CHARACTERIZING IMAGE QUALITY: BLIND ESTIMATION OF THE POINT SPREAD FUNCTION FROM A SINGLE IMAGE

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ABSTRACT

This paper describes a method for blind estimation of sharpness and resolving power from a single image. These measures can be used to characterize images in the context of the performance of image analysis procedures. The method assumes the point spread function (PSF) can be approximated by an anisotropic Gaussian. The width σ of the PSF is determined by the ratio $\sigma_g/\sigma_{g'}$ of the standard deviations of the intensity and of its derivative at edges. The contrast sensitivity function (CSF) is based on an optimal model for detecting straight edges between homogeneous regions in noisy images. It depends on the signal to noise ratio and is linear in the frequency. The method is applied to artificial and real images proving that it gives valuable results.

1 INTRODUCTION

The usability of images for interpretation, orientation or object reconstruction purposes highly depends on the image quality. In principle it makes no difference whether image analysis is performed manually by a human operator or whether digital images are analyzed automatically: The reliability, accuracy and precision of results of image analysis procedures directly is influenced by the quality of the underlying image data.

Image quality can be characterized by a large number of measures, e. g. contrast, brightness, noise variance, sharpness, radiometric resolution, granularity, point spread function (PSF), modulation and contrast transfer function (MTF, CTF), resolving power, etc. (cf. (Lei and Tiziani, 1989), (Zieman, 1997)), all referring to the radiometry of the images.

As aerial cameras and films are designed to obtain highest image quality, the user, based on his/her experience normally just decides on whether the images can be used or not, e. g. due to motion blur. In the following process, image quality is not referred to using classical quality measures. With digital or digitized images the situation changes, especially because automatic image analysis procedures can be applied and their performance can be much better described as a function of image quality.

In (Förstner, 1996) it is shown that the performance characteristics of vision algorithms can be used to select the set (a,t) of algorithms a with tuning parameters t applied to image data d leading to a quality $q(r|d,a,t)$ of the result r from

$$(\widehat{a},\widehat{t}) = \{(a,t)|P(\underline{q}(\underline{r}|\underline{d},a,t) > q_0) > P_0\}.$$

Thus the probability P of obtaining a quality $\underline{q}$ being better than a pre-specified minimum quality q_0 should be larger than a pre-specified minimum probability P_0. The most difficult part in evaluating this equation is the characterization of the domain $\mathcal{D}$ of all the images d which one expects. Therefore one needs to be able to characterize images to that extent which is relevant for the task of performance characterization or more specifically for the selection of appropriate algorithms a and tuning parameters t. As an example, fig. 1 shows the effect of two different edge detectors on two aerial images of different sharpness. The final goal would be to predict the quality of the result of these edge detectors as a function of the image sharpness as one of the decisive parameters.

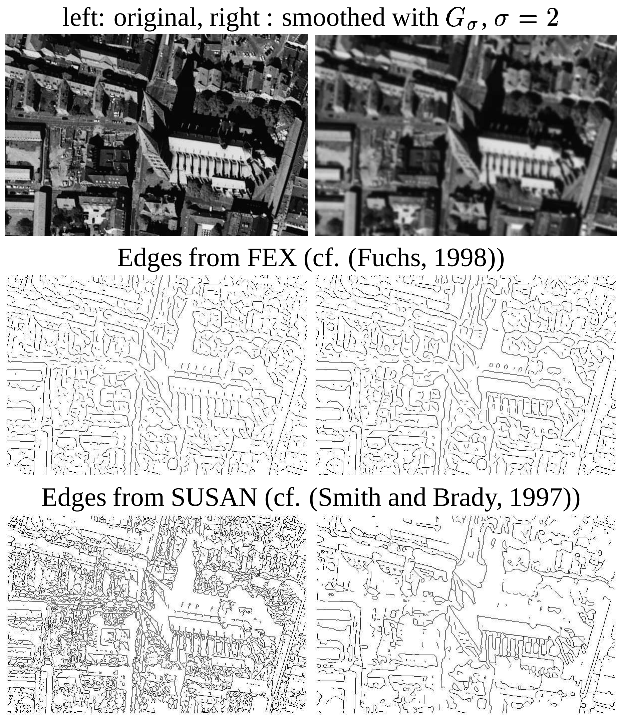

Figure 1: Effect of two different edge detectors on aerial images of different sharpness. The same parameters were taken for both images, no attempt was made to obtain the best results in all four cases.

