$$\begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix} = \begin{pmatrix} r \cos\varphi_p \cos\lambda_p \\ r \cos\varphi_p \sin\lambda_p \\ r \sin\varphi_p \end{pmatrix}$$

$$\begin{pmatrix} x_p \\ y_p \end{pmatrix} = \begin{pmatrix} r\lambda_p \\ r\varphi_p \end{pmatrix}$$

for spherical coordinates and plate carrée

Cylindrical Projections

- plate carrée ($x = r\lambda$, $y = r\varphi$) is *one* example



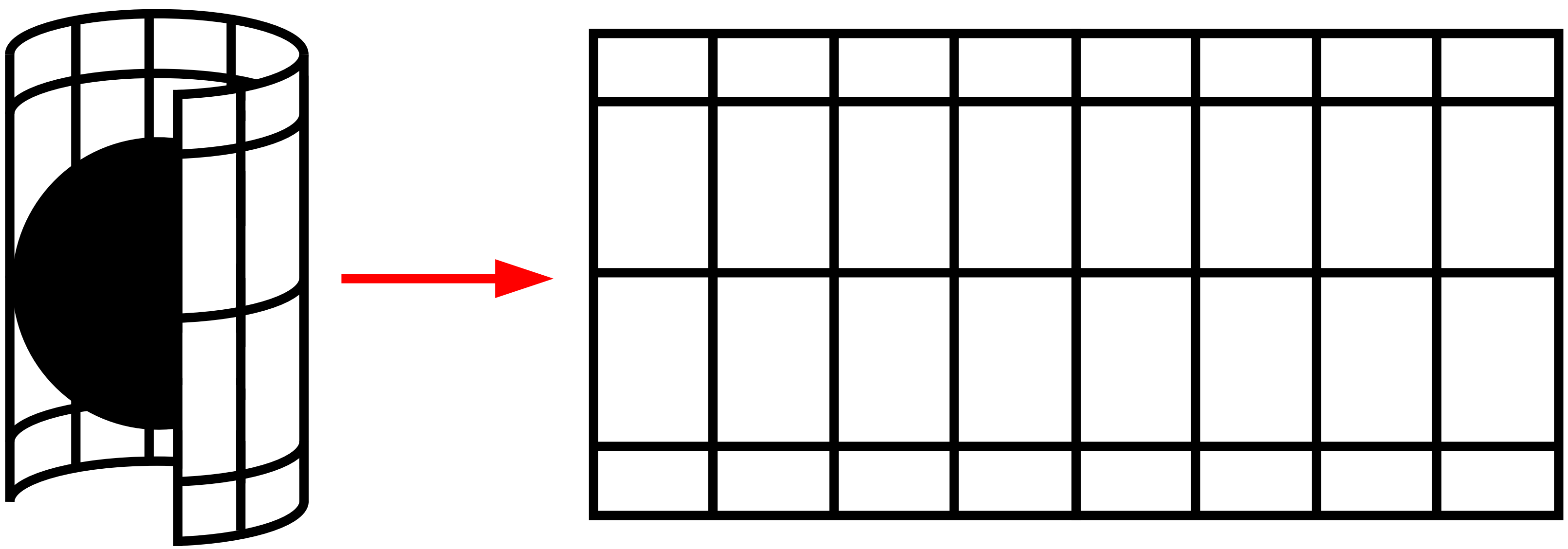
Cylindrical Projections

- plate carrée ($x = r\lambda$, $y = r\varphi$) is *one* example

For **all** cylindrical projections in normal position:

- Lines of equal latitude are mapped to horizontal lines.
- Meridians are mapped to vertical lines.
- The equator is scaled with a constant factor (often with 1).

The distances between lines of equal latitude can be nonuniform.



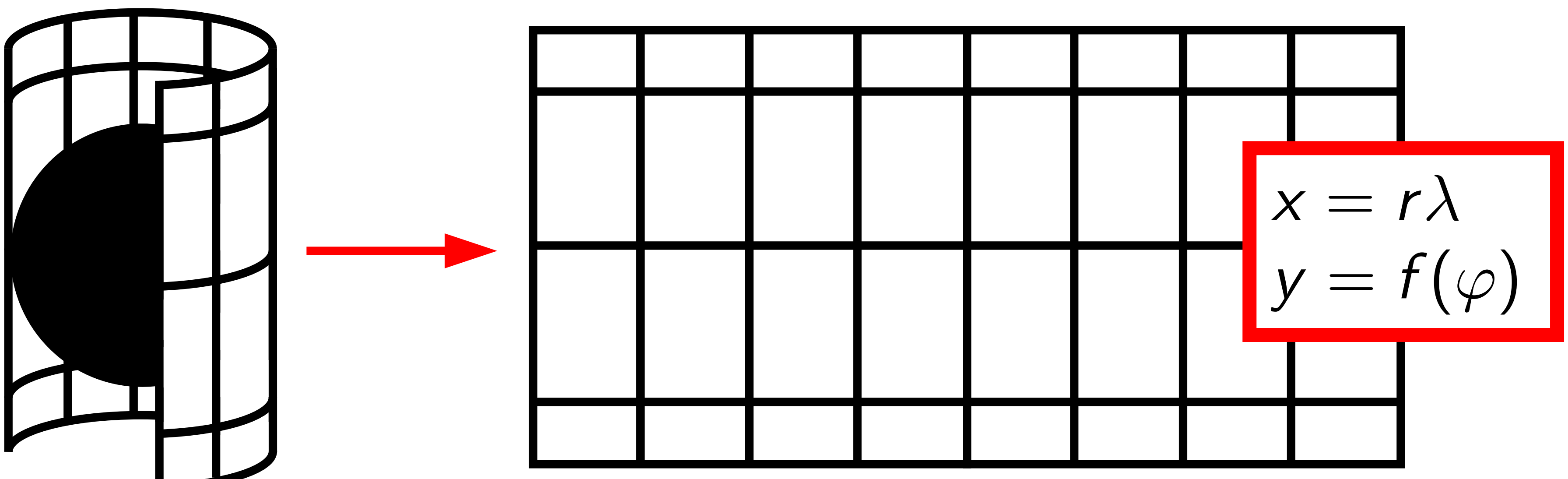
Cylindrical Projections

- plate carrée ($x = r\lambda$, $y = r\varphi$) is *one* example

For **all** cylindrical projections in normal position:

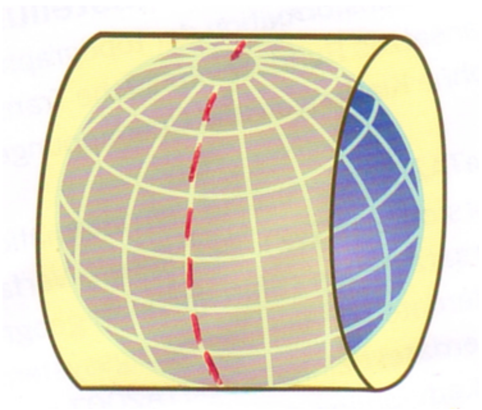
- Lines of equal latitude are mapped to horizontal lines.
- Meridians are mapped to vertical lines.
- The equator is scaled with a constant factor (often with 1).

The distances between lines of equal latitude can be nonuniform.



Cylindrical Projections

- For *transverse* cylindrical projections, consider one meridian as the Earth's equator.



source: Spata (2010)

Mercator Projection

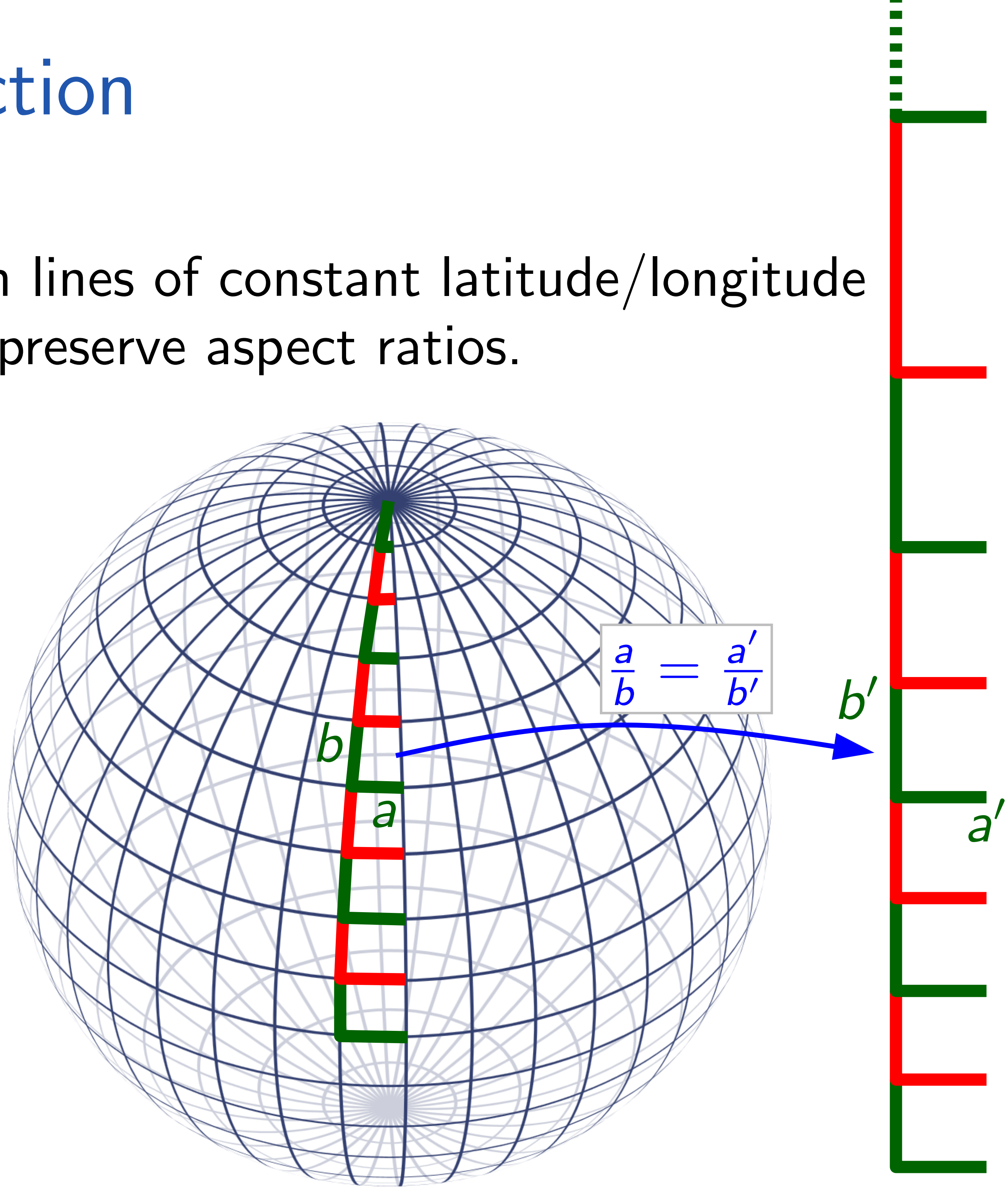
- arguably, the most important cylindrical projection

Gerhard Krämer
(1512-1594)

Mercator Projection

Idea:

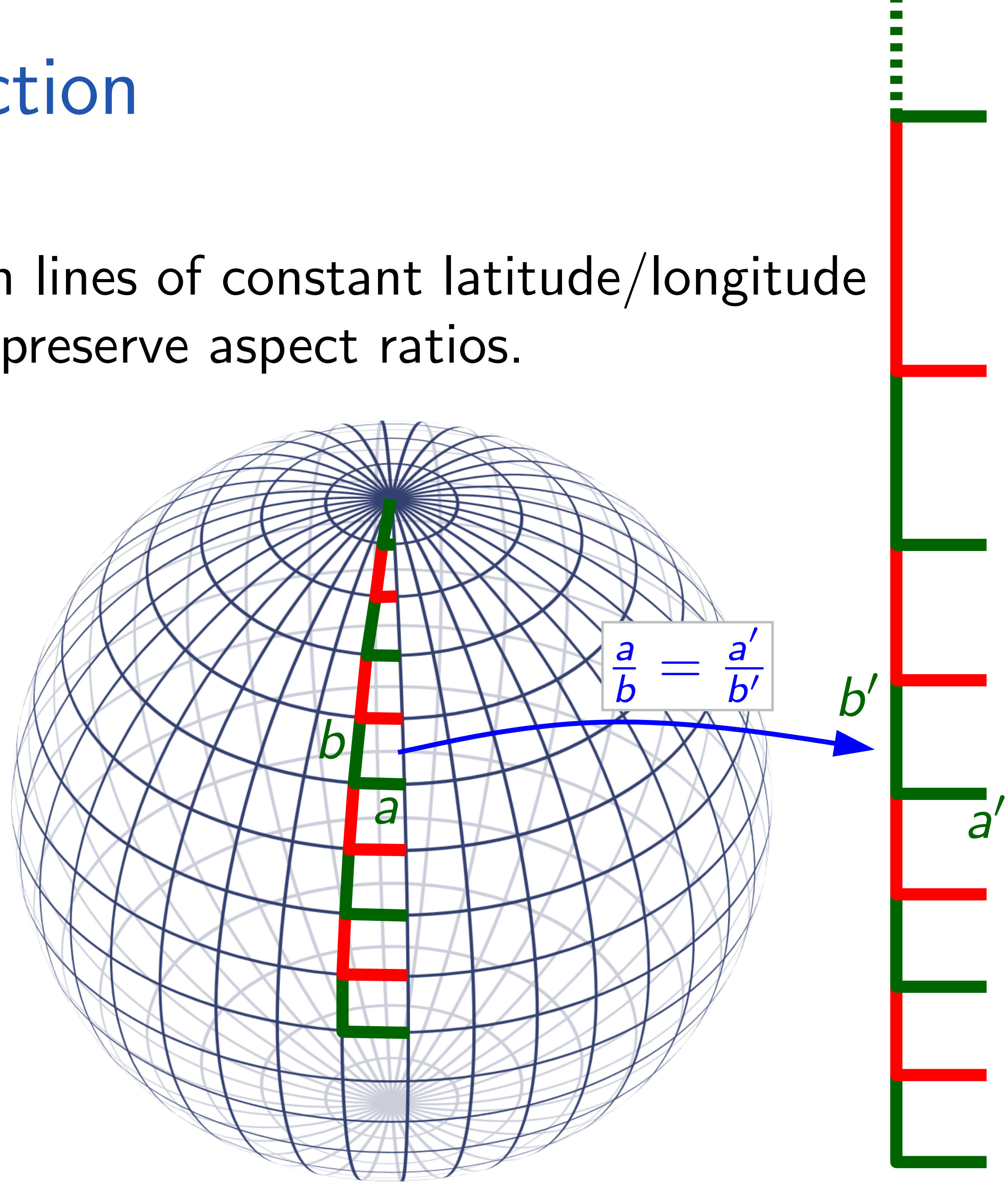
- Map areas between lines of constant latitude/longitude to rectangles that preserve aspect ratios.



