

# On the Theory

## 2. Probability Theory and Random Variables

- p. 54, Eqs (2.220) and (2.221) should read

$$\underline{x}_n'' := \underline{x}_{n-1} - 2\underline{x}_n + \underline{x}_{n+1} = \sum_{p=1}^P a_p \underline{x}_{n-p}'' + \underline{\varepsilon}_n,$$

and

$$\underline{x}_{n+1} = -(\underline{x}_{n-1} - 2\underline{x}_n) + \sum_{p=1}^P a_p \underline{x}_{n-p}'' + \underline{\varepsilon}_n$$

## 4. Estimation

- p. 84: after the text after Eq. (4.37) add:  
"Care has to be taken when naming the weighted sum of the squared residuals being a Mahalanobis distance, since the inverse of  $\Sigma_{\hat{v}\hat{v}}$  does not exist. However, the weight matrix  $W_{ll}$  is what is called a *generalized inverse* of  $\Sigma_{\hat{v}\hat{v}}$ . Therefore we need a definition of a generalized Mahalanobis distance if the covariance matrix is singular. Then we would define  $\Omega = \hat{v}^T \Sigma_{\hat{v}\hat{v}}^- \hat{v}$  with the generalized inverse  $W_{ll} = \Sigma_{\hat{v}\hat{v}}^-$ .  
A generalized inverse  $A^-$  of a general matrix  $A$  fulfills  $AA^-A = A^-$ . Therefore a generalized Mahalanobis distance can be defined as  $\underline{d}^2 = (\underline{x} - \underline{\mu}_x)^T \Sigma_{xx}^- (\underline{x} - \underline{\mu}_x)$ , which can be shown to be  $\chi^2$ -distributed with  $\text{rk}\Sigma_{xx}$  degrees of freedom if  $\underline{x} \sim \mathcal{N}(\underline{\mu}_x, \Sigma_{xx})$ ."
- p. 92: before (4.94) add the reference: [Förstner \(1979\)](#).
- p. 93, 138, 139, and 151 in Eqs. (4.105), (4.350), (4.351), (4.345) (4.346), and (4.389) it must read ... = argmin... instead of ... = argmax....
- p. 119, second paragraph. It should read: 'Let us therefore assume that  $I$  reference values, collected in the vector  $\mathbf{y}_r$ , for ...'
- p. 132, Fig. 4.11, line three of the heading: The text should read: '... with all points and a statistical test is applied to identify outliers, a ...'

## 5. Homogeneous Representations of Points, Lines and Planes

- p. 212, 3rd line before Eq. (5.52): It should read: ‘..., and we will generally not distinguish between them.’
- p. 215, Definition 5.3.9: It must start with ”The projective space  $\mathbb{P}^n(\mathbb{R})$  contains all  $n$ -dimensional points  $\chi$  with homogeneous real-valued coordinates  $\mathbf{x} \in \mathbb{R}^{n+1} \setminus \mathbf{0}, \dots$ ”

## 6. Transformations

- p. 253, after (6.23): The text should read: ‘...we have the parameters  $c$  and  $f$  which cause the typical effects ...’.

## 7. Geometric Operations

- p. 322: Add a section on directly estimating conics and quadrics.

### 7.4.4 Minimal Solutions for Conics and Quadrics

Given are  $I = 5$  points  $\chi_i(\mathbf{x}_i)$ . They are assumed to lie on a conic. From the vector representation of conics and quadrics (7.117), p. 316 we directly obtain the unknown parameters  $a_{ij}$  of the conic  $a_{11}u_i^2 + 2a_{12}u_iv_i + 2a_{13}u_iw_i + a_{22}v_i^2 + 2a_{23}v_iw_i + a_{33}w_i^2 = 0$  from the nullspace of the  $5 \times 6$ -matrix

$$A_{5 \times 6} = \begin{bmatrix} u_1^2, & 2u_1v_1, & v_1^2, & 2u_1w_1, & v_1^2, & 2v_1w_1, & w_1^2 \\ & & & \dots & & & \\ u_i^2, & 2u_iv_i, & v_i^2, & 2u_iw_i, & v_i^2, & 2v_iw_i, & w_i^2 \\ & & & \dots & & & \\ u_5^2, & 2u_5v_5, & v_5^2, & 2u_5w_5, & v_5^2, & 2v_5w_5, & w_5^2 \end{bmatrix},$$

see Sect. 4.8.2.5, p. 182, (5.434), there with a different notation for coordinates and parameters. The nullspace efficiently can be calculated using the QR decomposition, see (A.116), p. 778. The solution generalizes to a direct algebraic solution for  $I > 5$  points. The direct determination of quadrics can be achieved analogously. Then 9 or more points  $\chi_i$  are necessary. Conditioning of the given coordinates is only necessary if more than the minimum number of points is used, see Sect. 6.9, p. 286.

## 10. Reasoning with Uncertain Geometric Entities

- p. 432, Eqs. (10.330), (10.333) it must read

$$\sigma_\alpha = 0.52^\circ$$

- p. 432, the second last paragraph: it must read " The estimated variance factor  $\hat{\sigma}_0^2 = 1.1219^2$ , ..."

### 13. Geometry and Orientation of the Image Pair

- p. 557, the last sentence of the proof needs to be deleted.

### 14. Geometry and Orientation of the Image Triplet

- p. 632, Table 14.4, last row, second column: It should read

$$S^{sT}(\mathbf{x}''') \top_\diamond(\mathbf{x}') S^{(s)T}(\mathbf{x}'') = 0$$

- p. 640, Fig. 14.4: The point in the left image with projection centre  $O'$  should be  $\chi'$ , not  $\chi'''$ .

### 15. Bundle Adjustment

p. 648, eq. (15.13) should read

$$p(\mathbf{k}_i, \mathbf{p}_t | \mathbf{l}_{it}) \propto p(\mathbf{l}_{it} | \mathbf{k}_i, \mathbf{p}_t) p(\mathbf{k}_i) p(\mathbf{p}_t).$$

### Appendix

- p. 772, Eq. (A.49) must read

$$R\mathbf{x} \times R\mathbf{y} = R(\mathbf{x} \times \mathbf{y}) \quad \text{and} \quad S(R\mathbf{x})R = RS(\mathbf{x}),$$

- p. 781, text before (A.139) must read: "Taking the decomposition of the not necessarily symmetric matrix  $A$ ,"
- p. 781, after Eq. (A.144) add: "The relations (A.140) to (A.144) also hold for matrices which cannot be decomposed as (A.139)."



## References

Förstner, W. (1979). Ein Verfahren zur Schätzung von Varianz- und Kovarianzkomponenten. *Allgemeine Vermessungs-Nachrichten 11-12*, 446-453. [1](#)