Among other measures, such as power spectrum or edge density, image sharpness is important for characterizing images. Image blur, which limits the visibility of details, can be objectively measured by the point spread function

(PSF) or its amplitude spectrum, the modulation transfer function (MTF). Together with the contrast sensitivity function (CSF), giving the least detectable contrast at an edge as a function of the spatial frequency of intensity changes, one can derive the resolving power. It is the maximum frequency of a periodic signal which can be detected with a given certainty.

Now, the precise determination of the PSF is quite involving, and usually derived from the intensity transition at edges, yielding the cumulative distribution of the PSF, interpreted as probability density function. Moreover, the classical CSF refers to a human observer.

This paper assumes the PSF to be a Gaussian function. We will introduce a simple procedure for measuring the main characteristics of the PSF, namely its width. We give a definition for the CSF based on an ideal edge detector for straight edges between noisy homogeneous regions. It therefore allows to fully automatically determine the resolving power of such an ideal edge detector. Experiments with synthetic and real data demonstrate the usefulness of the proposed approach.

2 THEORETICAL BASIS

As we are interested in simplifying the characteristic measures of image quality we summarize the basic relations.

2.1 Point and edge spread function

The quality of an imaging system may be evaluated using the un-sharpness or blur at edges. The edge spread function of a 1-dimensional signal is the response $\bar{s}(x)$ of the system to an ideal edge $s(x)$ of height 1 (cf. the first row in fig. 2).

The quality of an imaging system usually is described by the point spread function (cf. the second row of fig. 2), being the response $h(x)$ of the system to a delta function $\delta(x)$. As the imaging system is assumed to be linear and the ideal edge $s(x)$ is the integral of the δ-function, the point spread function is the first derivative of the edge spread function: $\bar{s}(x) = h'(x)$. Observe, we may interprete the point spread function as a probability density function and the corresponding edge response function as its cumulative distribution function resp. distribution function.

In two dimensions the situation is a bit more involving. If we differentiate the 1-dimensional cross section of the response $\bar{s}(u)$ to an ideal two dimensional edge $s(u)$ we obtain a bell shaped function. It is the marginal distribution of the point spread function along the edge direction. Fusing a large number of such marginal distributions of the PSF can only be done in the Fourier domain using tomographic reconstruction techniques (cf. (Rosenfeld and Kak, 1982)).

The situation becomes much easier in case we can approximate the 2-dimensional PSF by a Gaussian. Then the edge spread function, i. e. the response to an arbitrary edge is an integrated Gaussian function.

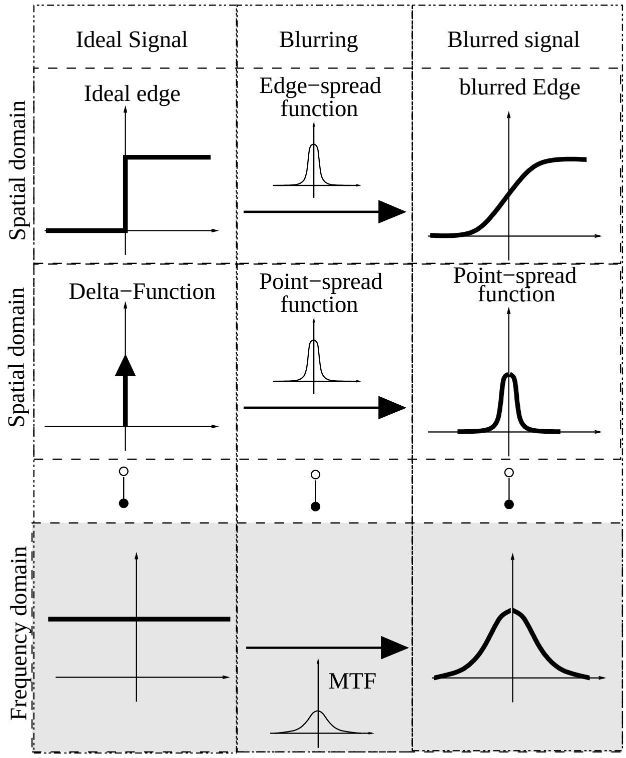

Figure 2: Edge spread function, point spread function and modulation transfer function.