Mercator Projection

Idea:

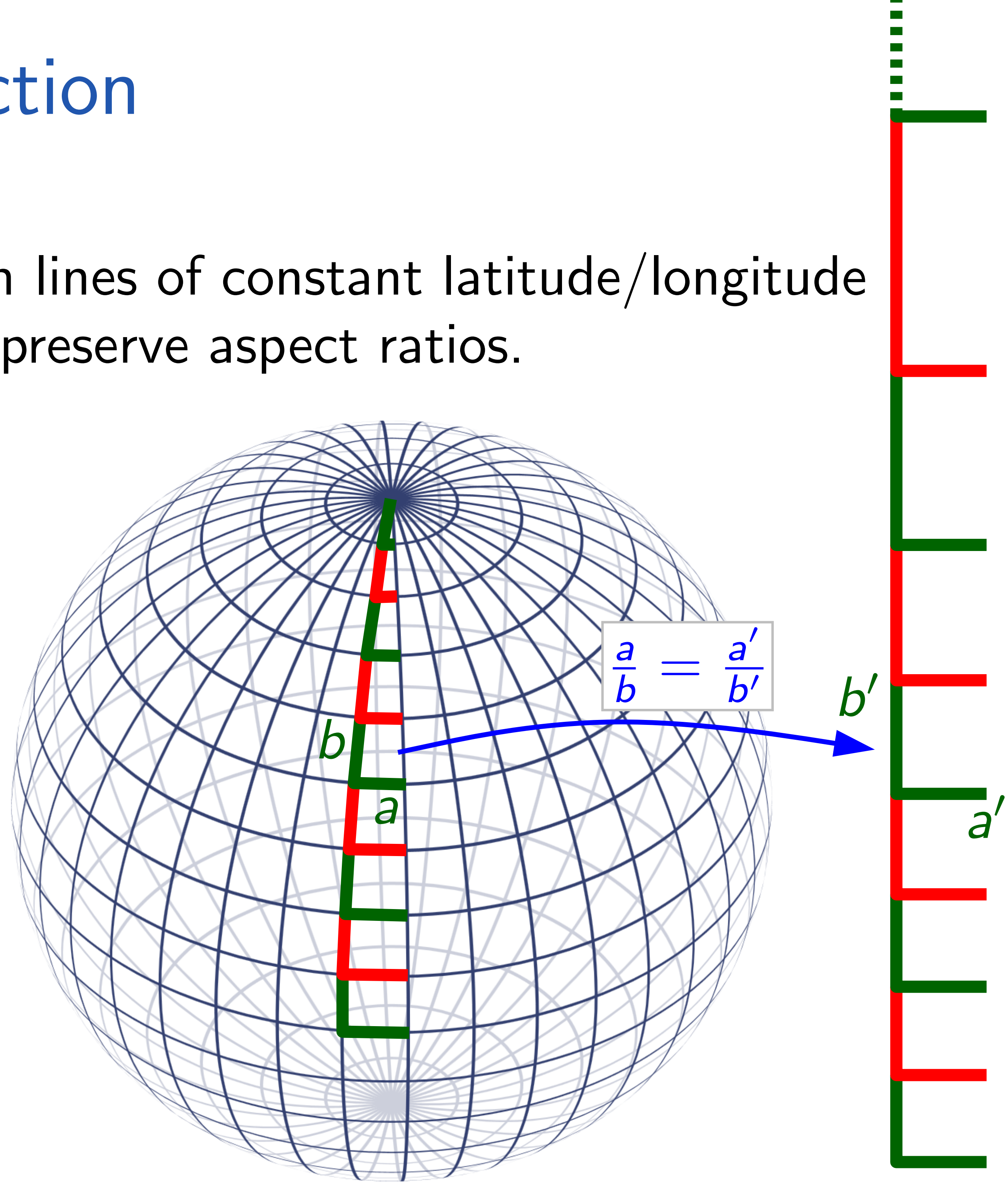
- Map areas between lines of constant latitude/longitude to rectangles that preserve aspect ratios.
- here shown for $\Delta\lambda = \Delta\varphi = 10^\circ$



Mercator Projection

Idea:

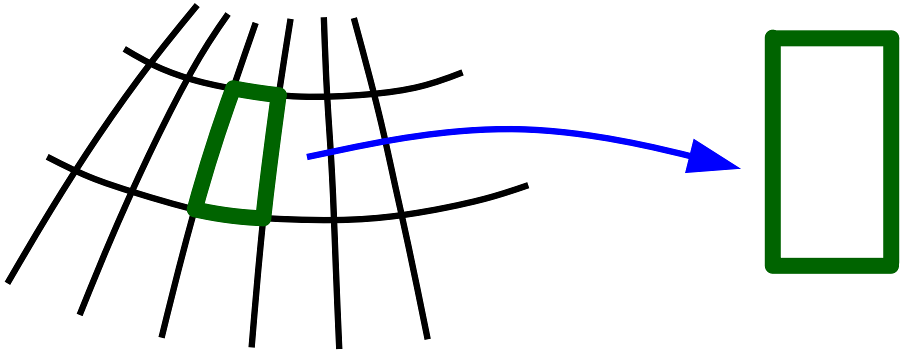
- Map areas between lines of constant latitude/longitude to rectangles that preserve aspect ratios.
- here shown for $\Delta\lambda = \Delta\varphi = 10^\circ$
- For an exact construction, choose $\Delta\lambda, \Delta\varphi$ infinitely small.



Mercator Projection

Advantage:

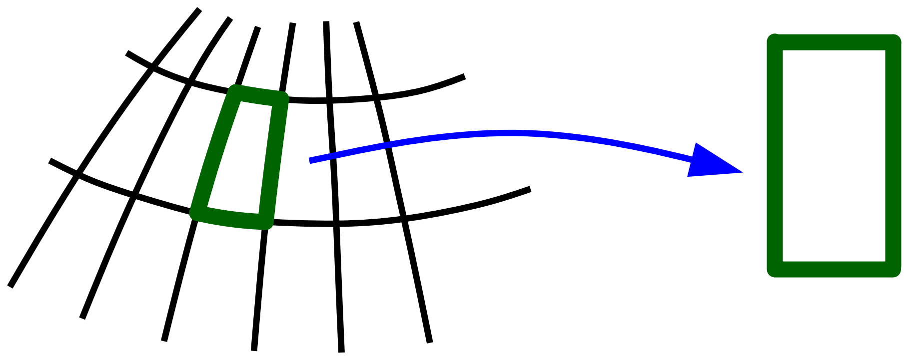
- Aspect ratios are preserved.



Mercator Projection

Advantage:

- Aspect ratios are preserved.

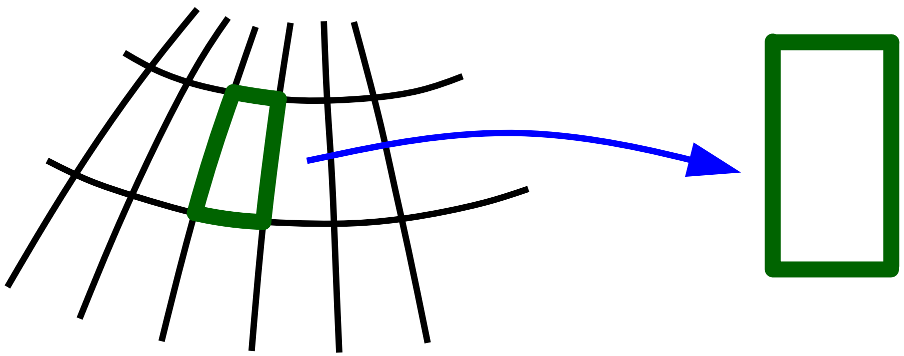


- This implies that angles are preserved!
- Thus, the Mercator projection is conformal.

Mercator Projection

Advantage:

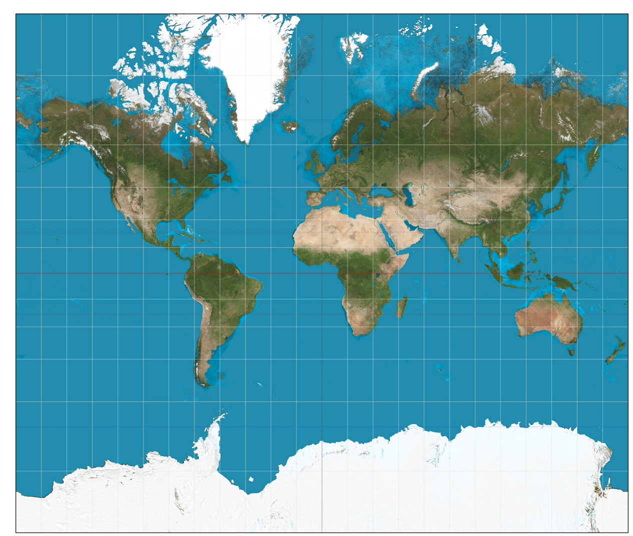
- Aspect ratios are preserved.



- This implies that angles are preserved!
- Thus, the Mercator projection is conformal.

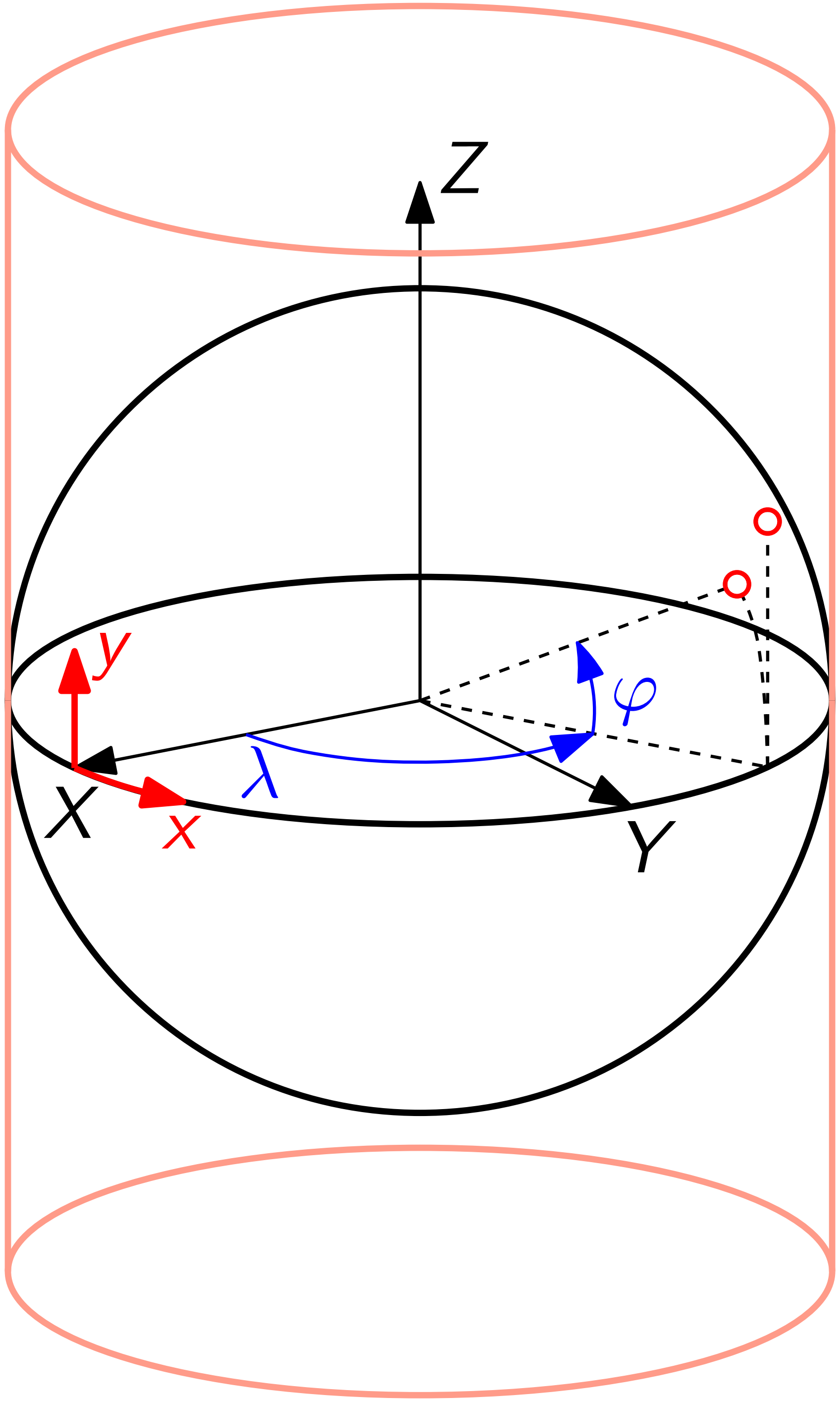
Disadvantages:

- Areas are distorted (esp. close to poles).
- Lengths of meridians are not preserved.