In detail we assume

$$h(\boldsymbol{x}) = \frac{1}{2\pi|\boldsymbol{\Sigma}|} \mathrm{e}^{-\frac{1}{2}\boldsymbol{x}^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}}$$

where the matrix $\boldsymbol{\Sigma}$ can be written as

$$\boldsymbol{\Sigma} = \boldsymbol{R} \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2 \end{pmatrix} \boldsymbol{R}^{\mathsf{T}}.$$

Here the two parameters σ_1 and σ_2 represent the width of the PSF in two orthogonal directions and $\boldsymbol{R}$ is the corresponding rotation matrix. In case we have two edges on the principle directions ξ and η of the PSF we obtain the two edge response functions

$$\bar{s}_1\xi) = \frac{1}{\sigma_1}\mathrm{erf}\left(\frac{\xi}{\sigma_1}\right) \qquad \bar{s}_2(\eta) = \frac{1}{\sigma_2}\mathrm{erf}\left(\frac{\eta}{\sigma_2}\right)$$

with the error function erf$(x) = \int_{-\infty}^{x} G_1(t)dt$.

We refer to the individual values σ as *local scale* as it corresponds to the notion of scale in a multi-scale analysis of an image. The matrix $\boldsymbol{\Sigma}$ is called *scale matrix*.

2.2 Modulation Transfer Function (MTF)

It is convenient to describe the characteristics of the imaging system by its response to periodic patterns, leading to the modulation transfer function $H(u, v)$. It is the amplitude spectrum of the point spread function,

$$h(x, y) \circ\!\!-\!\!\bullet\, H(u, v),$$

explicitely $h(x,y) = \int\int H(u,v) \mathrm{e}^{j2\pi(xu+yv}dudv$ or

$$h(\boldsymbol{x}) = \int H(\boldsymbol{u}) \mathrm{e}^{j2\pi \boldsymbol{u}^\mathsf{T}\boldsymbol{x}} d\boldsymbol{u}$$

using the definition of the Fourier transform of (Castleman, 1979) .

In case we have a sinus-type pattern $s(x) = a\sin(2\pi ux) = a\sin(2\pi\frac{x}{\lambda})$ the response of the system is a sine-wave with contrast $\bar{a} = H(u)a$. As the MTF usually falls off for large frequencies, contrast of tiny details is diminished heavily.

In our special context we obtain the MTF for the Gaussian shaped PSF

$$G_\Sigma(\boldsymbol{x}) \circ\!\!-\!\!\bullet\ \mathrm{e}^{-2\pi^2\boldsymbol{u}^\mathsf{T}\boldsymbol{\Sigma}\boldsymbol{u}}$$

which again is a Gaussian, however, with the matrix $\boldsymbol{P} = \boldsymbol{\Sigma}^{-1}/4\pi^2$ as parameter. Observe that we have

$$\boldsymbol{P} = \boldsymbol{R} \begin{pmatrix} \frac{1}{4\pi^2\sigma_1^2} & 0 \\ 0 & \frac{1}{4\pi^2\sigma_2^2} \end{pmatrix} \boldsymbol{R}^\mathsf{T}.$$

2.3 Contrast Sensitivity Function

In order to evaluate the usefulness of the imaging system with a certain PSF or MTF the so called contrast sensitivity function (CSF) is used. The contrast sensitivity function gives the minimum contrast at a periodic edge pattern which can be perceived by a human. In our case we want to apply this notion to edge detectors.

Assume we have a periodic pattern of edges characterized by the wavelength λ and the contrast c. Further assume the image to be sampled with a pixel size of Δx and the noise has standard deviation σ_n. An ideal edge detector would adapt to the wavelength of the pattern and perform an optimal test whether an edge exists or not. For simplicity we assume that the pattern is parallel to one of the two coordinate systems and that the edge detector uses the maximum possible square of size $\lambda \times \lambda$. The difference Δg between the means μ_1 and μ_2 of the two neighboured areas can be determined from the $N/2 = (\lambda/\Delta x)^2/2$ pixels in the two areas. It has standard deviation

$$\sigma_{\Delta g} = \sqrt{\sigma_{\mu_1}^2 + \sigma_{\mu_2}^2} = \sqrt{2}\sigma_\mu = \sqrt{2}\cdot\sqrt{\frac{2}{N}}\sigma_n = \frac{2\Delta x}{\lambda}\sigma_n.$$

Thus in case we perform the test with a significance number α and require a minimum probability β_0 for detecting the edge we can detect edges with a minimum height

$$\Delta_0 g = \delta_0(\alpha,\beta_0)\sigma_{\Delta g} = \delta_0(\alpha,\beta_0)\frac{2\Delta x}{\lambda}\sigma_n.$$

The factor $\delta_0(\alpha,\beta_0)$ depends on the significance level of the test and the required probability of detecting an edge. It is reasonable to fix it; in case we choose a small significance number $\alpha = 0.001$ and a minimum detectability $\beta_0 = 0.8$ we have $\delta_0 = 4.17 \approx 4$. The minimum detectable contrast in a reasonable manner depends on the size of the window and the noise level: The larger the noise standard deviation and the smaller the window the larger the contrast of the edge needs to be in order to be detectable.

As we finally want to relate the contrast sensitivity to the frequency $u = 1/\lambda$ and obtain the contrast sensitivity function

$$\mathrm{CSF}(u) \doteq \Delta_0 g(u) = 2\delta_0\ \Delta x\ u\ \sigma_n$$

It goes linear with the frequency, indicating higher frequency edge patterns require higher contrast.

2.4 Resolving power

The resolving power RP usually is defined as that frequency u where the contrast is too small due to the properties of the imaging system to be detectable. As periodic patterns with small wave length will loose contrast heavily they may not be perceivable any more.

The MTF has maximum value 1 and measures the ratio in contrast $\mathrm{MTF}(u) = \bar{a}(u)/a(u)$, whereas the CSF measures the minimum contrast being detectable. In order to be able to compare the MTF with the CSF we need to normalize the CSF. This easily can be done in case we introduce the signal to noise ratio

$$\mathrm{SNR} = \frac{k}{\sigma_n},$$

with k being the contrast. Then the relative contrast sensitivity function reads as

$$\mathrm{rCSF}(u) \doteq \frac{\mathrm{CSF}(u)}{k} = \frac{2\delta_0\ \Delta x\ u\ \sigma_n}{k} = \frac{2\delta_0 \Delta x\ u}{\mathrm{SNR}}$$

which immediately can be compared with the MTF.

One usually argues, that the resolving power is the frequency where the relative contrast, measured by the MTF, is identical to the minimum relative contrast being detectable (cf. fig. 3). Thus the resolving power RP=u_0 is implicitly given by

$$\mathrm{MTF}(u_0) = \mathrm{rCSF}(u_0).$$

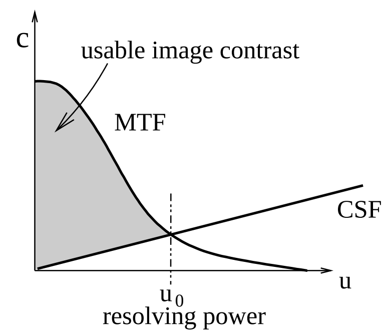

Figure 3: Relations between the modulation transfer function (MTF), the contrast sensitivity function (CSF) and the resolving power (RP).

In the 1-dimensional case we can explicitely give u_0

$$u_0 = \frac{1}{2\pi\sigma}\sqrt{\mathrm{LambertW}\left(\frac{\pi^2}{\delta_0^2}\frac{\sigma^2}{\Delta x^2}\mathrm{SNR}^2\right)}.$$

The LambertW-function is defined implicitly by (c.f. (Corless et al., 1996))

$$\text{LambertW}(x) \cdot \exp(\text{LambertW}(x)) = x.$$

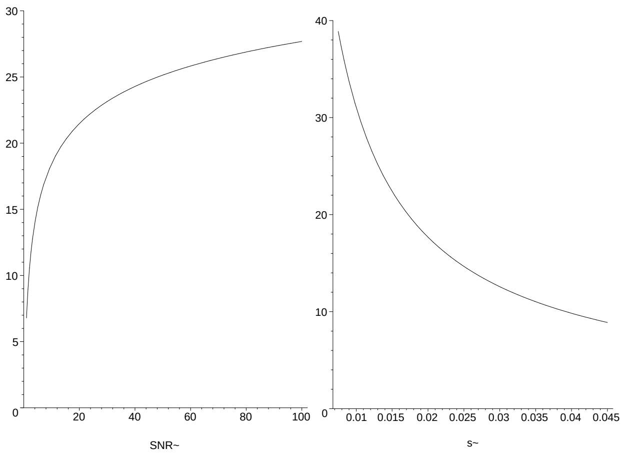

Figure 4: Resolving power in lines/mm for aerial images with a pixel size of 15 μm as a function of SNR (left, $\sigma = 1$) and of the width σ of the PSF (right, SNR=10)

Figure 4 shows the resolving power of our ideal edge detector in lines/mm for aerial images as a function of the signal to noise ratio and of the width σ of the point spread function. The resolving power increases with increasing SNR and reaching 25-30 lines/mm for good SNRs. It decreases with increasing blur, falling below 10 lines/mm for $\sigma > 45\mu m$. These results are reasonable, as they are confirmed by practical experiences with digital aerial images (c.f. (Albertz, 1991)).