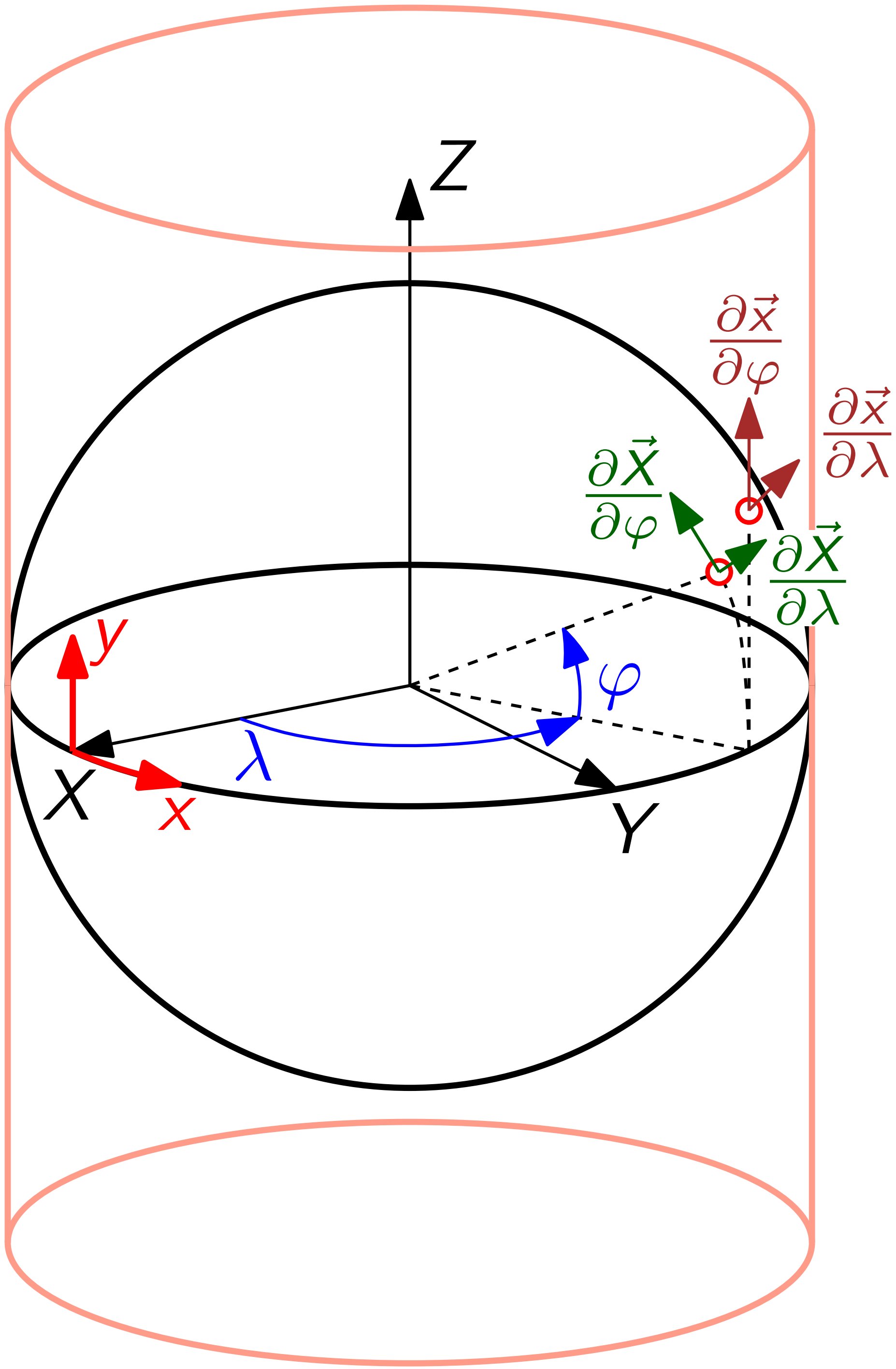


source: Wikipedia

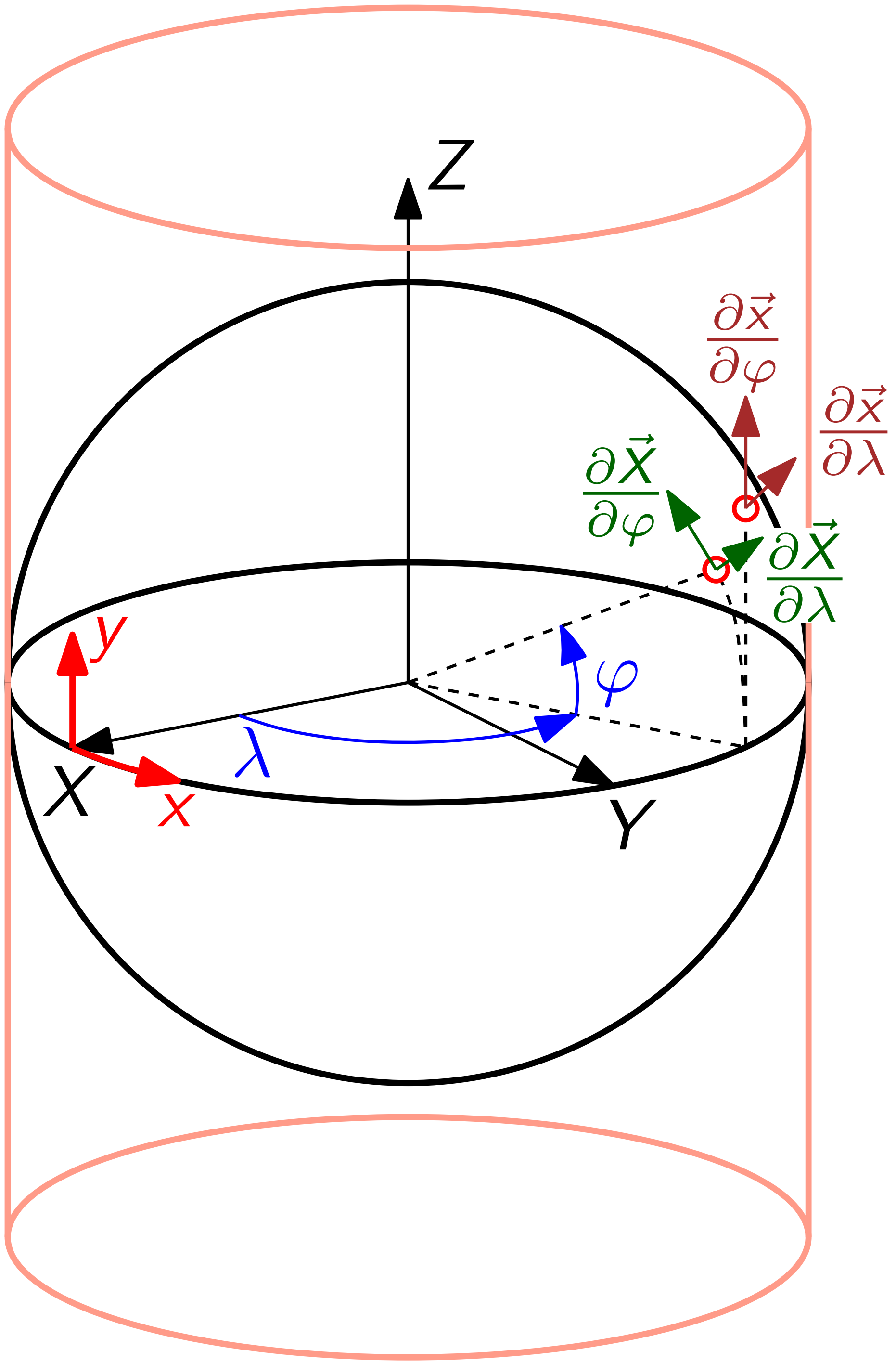
Mercator Projection – Maths



Mercator Projection – Maths



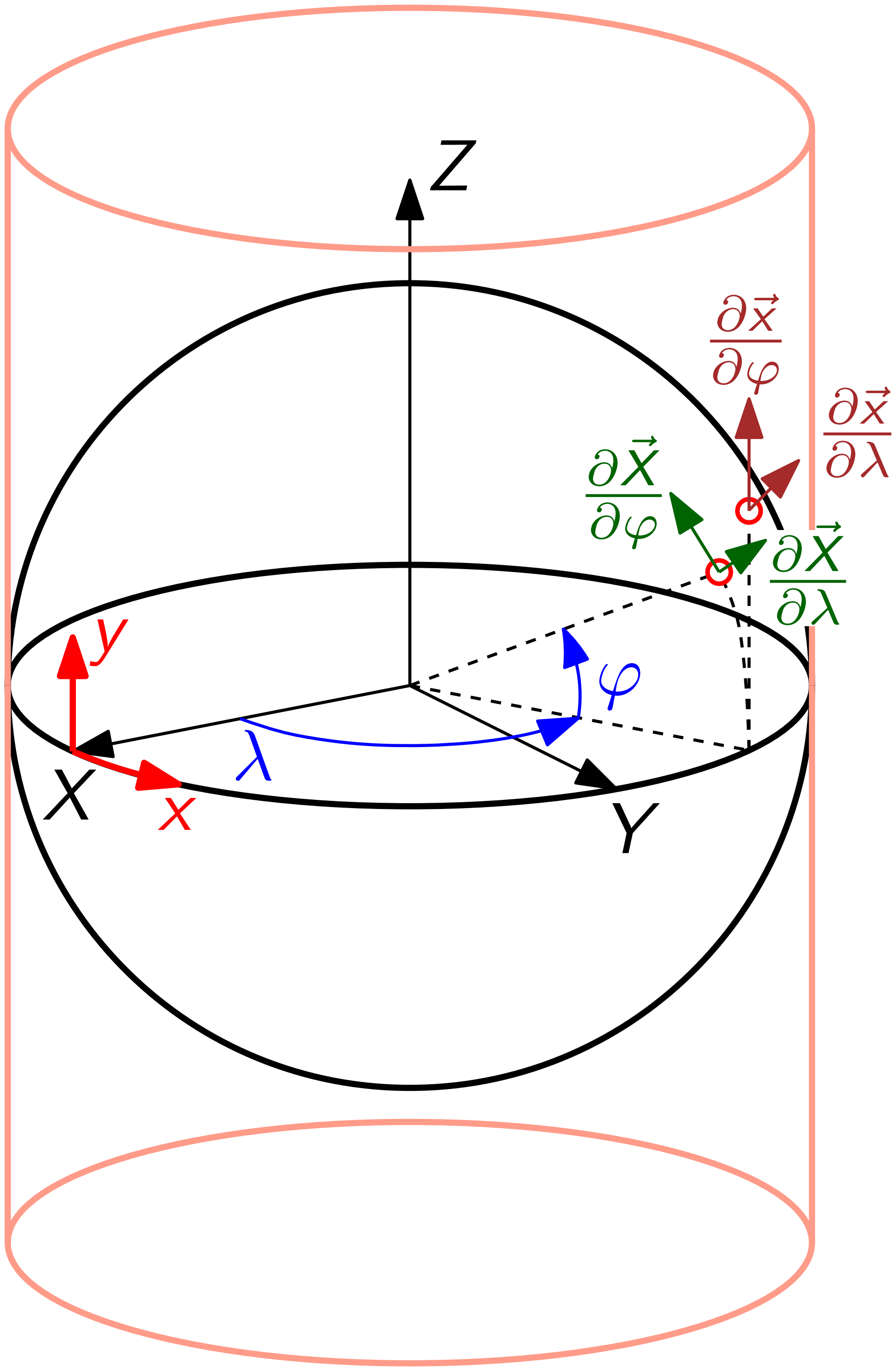
Mercator Projection – Maths



Requirement:
same aspect ratios

$$\frac{\left|\frac{\partial \vec{x}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{x}}{\partial \lambda}\right|} = \frac{\left|\frac{\partial \vec{X}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{X}}{\partial \lambda}\right|}$$

Mercator Projection – Maths



Requirement:
same aspect ratios

$$\frac{\left|\frac{\partial \vec{x}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{x}}{\partial \lambda}\right|} = \frac{\left|\frac{\partial \vec{X}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{X}}{\partial \lambda}\right|}$$

$$\Leftrightarrow \frac{\left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2} = \frac{\left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2}$$

Mercator Projection – Maths

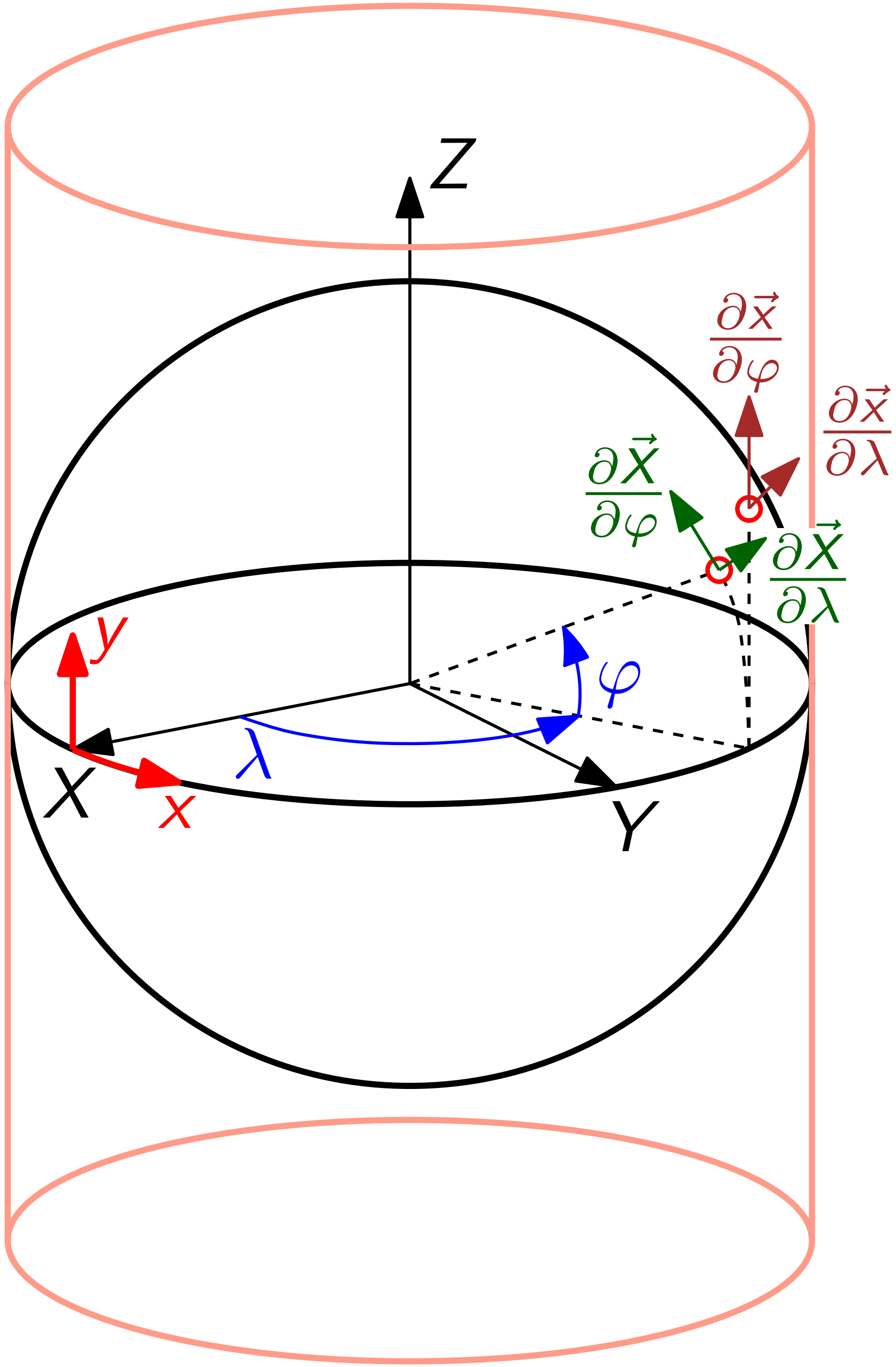
$$\left(\frac{\partial\vec{x}}{\partial\varphi}\right)^2 = \begin{pmatrix} 0 \\ f'(\varphi) \end{pmatrix}^2 = (f'(\varphi))^2$$

$$\left(\frac{\partial\vec{x}}{\partial\lambda}\right)^2 = \begin{pmatrix} r \\ 0 \end{pmatrix}^2 = r^2$$

$$\left(\frac{\partial\vec{X}}{\partial\varphi}\right)^2 = \begin{pmatrix} -r\sin\varphi\cos\lambda \\ -r\sin\varphi\sin\lambda \\ r\cos\varphi \end{pmatrix}^2 = r^2$$

$$\left(\frac{\partial\vec{X}}{\partial\lambda}\right)^2 = \begin{pmatrix} -r\cos\varphi\sin\lambda \\ r\cos\varphi\cos\lambda \\ 0 \end{pmatrix}^2 = r^2\cos^2\varphi$$

Mercator Projection – Maths

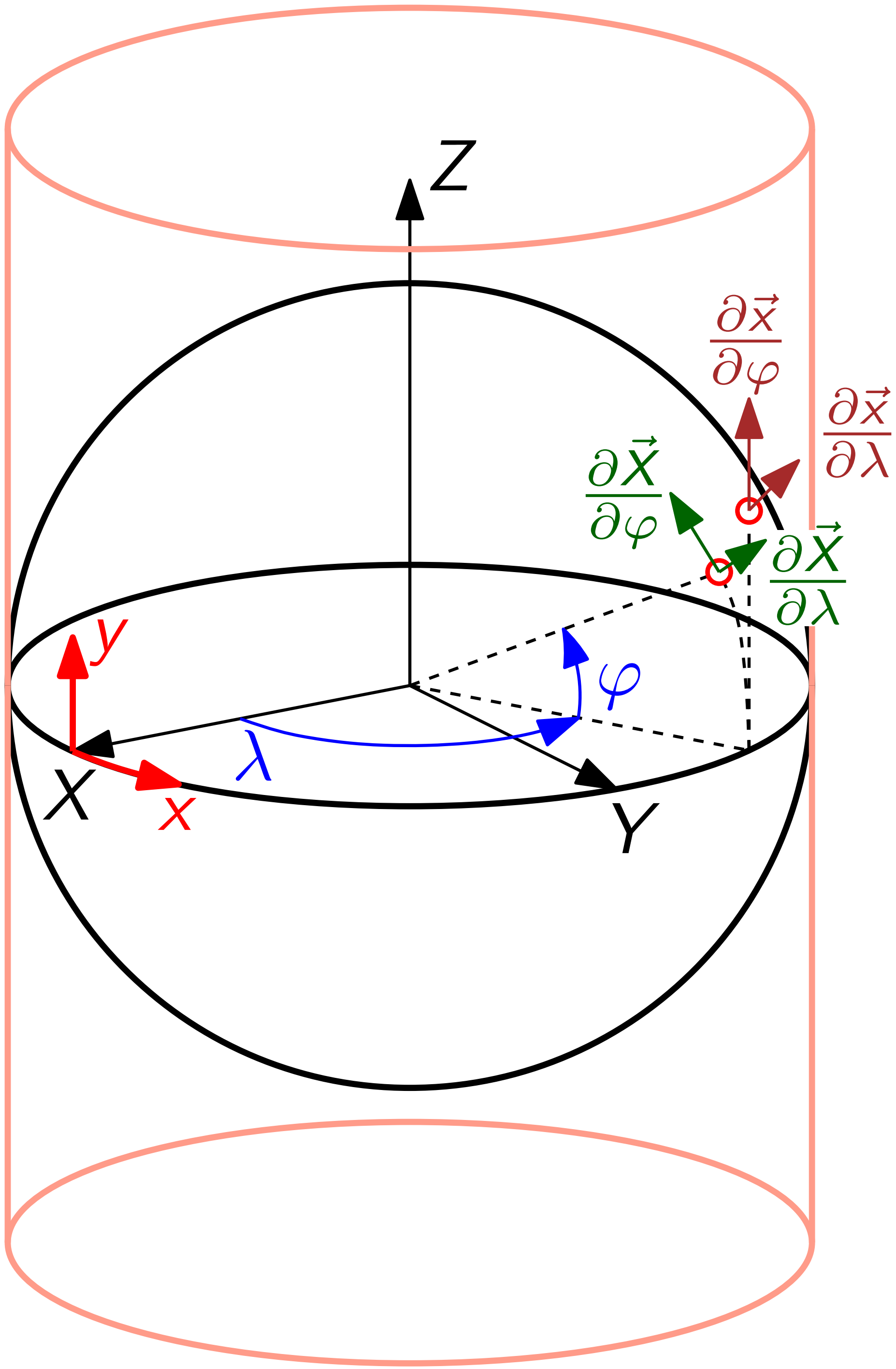


Requirement:
same aspect ratios

$$\frac{\left|\frac{\partial \vec{x}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{x}}{\partial \lambda}\right|} = \frac{\left|\frac{\partial \vec{X}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{X}}{\partial \lambda}\right|}$$

$$\Leftrightarrow \frac{\left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2} = \frac{\left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2}$$

Mercator Projection – Maths



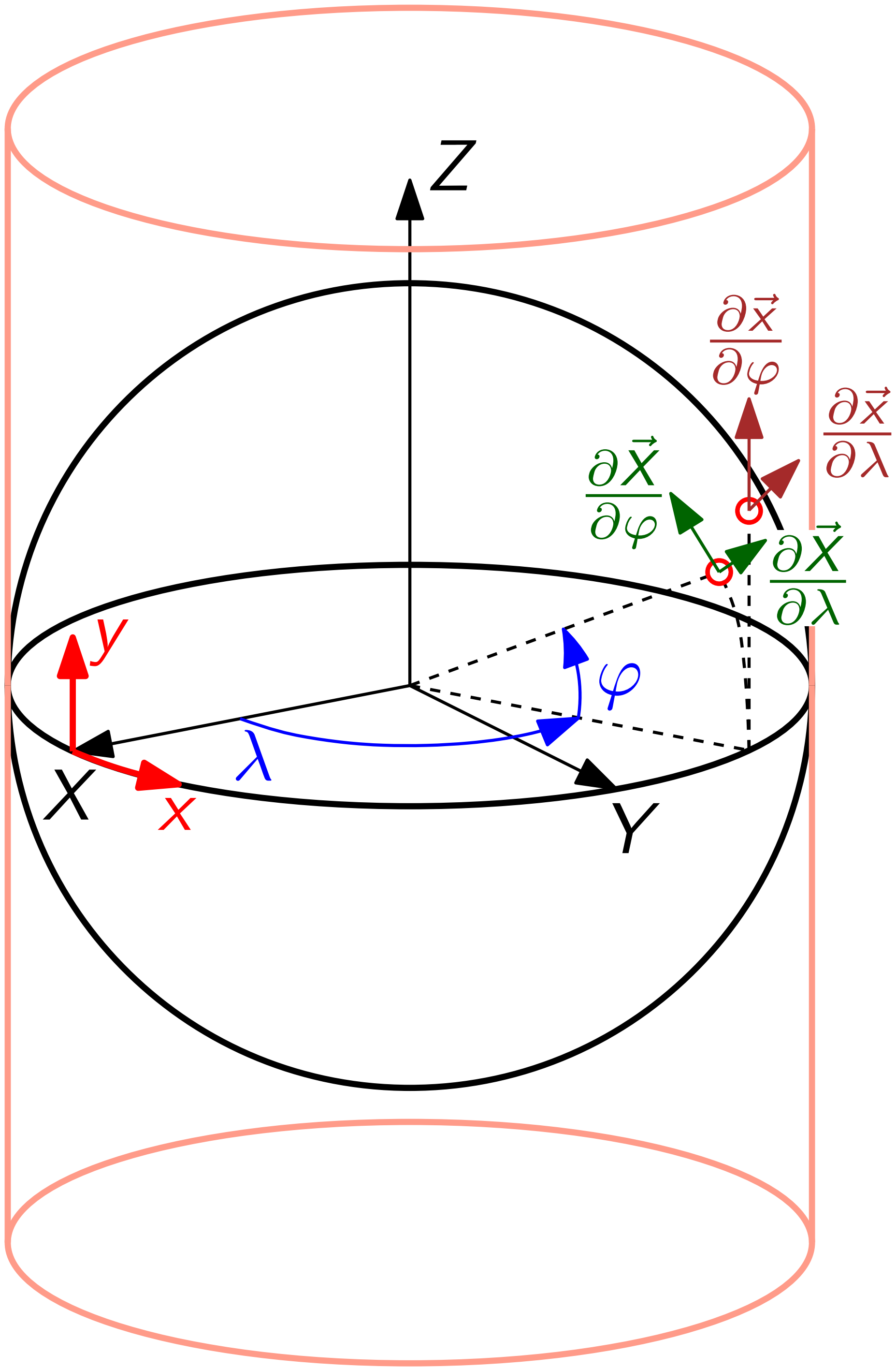
Requirement:
same aspect ratios

$$\frac{\left|\frac{\partial\vec{x}}{\partial\varphi}\right|}{\left|\frac{\partial\vec{x}}{\partial\lambda}\right|} = \frac{\left|\frac{\partial\vec{X}}{\partial\varphi}\right|}{\left|\frac{\partial\vec{X}}{\partial\lambda}\right|}$$

$$\Leftrightarrow \frac{\left(\frac{\partial\vec{x}}{\partial\varphi}\right)^2}{\left(\frac{\partial\vec{x}}{\partial\lambda}\right)^2} = \frac{\left(\frac{\partial\vec{X}}{\partial\varphi}\right)^2}{\left(\frac{\partial\vec{X}}{\partial\lambda}\right)^2}$$

$$\Leftrightarrow \frac{\left(f'(\varphi)\right)^2}{r^2} = \frac{r^2}{r^2\cos^2\varphi}$$

Mercator Projection – Maths



Requirement:
same aspect ratios

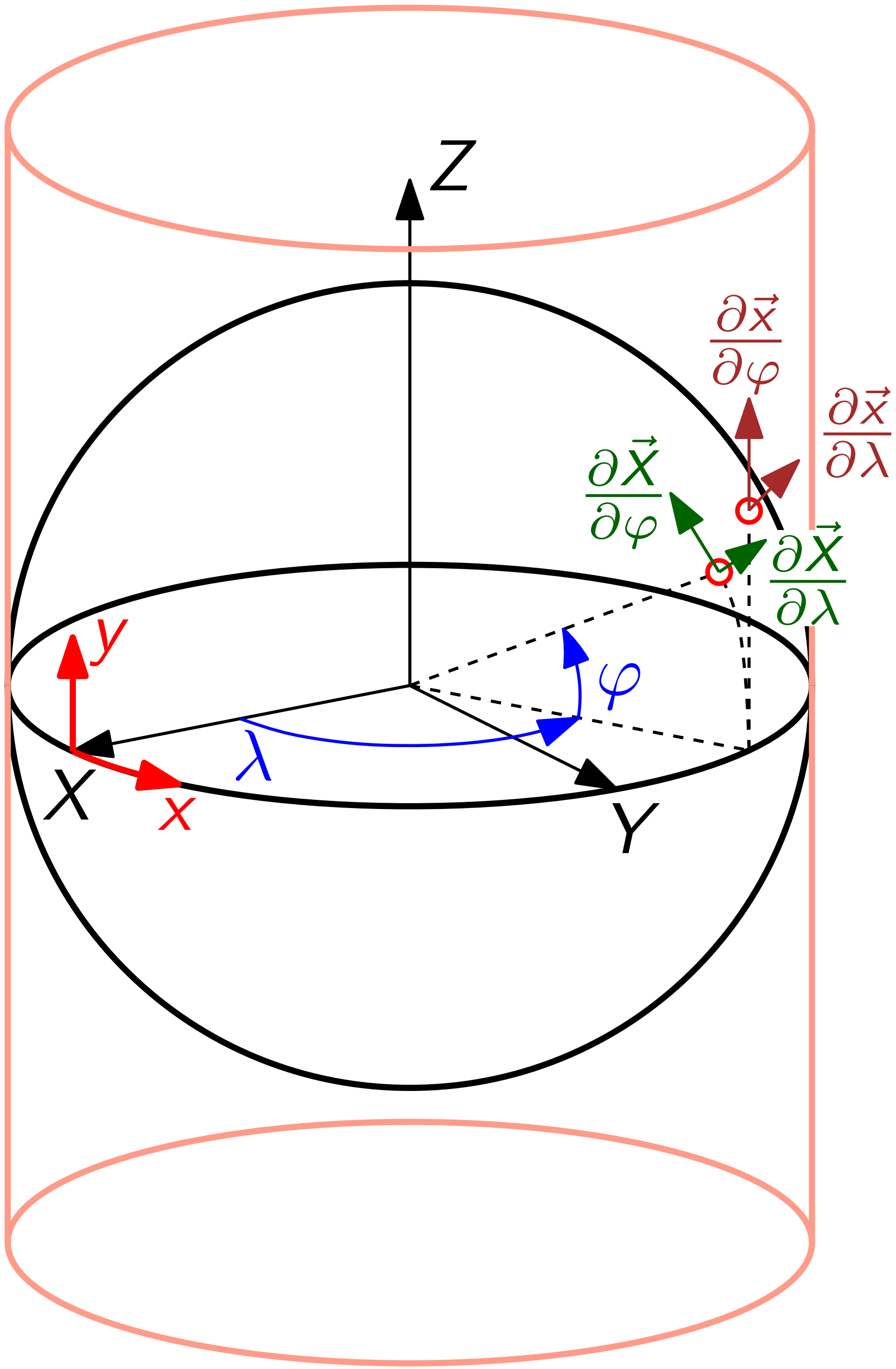
$$\frac{\left|\frac{\partial \vec{x}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{x}}{\partial \lambda}\right|} = \frac{\left|\frac{\partial \vec{X}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{X}}{\partial \lambda}\right|}$$

$$\Leftrightarrow \frac{\left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2} = \frac{\left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2}$$

$$\Leftrightarrow \frac{\left(f'(\varphi)\right)^2}{r^2} = \frac{r^2}{r^2 \cos^2 \varphi}$$

$$\Leftrightarrow f'(\varphi) = \frac{r}{\cos \varphi}$$

Mercator Projection – Maths



Requirement:
same aspect ratios

$$\frac{\left|\frac{\partial \vec{x}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{x}}{\partial \lambda}\right|} = \frac{\left|\frac{\partial \vec{X}}{\partial \varphi}\right|}{\left|\frac{\partial \vec{X}}{\partial \lambda}\right|}$$

$$\Leftrightarrow \frac{\left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2} = \frac{\left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2}{\left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2}$$

$$\Leftrightarrow \frac{\left(f'(\varphi)\right)^2}{r^2} = \frac{r^2}{r^2 \cos^2 \varphi}$$

$$\Leftrightarrow f'(\varphi) = \frac{r}{\cos \varphi}$$

$$\Leftrightarrow f(\varphi) = r \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

$f(0) = 0$

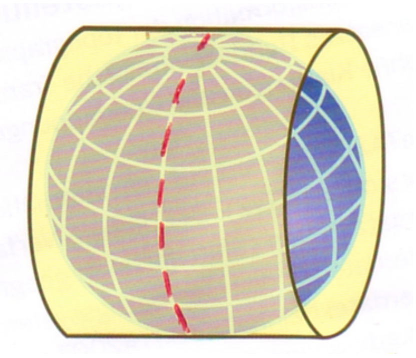
Mercator Projection

Mapping function:

$$x_p = r\lambda_p$$
$$y_p = r \ln \tan\left(\frac{\pi}{4} + \frac{\varphi_p}{2}\right)$$

r = Earth's radius

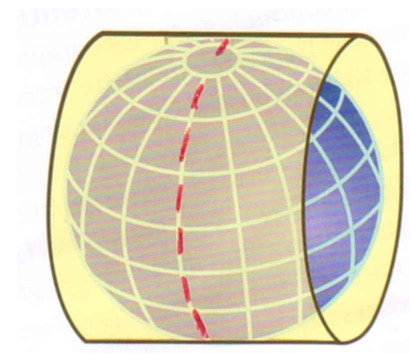
Mercator Projection



- often the transverse version is used; central meridian can be chosen arbitrarily.