2.5 Contrast, Gradient and Local Scale

We now derive a simple relation between the contrast, the gradient and the local scale, which we will use to determine the local scale at an edge. We assume an edge in an image to be a blurred version of an ideal edge. In case the PSF is a Gaussian $G_\sigma(x)$ the edge follows

$$s(x) = \text{erf}\sigma(x) = k\frac{1}{\sigma}\text{erf}\left(\frac{x}{\sigma}\right) + m$$

where m is the mean intensity and k is the contrast. Following (Fuchs, 1998) the contrast can be determined from the standard deviation σ_g of the signal around the edge, $k = 2\sigma_g$. The gradient magnitude of the edge is given by the first derivative of the edge function, which in our case is $kG_\sigma(0) = k/(\sqrt{2\pi}\sigma)$. Thus we have the relation

$$|\nabla g| = \frac{k}{\sqrt{2\pi}\sigma}$$

From this and $k = 2\sigma_g$ we can easily derive

$$\sigma = \sqrt{\frac{2}{\pi}}\frac{\sigma_g}{|\nabla g|}$$

The practical procedure determines the variance of the signal from

$$\sigma_g^2 = E(g^2) - (E(g))^2 = g^2 * G_{\sigma_l} - (g * G_{\sigma_l})^2$$

where the kernel width l is chosen to be large enough to grasp the neighbouring regions. We use a kernel size of $l = 20$. The gradient magnitude should be estimated robustly from a small neighborhood. We use a Gaussian kernel with $\sigma = 1$ for estimating the gradient magnitude.

2.6 Blind estimating the PSF from a single image

We are now prepared to develop a procedure for blindly estimating the PSF from a single image. Blind estimation means, we do not assume any test pattern to be available.

As the PSF is derived via the sharpness of the edges, and the PSF is the image of an ideal point, a δ-function, we need to assume that the image contains edges which in the original are very sharp, thus close to ideal step-edges. This can e. g. be assumed for images of buildings or other man-made objects, as the sharpness of the edges in object space is much higher than the resolution of the imaging system can handle. Formally, if the image scale is $1 : S$, the width σ_i of the image of the sharp edge would be $\sigma_i = \sigma_o/S$ and we assume that this value is far beyond what the optics or the sensor can handle.

Now, for each edge we obtain a single value σ_e. In case it would be the image of an ideal edge in object space it can be interpreted as an edge with the expected mean frequency $1/\sigma_e$ in the MTF in that direction. Thus we obtain a histogram from all edges with

$$\boldsymbol{u}_e = \frac{1}{\sigma_e}\begin{pmatrix}\cos\phi \\ \sin\phi\end{pmatrix} \quad \text{and} \quad \boldsymbol{u}_e = -\frac{1}{\sigma_e}\begin{pmatrix}\cos\phi \\ \sin\phi\end{pmatrix}$$

where the direction vector points across the edge. We use two values, as we do not want to distinguish between edges having different sign.

In case the edge is already fuzzy in object space, the estimated value σ_e of the edge will be larger, thus the $1/\sigma_e$ will be smaller. Therefore we search for the ellipse which contains all points u_e and has smallest area. This ellipse is an estimate for the shape of the ellipse $\boldsymbol{u}^\mathsf{T}\boldsymbol{\Sigma}\boldsymbol{u} = 1$, thus for $\boldsymbol{\Sigma}$ of the PSF.

3 EXPERIMENTAL RESULTS

The following examples want to show the usefulness of the approach. In detail we do the following:

1. Using an ideal test image (Siemens star) with known sharpness we compare our estimation with given ground truth (cf. fig. 5).

2. Using the same test image but with noise we check the sensitivity of the method is with respect to noise (cf. fig. 6).