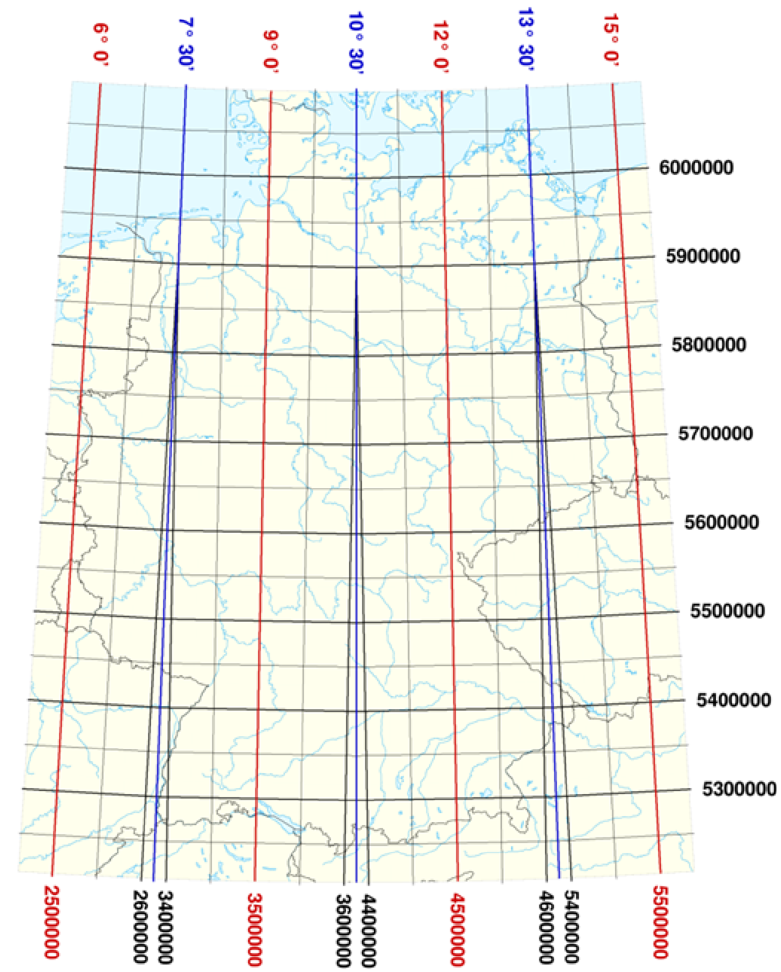
Mercator Projection

- often the transverse version is used; central meridian can be chosen arbitrarily.



Gauß Krüger projection:

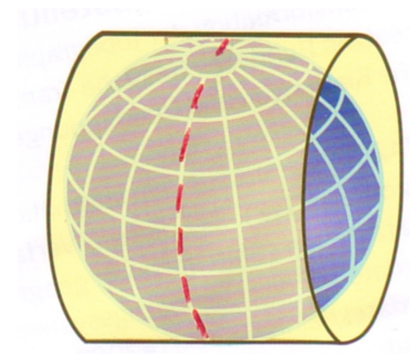
- Earth is subdivided into zones of width 3°.
- For different zones, different projections are used (central meridian = center of zone)



Quelle: Wikipedia

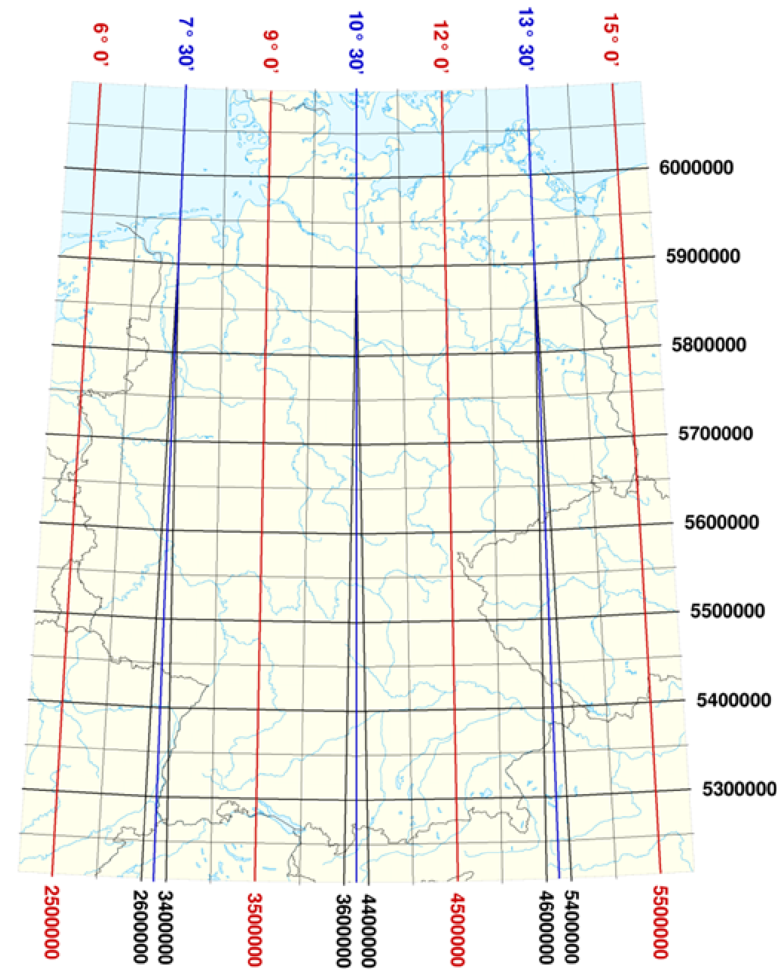
Mercator Projection

- often the transverse version is used;
  central meridian can be chosen arbitrarily.



Gauß Krüger projection:

- Earth is subdivided into zones of width 3°.
- For different zones, different projections are used (central meridian = center of zone)
- Usage: official surveys in Germany until 1991, then replaced by UTM

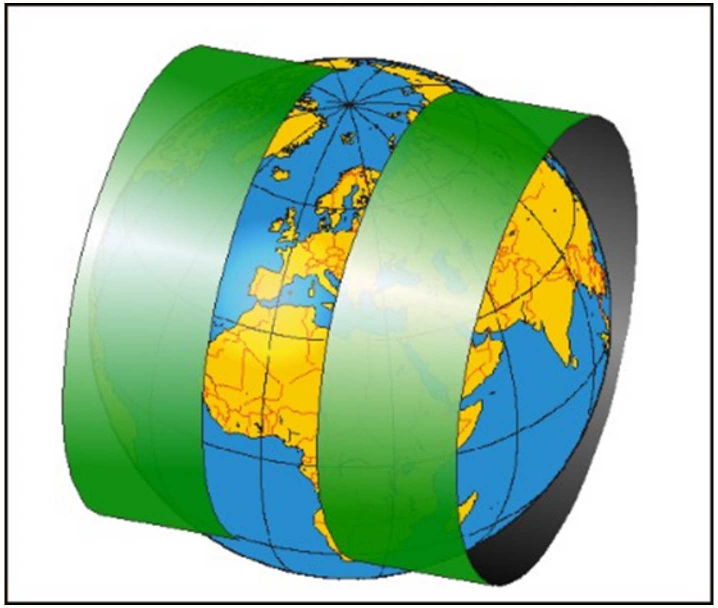


Quelle: Wikipedia

Mercator Projection

Universal Transverse Mercator (UTM):

- Similar to Gauß Krüger
- width of zone 6°
- scale = 0.9996



Quelle: Wikipedia

Mercator Projection

Mapping function:

$$x_p = r\lambda_p$$
$$y_p = r \ln \tan\left(\frac{\pi}{4} + \frac{\varphi_p}{2}\right)$$

$r =$ Earth's radius

Mercator Projection

Mapping function:

$$x_p = r\lambda_p$$
$$y_p = r \ln \tan\left(\frac{\pi}{4} + \frac{\varphi_p}{2}\right)$$

r = Earth's radius

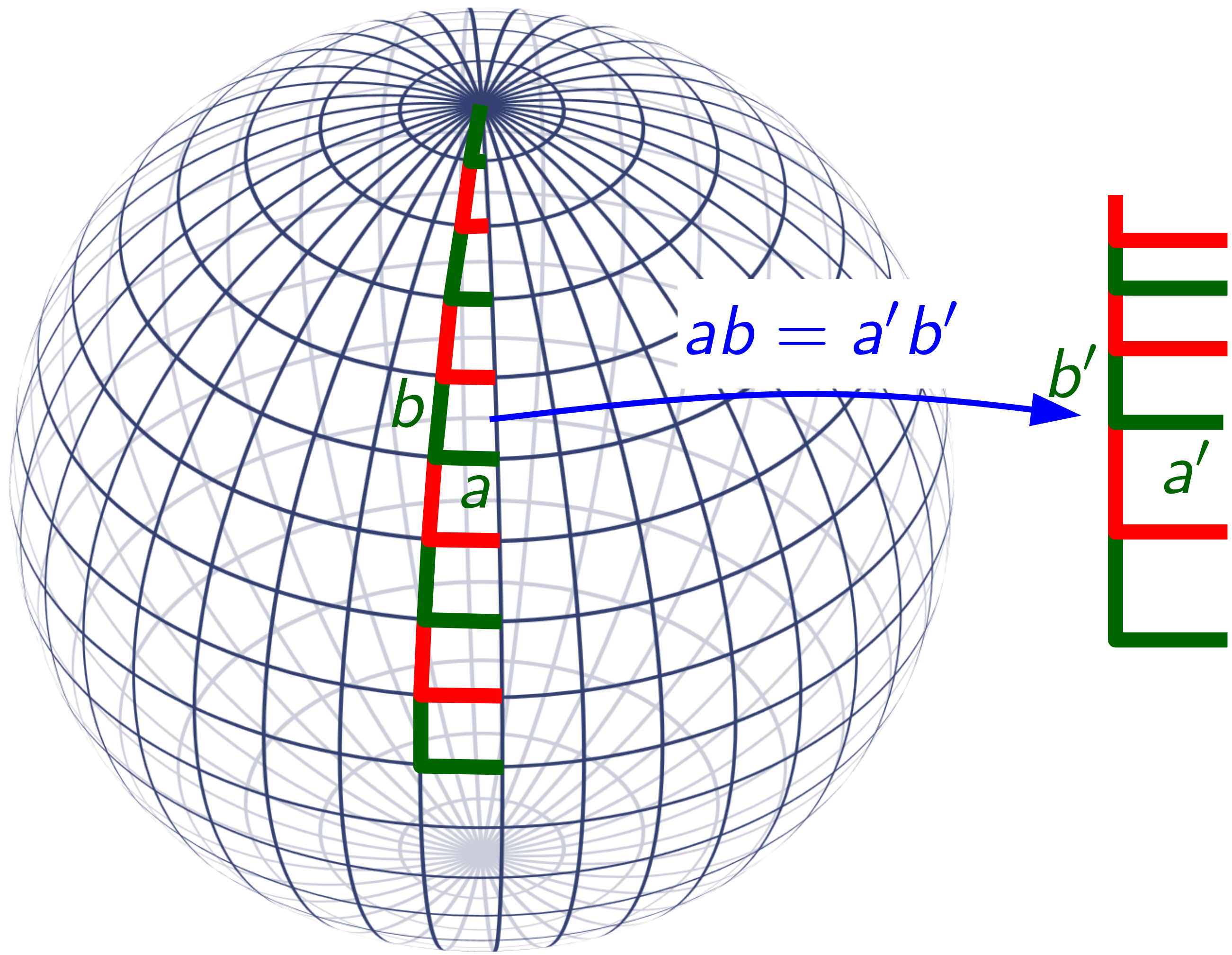
Conclusion:

- Mercator projection is not appropriate for world maps.
- Frequently used for smaller zones
  - close to equator or
  - close to a central meridian (transverse version).

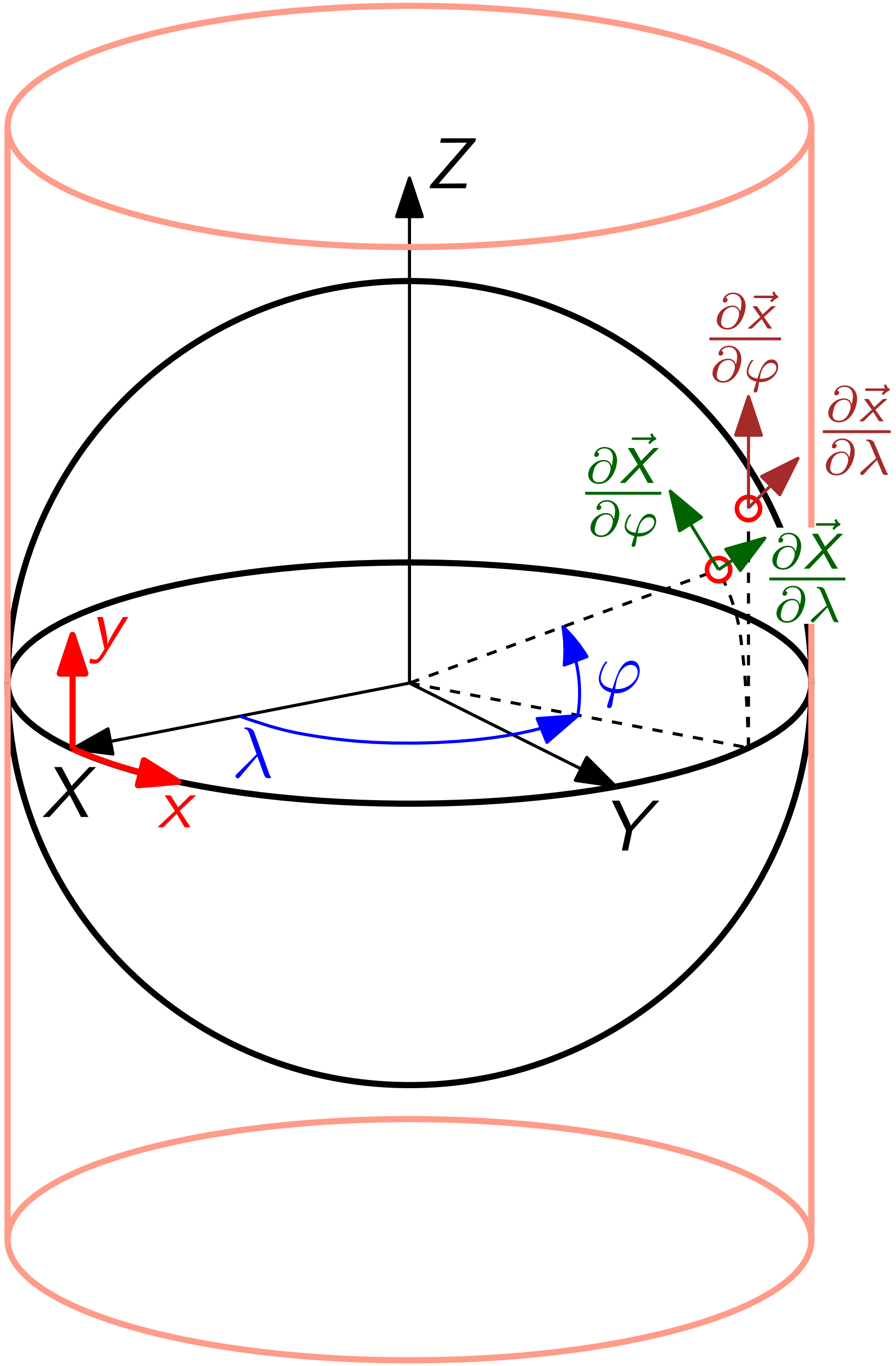
Cylindrical Equal-area Projection – Idea

Idea:

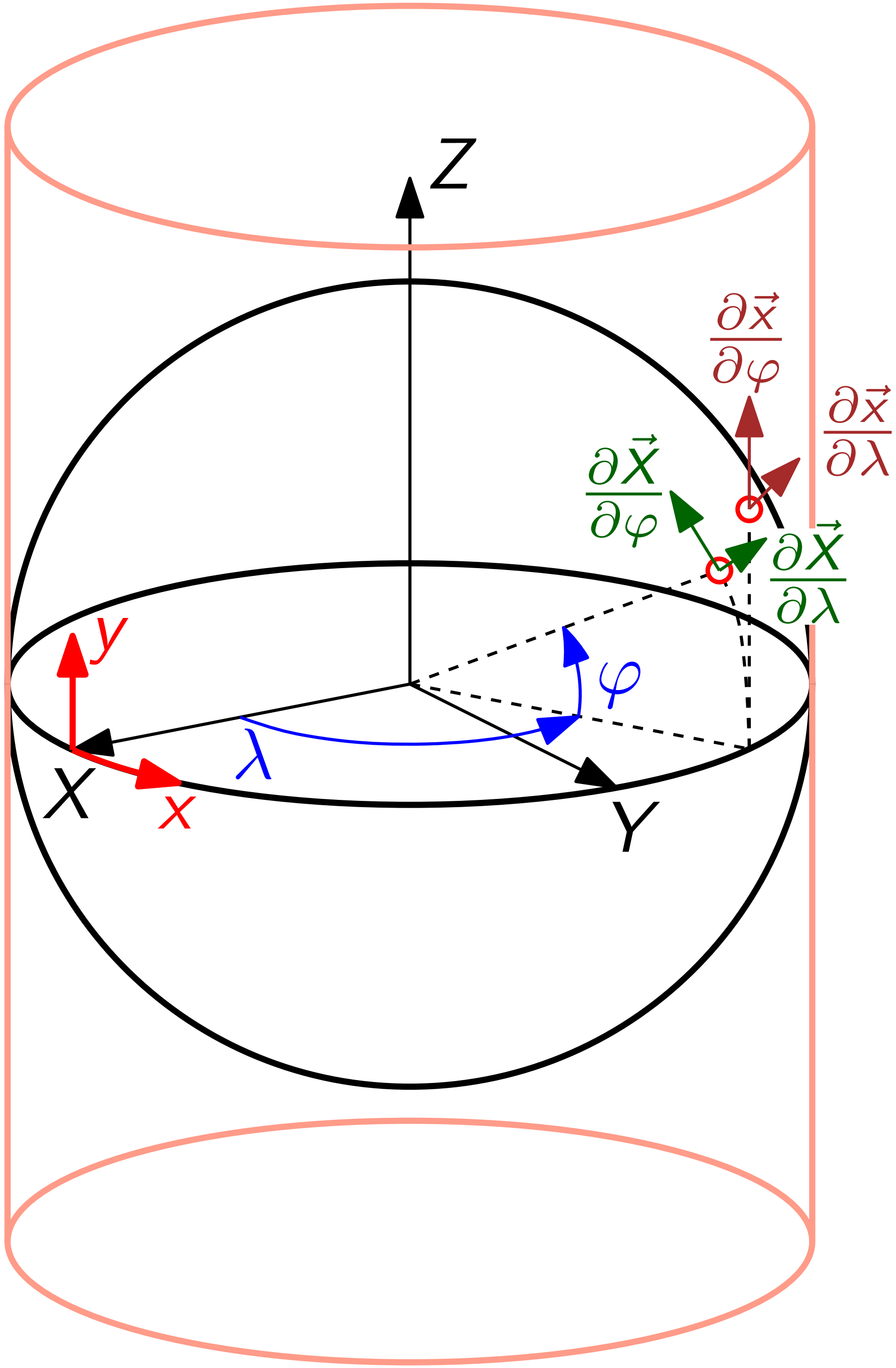
- Map areas between lines of constant latitude/longitude to rectangles of the same sizes.
- here shown for $\Delta\lambda = \Delta\varphi = 10^\circ$
- For an exact construction, choose $\Delta\lambda, \Delta\varphi$ infinitely small.



Cylindrical Equal-area Projection – Maths



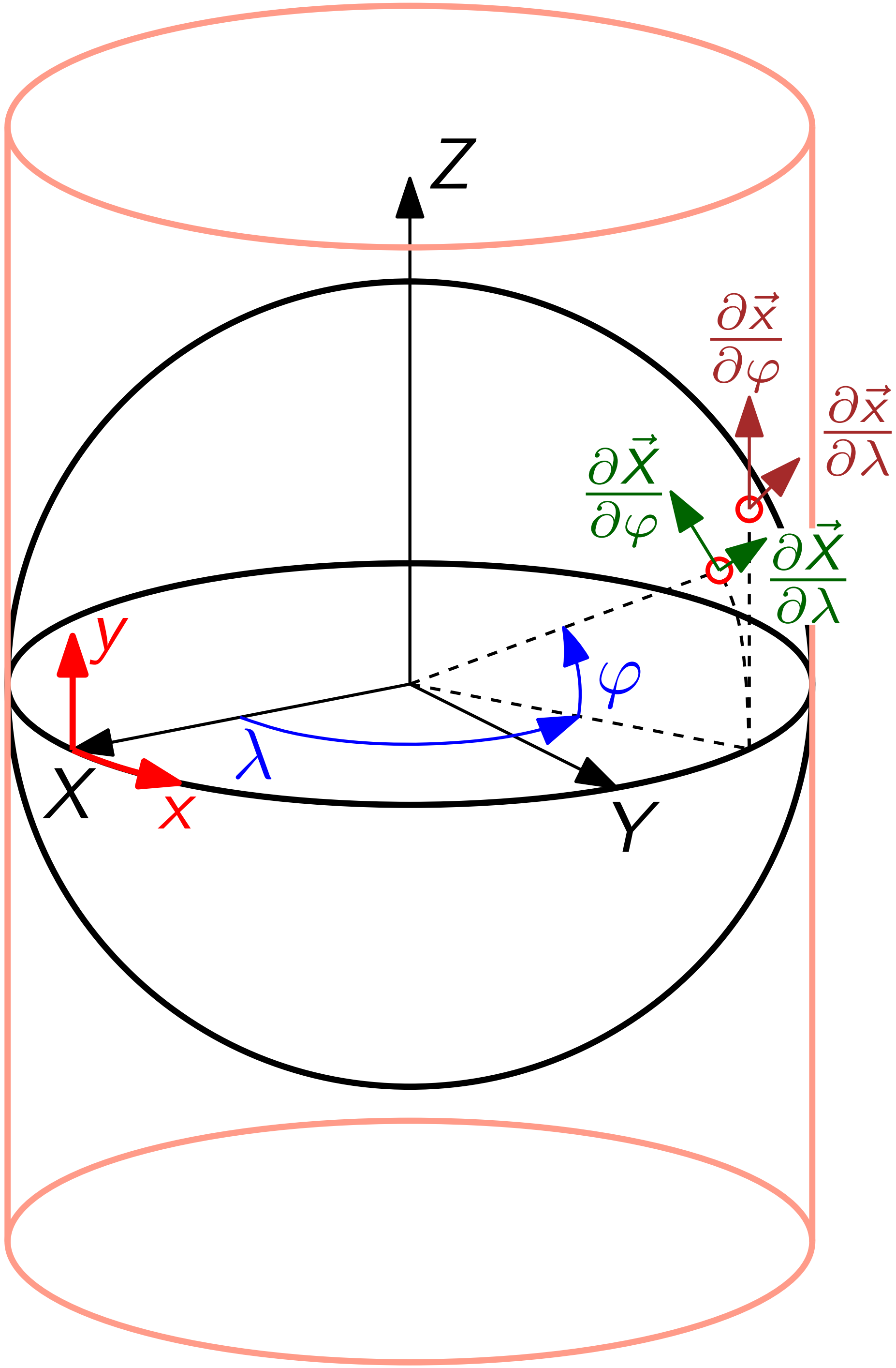
Cylindrical Equal-area Projection – Maths



Requirement:
same sizes

$$\left|\frac{\partial \vec{x}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{x}}{\partial \lambda}\right| = \left|\frac{\partial \vec{X}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{X}}{\partial \lambda}\right|$$

Cylindrical Equal-area Projection – Maths

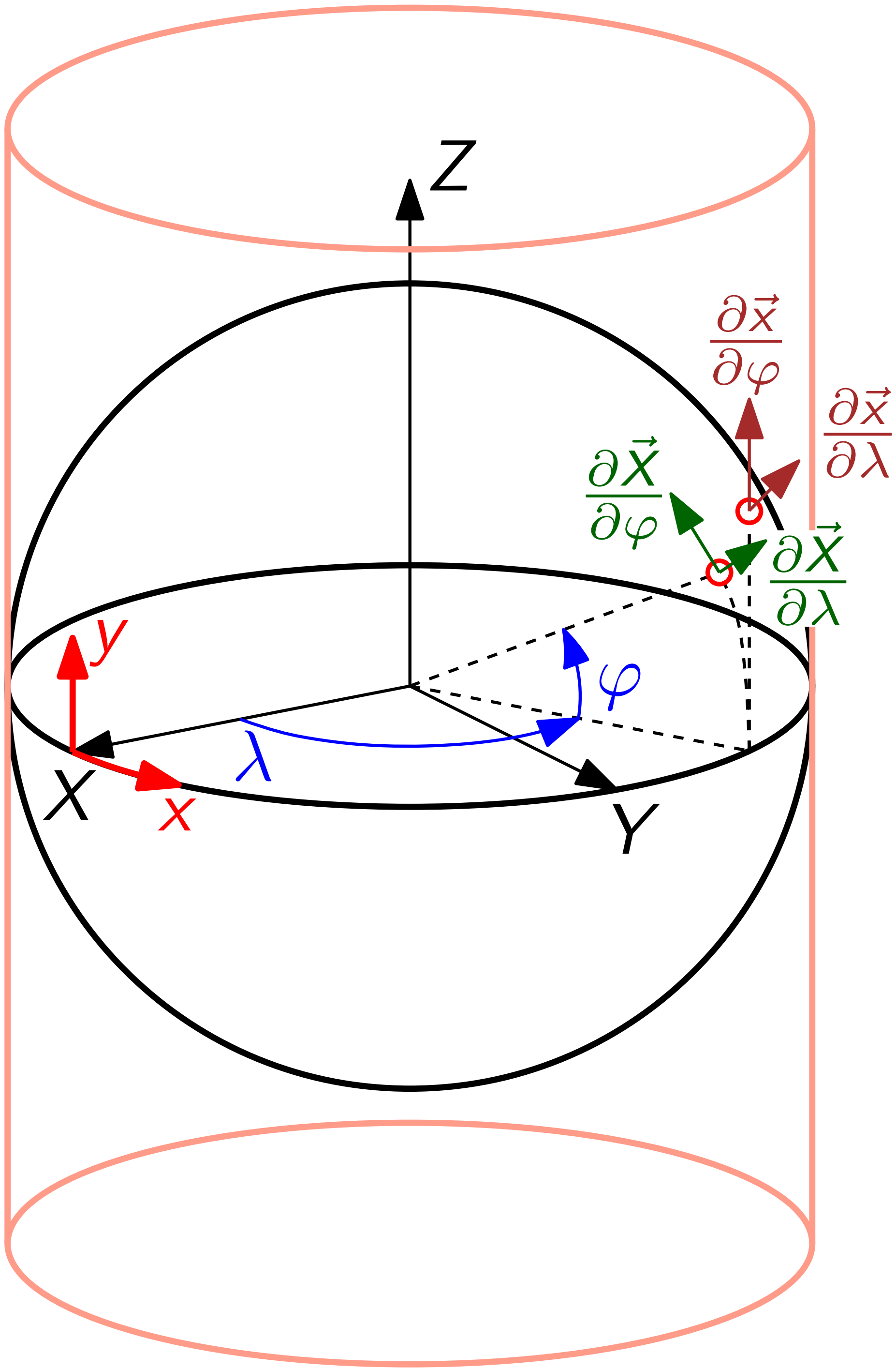


Requirement:
same sizes

$$\left|\frac{\partial \vec{x}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{x}}{\partial \lambda}\right| = \left|\frac{\partial \vec{X}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{X}}{\partial \lambda}\right|$$

$$\Leftrightarrow \left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2 \left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2 = \left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2 \left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2$$