3. Using real images with known artificial blur we check whether the method works in case the edge distribution is arbitrary (cf. fig 7).

4. Using scanned aerial imagery with different sharpness, caused by the scanning procedure, we test whether the method also reacts to natural differences in sharpness (cf. fig. 8).

In all cases the minimum resolving power of an ideal edge detector is given. In the case of digital images we refer to a pixel size of 15 μm.

3.1 Demonstration on synthetic Data

Test on noiseless data. The following sequence of gradually blurred images was used to test the proposed method to determine the point spread function and the resolving power with respect to correctness of the implemented algorithm.

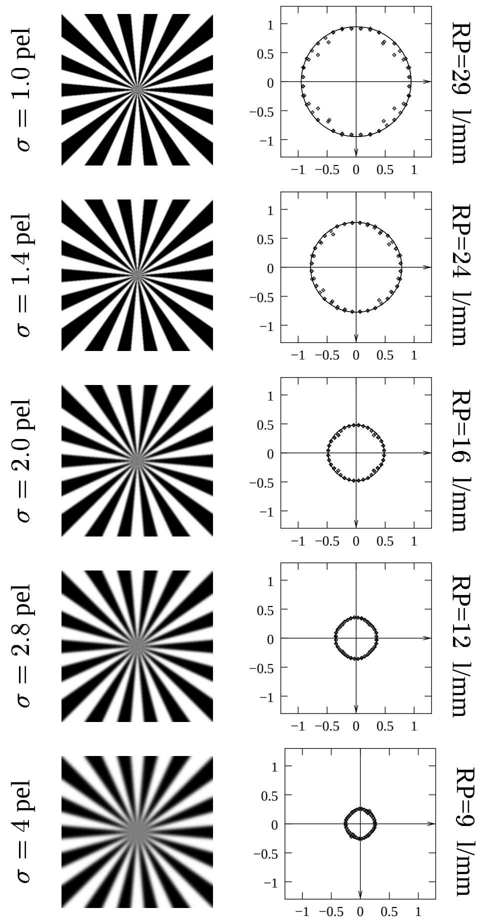

Figure 5: Siemens - star at various steps of image sharpness ($\Delta x = 15\mu$m, $\sigma_n = 1gr$, SNR$= 255$). left: test image, right: histogram of edges, resolving power of optimal edge detector.

The method gives reasonable results: For each test image, the histogram of edges is a circle with the correct radius $1/\sigma$, being the reciprocal width σ of the point spread function used to generate the image.

Test on noisy data. To test the sensitivity of the algorithm with respect to image noise the Siemens star $\sigma =$ 2.8 pel from fig. 5 was speckled with Gaussian noise, the noise variance being σ_n^2.

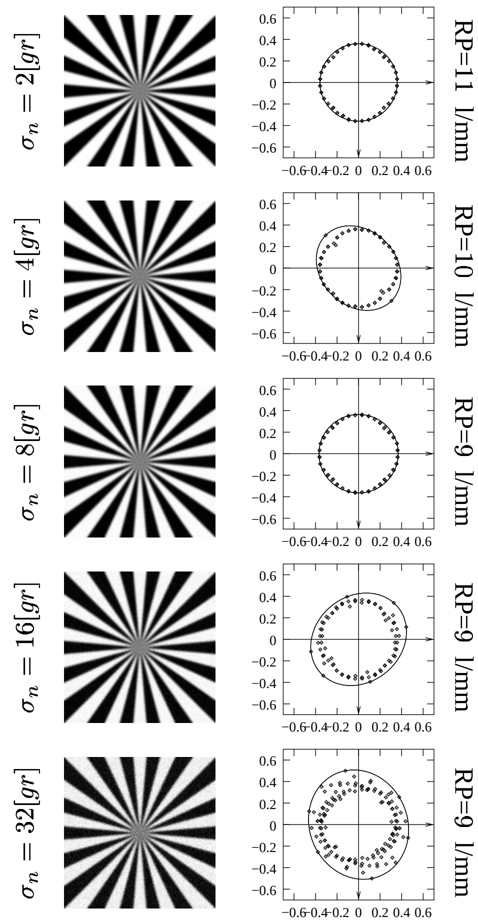

Figure 6: Siemens star with $\sigma = 2.8$ pel at various steps of image noise (SNR=$128, 64, 32, 16, 8$).