Cylindrical Equal-area Projection – Maths



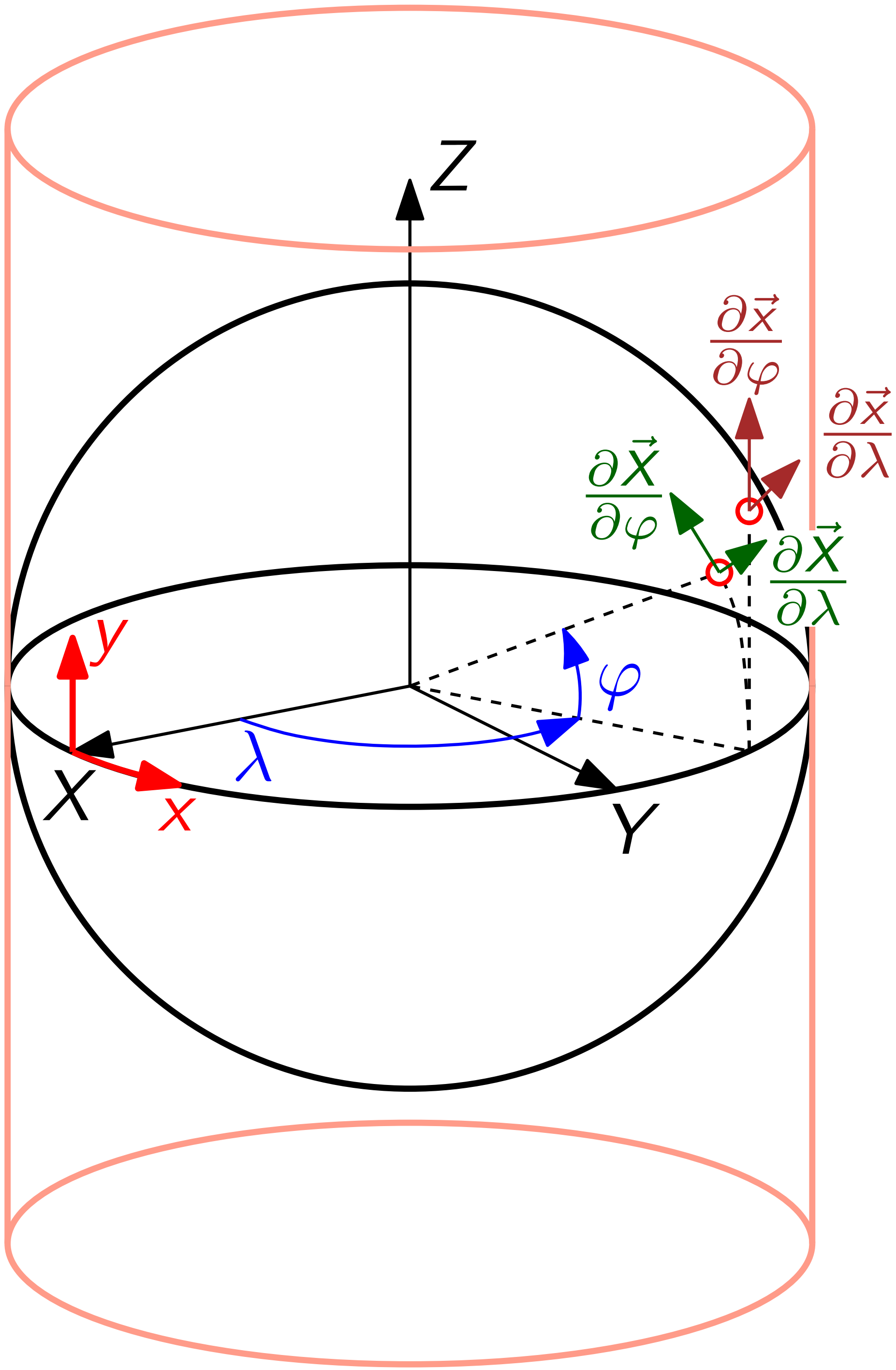
Requirement:
same sizes

$$\left|\frac{\partial \vec{x}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{x}}{\partial \lambda}\right| = \left|\frac{\partial \vec{X}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{X}}{\partial \lambda}\right|$$

$$\Leftrightarrow \left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2 \left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2 = \left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2 \left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2$$

$$\Leftrightarrow \left(f'(\varphi)\right)^2 r^2 = r^4 \cos^2 \varphi$$

Cylindrical Equal-area Projection – Maths



Requirement:
same sizes

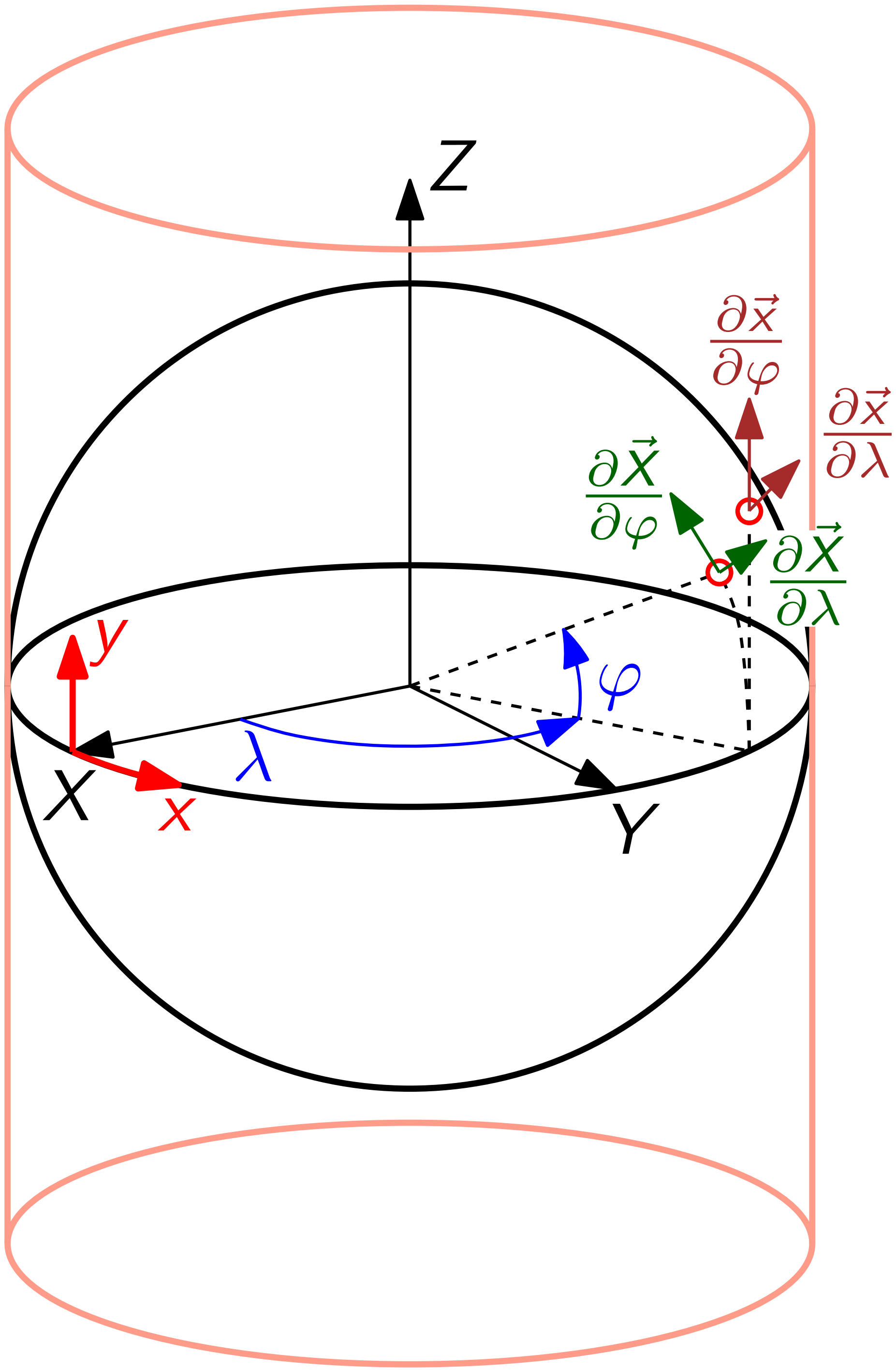
$$\left|\frac{\partial \vec{x}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{x}}{\partial \lambda}\right| = \left|\frac{\partial \vec{X}}{\partial \varphi}\right| \cdot \left|\frac{\partial \vec{X}}{\partial \lambda}\right|$$

$$\Leftrightarrow \left(\frac{\partial \vec{x}}{\partial \varphi}\right)^2 \left(\frac{\partial \vec{x}}{\partial \lambda}\right)^2 = \left(\frac{\partial \vec{X}}{\partial \varphi}\right)^2 \left(\frac{\partial \vec{X}}{\partial \lambda}\right)^2$$

$$\Leftrightarrow (f'(\varphi))^2 \, r^2 = r^4 \cos^2 \varphi$$

$$\Leftrightarrow f'(\varphi) = r \cos \varphi$$

Cylindrical Equal-area Projection – Maths



Requirement:
same sizes

$$\left|\frac{\partial\vec{x}}{\partial\varphi}\right| \cdot \left|\frac{\partial\vec{x}}{\partial\lambda}\right| = \left|\frac{\partial\vec{X}}{\partial\varphi}\right| \cdot \left|\frac{\partial\vec{X}}{\partial\lambda}\right|$$

$$\Leftrightarrow \left(\frac{\partial\vec{x}}{\partial\varphi}\right)^2 \left(\frac{\partial\vec{x}}{\partial\lambda}\right)^2 = \left(\frac{\partial\vec{X}}{\partial\varphi}\right)^2 \left(\frac{\partial\vec{X}}{\partial\lambda}\right)^2$$

$$\Leftrightarrow (f'(\varphi))^2\, r^2 = r^4 \cos^2\varphi$$

$$\Leftrightarrow f'(\varphi) = r\cos\varphi$$

$$\Leftrightarrow f(\varphi) = r\sin\varphi$$

$f(0) = 0$

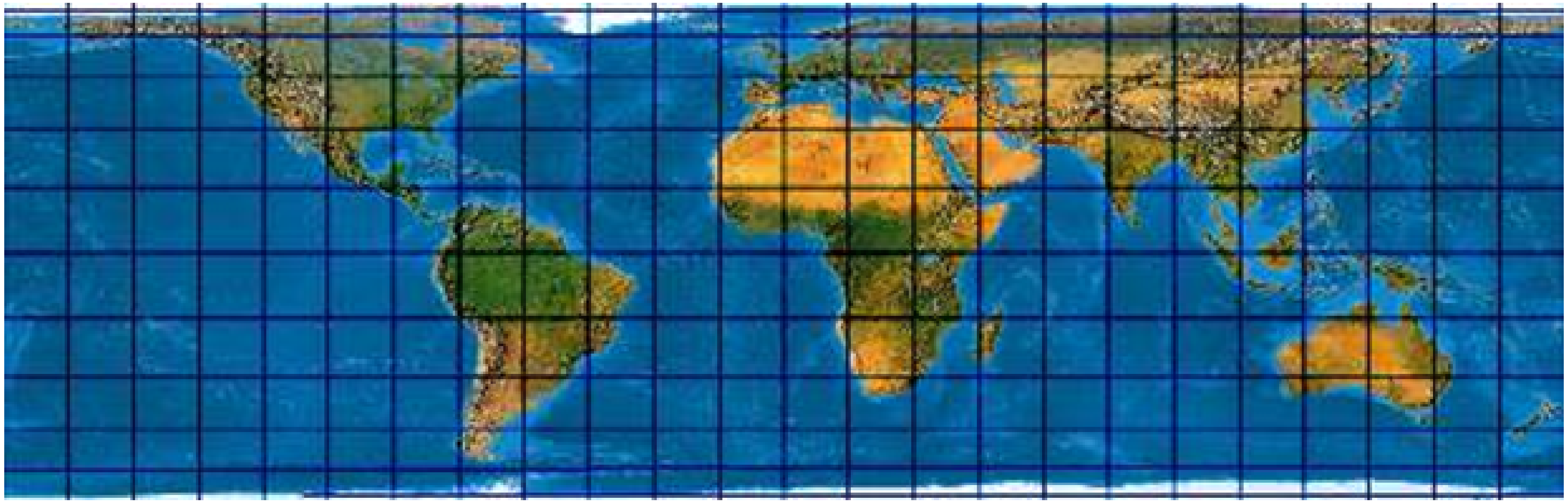
Cylindrical Equal-area Projection

Mapping function:

$$x_p = r\lambda_p$$
$$y_p = r \sin \varphi_p$$

r = Earth's radius

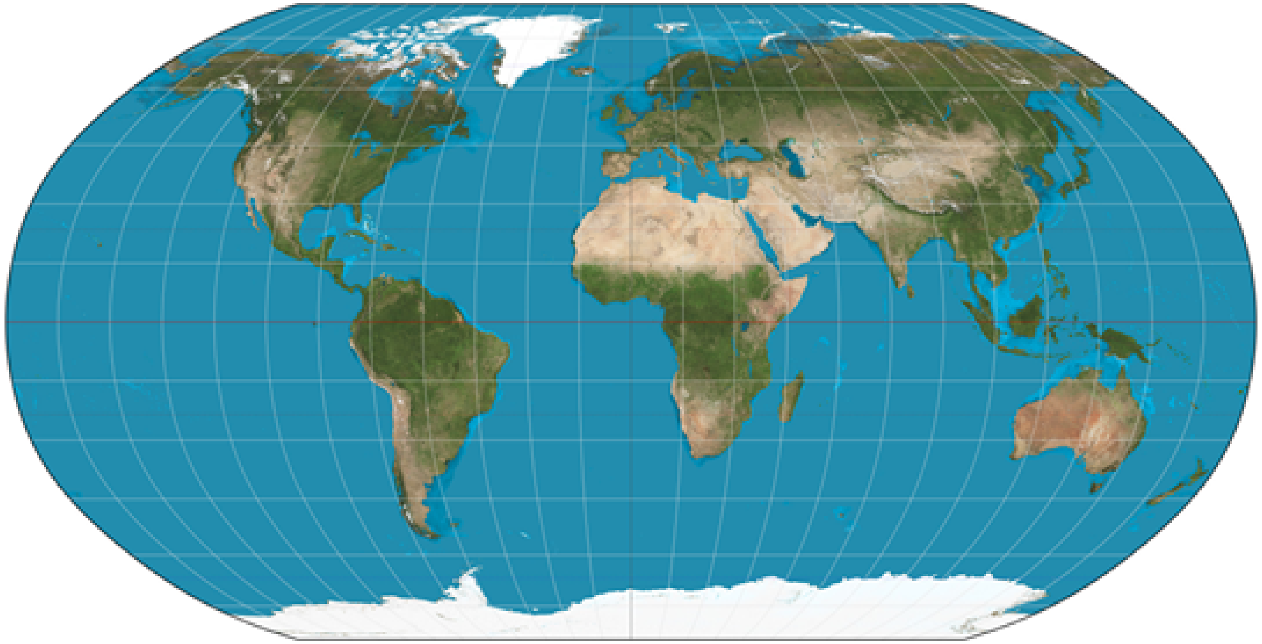
Cylindrical equal-area projection leads to large distortions of aspect rations and angles



source: Wikipedia

More Map Projections

- For a cartographic visualization of the whole world, it is better to use a map projection that tries to compromise between area distortion and angle distortion.
- e.g. Robinson projection:

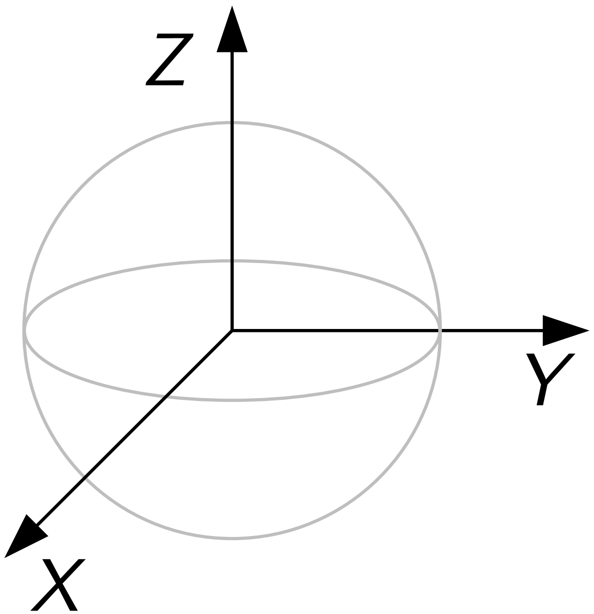


source: Wikipedia

- No map projection preserves both areas and angles!

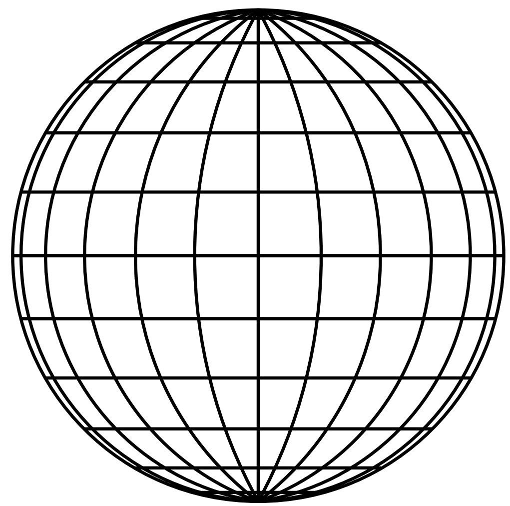
Coordinate Systems for Geoinformation

Earth-centered Cartesian coordinates

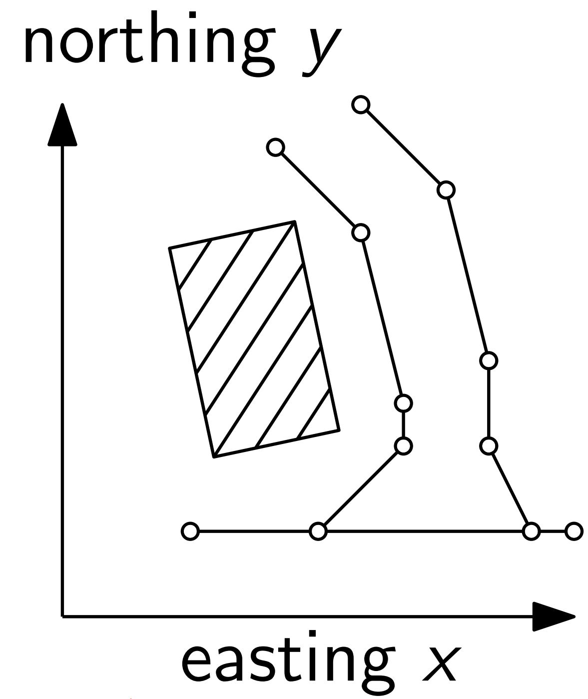


spherical/ellipsoidal coordinates

latitude φ, longitude λ



projected coordinates

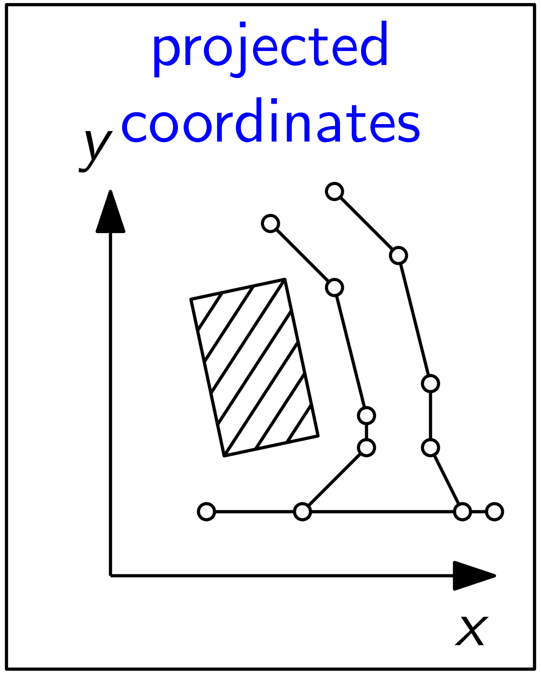


$$\begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix} = \begin{pmatrix} r \cos\varphi_p \cos\lambda_p \\ r \cos\varphi_p \sin\lambda_p \\ r \sin\varphi_p \end{pmatrix}$$

$$\begin{pmatrix} x_p \\ y_p \end{pmatrix} = \begin{pmatrix} r\lambda_p \\ r \ln\tan\left(\frac{\pi}{4} + \frac{\varphi_p}{2}\right) \end{pmatrix}$$

for spherical coordinates and Mercator projection

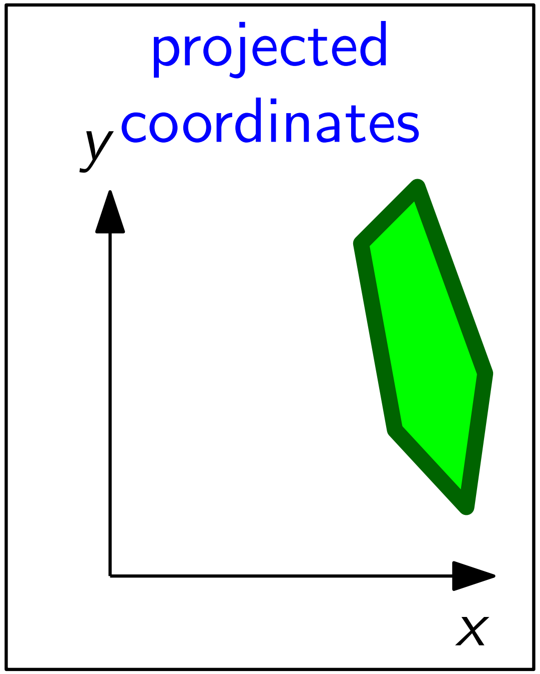
Coordinate Transformation



Earth-centered system: *ETRS89*

ellipsoid: *GRS80*

projection: *UTM*



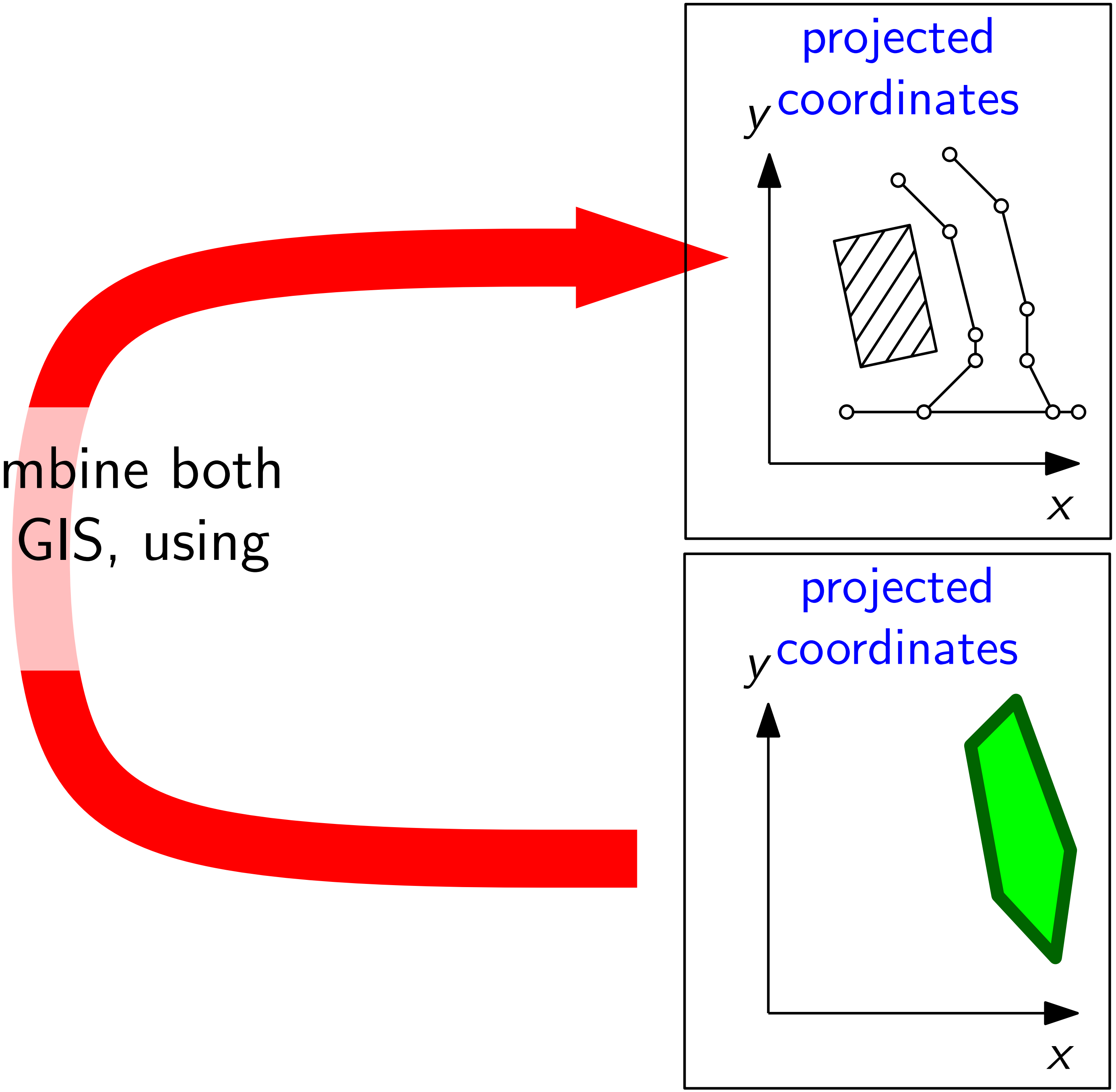
Earth-centered system: *WGS84*

ellipsoid: *WGS84*

projection: *Mercator*

Coordinate Transformation

Aim: Combine both layers in GIS, using UTM



Earth-centered system: *ETRS89*

ellipsoid: *GRS80*

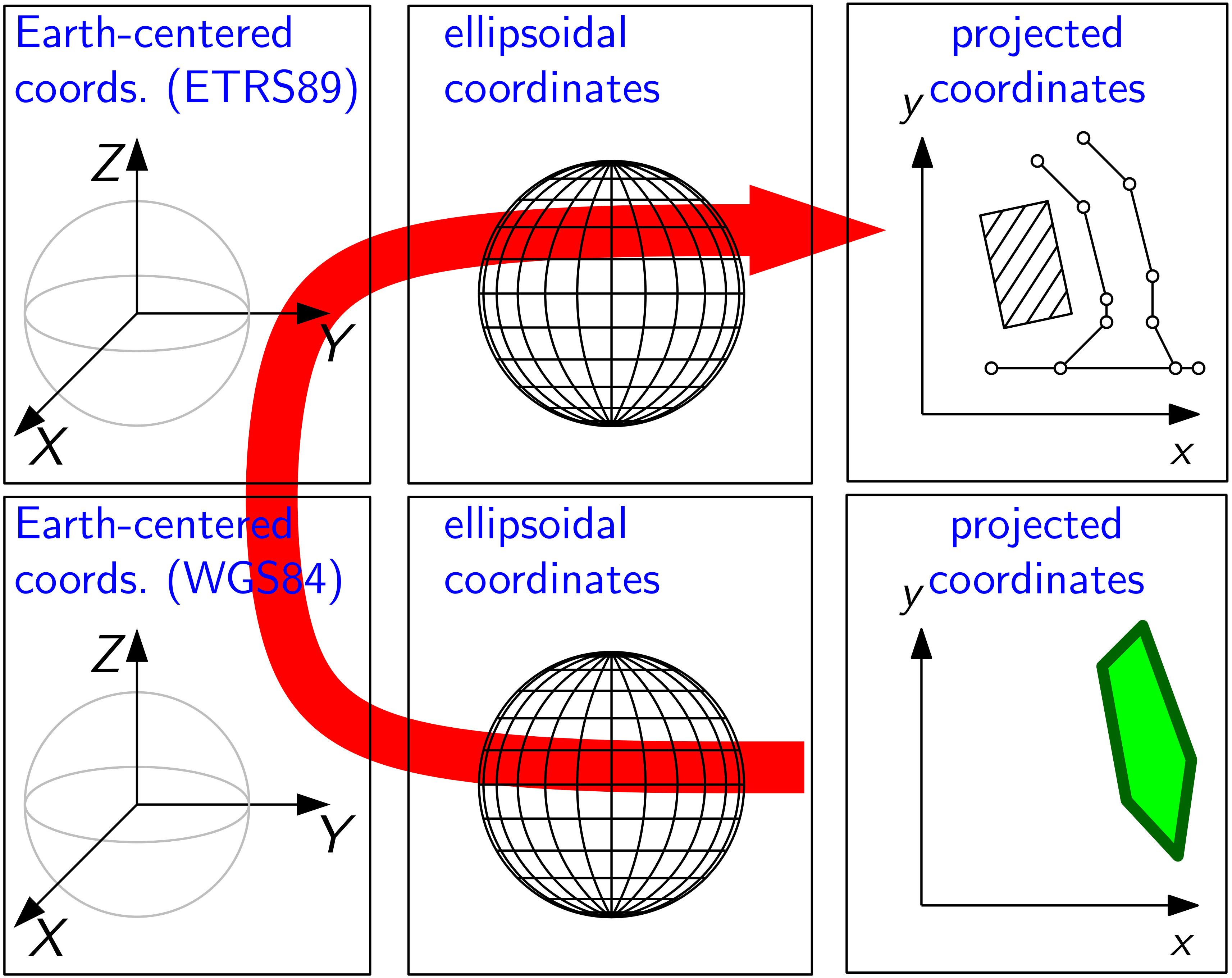
projection: *UTM*

Earth-centered system: *WGS84*

ellipsoid: *WGS84*

projection: *Mercator*

Coordinate Transformation



Earth-centered system: *ETRS89*

ellipsoid: *GRS80*

projection: *UTM*

Earth-centered system: *WGS84*

ellipsoid: *WGS84*

projection: *Mercator*