The results in fig. 6 show that the method is quite robust with respect to image noise. Note that the slightly decreasing resolving power of the ideal edge from the first to the last row is caused by the increasing image noise.

3.2 Results on real data

Real data with artificial blur. The method was also tested on a real image of the MIT building which was gradually blurred by convolution with Gaussian filters of increasing filter width (cf. fig 7).

We see that the method seems to yield correct results. In almost each histogram of edges the ellipse containing all points is elongated, indicating anisotropy of the image sharpness for the given image.

Aerial image with various sharpness. Finally, the method was applied to digitized versions of an aerial image (cf. fig. 8, top row) scanned three times with a pixel size of 7μm. Various image sharpness has been realized physically by imposing layers of transparencies between the original and the scanner platform, thus exploiting the limited depth of view of the optical system of the scanner.

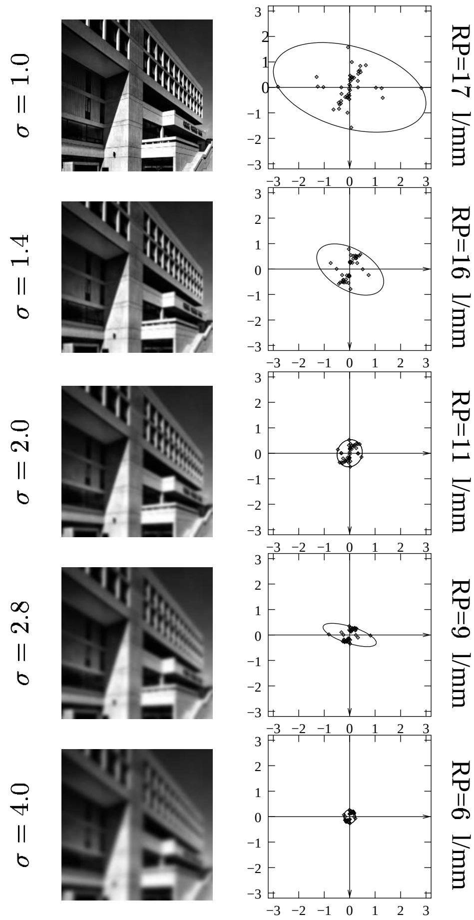

Figure 7: MIT building at various steps of image sharpness (SNR=20).

We see in fig. 8 that the method works quite well even on real data. The different sharpness of the three versions of the image sharpness is recognized. The good resolving power obtained for the ideal edge detector is plausible, as the scanned original was of excellent quality.

4 CONCLUSIONS AND OUTLOOK

We have developed a procedure for blindly estimating the point spread function. We define a contrast sensitivity function. This allows us to derive the resolving power as a function of the PSF, the pixel size and the signal to noise ratio. The PSF is assumed to be an anisotropic Gaussian function. We estimate the corresponding scale matrix $\mathbf{\Sigma}$ from the local scale at automatically extracted edges. We assume the image contains enough edges with different orientations which result from very sharp edges in the scene. The contrast sensitivity function which is based on an ideal adaptive edge detection scheme for straight edges between noisy homogeneous regions is derived. Experiments on artificial and real data demonstrate the usefulness of the approach.

The method is restricted to images with a sufficient number of edges and to Gaussian shaped PSF. An extension to general point spread functions is possible using tomographic techniques, based on the Radon-transformation (cf. (Rosenfeld and Kak, 1982)).

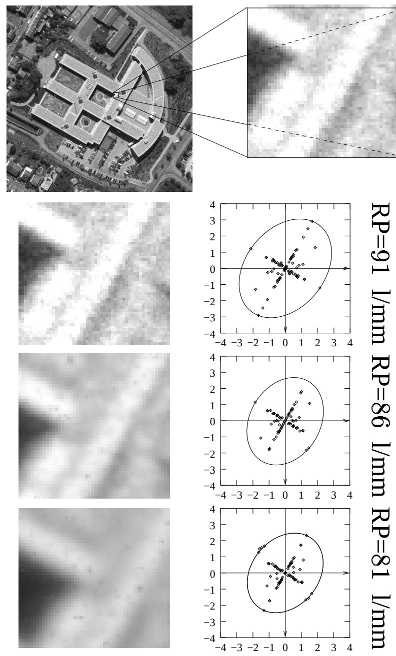

Figure 8: Aerial images with various image sharpness. Top: whole original image with image patch. Left: image patch at various steps of sharpness. Right: edge histogram, resolving power.

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