Model-based 2D-Shape Recovery

Ansgar Brunn, Uwe Weidner, and Wolfgang Förstner

Institut für Photogrammetrie
Nußallee 15, 53115 Bonn
email: ansgar,weidner,wf@ipb.uni-bonn.de

Abstract: The paper presents a new approach for the reconstruction of polygons using local and global constraints. The MDL-based solution is shown to be useful for analysing range and image data of buildings.

1 Introduction

Segmentation of the boundary of 2D-shapes is a basic prerequisite for reconstruction, recognition or matching tasks. The goal of the segmentation is to replace a low level description of the shape by a more structured one exploiting knowledge about the objects' boundaries. Parametric descriptions may be just dense sequences of points, splines or Fourier descriptors, all not being specific for a certain class of objects. Strucutured descriptions may be sequences of shape primitives, attributed skeletons, or – exploiting the dichotomy of boundary and region representation – overlapping sets of shape primitives.

Parametric and structural descriptions in a natural way may be associated with the type of the shape models implicitely or explicitely used. *Local* models of boundaries refer to properties like curvature of lines, average length or angle of polygons, *global* models refer to relations of non neighbouring shape primitives such as parallelity, collinearity or directly to global measures such as area or connectivity.

As there up to now seems to be no general theory of shape, techniques for shape recovery need to refer to a specific shape class. Our research interest is in building reconstruction from aerial images or range data. Therefore, we need techniques for recovering the boundary of image segments of roofs or of ground plans. In both cases the shapes show specific regularities such as parallelity and collinearity, and in case of range data also orthogonality. On the other side due to occlusions caused by interference with other objects or object parts and the great variety of real shapes even within this restricted class, we cannot use global measures for guiding the recovery process. Moreover, we should be able to deal with multiple boundaries of objects, i. e. objects with holes and groups of objects.

Whereas the number of papers dealing with the approximation of boundaries based on local models is quite large (cf. references in Fischler and Wolf 1994), and quite some algorithms exist for finding shapes represented with a fixed set of parameters (rectangles: e. g. Lin *et al.* 1994, snakes: Kass *et al.* 1988), no concept is known to the authors which is able to recover general polygons with global constraints. The work of Fua and Hanson 1987 probably is most closely

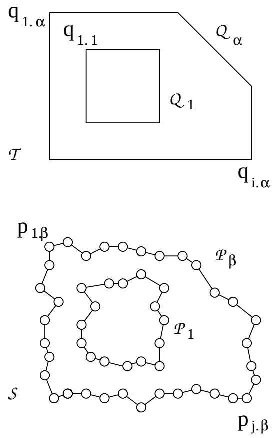

Fig. 1. True shape $\tilde{\mathcal{T}}$ and observed shape $\mathcal{S}$

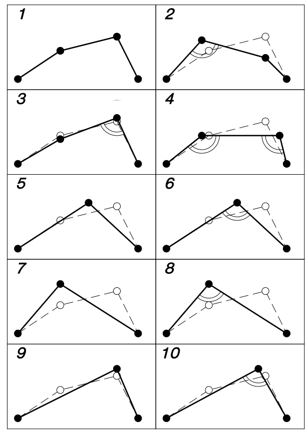

Fig. 2. 10 alternatives for local configuration

related to ours in terms of generality, but refers to structured aggregates of region primitives, whereas we refer to free boundaries.

The task to be solved can be stated as follows: The *true shape* $\tilde{\mathcal{T}}$ to be recovered from the observed shape $\mathcal{S}$ consists of a set of polygons, thus $\tilde{\mathcal{T}} = \{\tilde{\mathcal{Q}}_\alpha\}$ with closed cyclical lists $\tilde{\mathcal{Q}}_\alpha = [q_{i.\alpha}]$ of length $\| \tilde{\mathcal{Q}}_\alpha \| = \tilde{n}_\alpha$ containing the points $q_{i.\alpha}(x_{i.\alpha}, y_{i.\alpha})$. Edges $f_{i_1.\alpha_1}(q_{i_1-1.\alpha_1}, q_{i_1.\alpha_1})$ and $f_{i_2.\alpha_2}(q_{i_2-1.\alpha_2}, q_{i_2.\alpha_2})$ of the same or of different polygons $\mathcal{Q}_{\alpha_1}$ and $\mathcal{Q}_{\alpha_2}$ may show geometric relations $r(f_{i_1.\alpha_1}, f_{i_2.\alpha_2})$, especially being parallel or collinear, and in case of range data being perpendicular. The *observed shape* also consists of a set of polygons $\mathcal{S} = \{\mathcal{P}_\beta\}$ with lists $\mathcal{P}_\beta = [p_{j.\beta}]$ and $p_{j.\beta}(x_{j.\beta}, y_{j.\beta})$.

The goal is to reconstruct $\tilde{\mathcal{T}}$, thus to find an estimate $\hat{\mathcal{T}}$ for $\tilde{\mathcal{T}}$ from the given shape $\mathcal{S}$ (cf. Fig. 1), which at the same time optimally fits the data $p_{j.\beta}(x_{j.\beta}, y_{j.\beta})$, and shows local and global regularities. As the number of polygon sides $\sum_\alpha \tilde{n}_\alpha$ as well as the existence of geometric relations is unknown and the given data $\mathcal{P}_\beta$ are noisy, an approximation criterium which is able to incorporate parametric and structural knowledge is needed. We use the principle of minimal description length (MDL) (cf. Rissanen 1987, Förstner 1989) which can be related to Bayesian estimation.

The paper first describes the concept of our scheme for 2D-shape recovery (section 2), describes how we use the local and the global model (section 3), and closes with examples from image and range data analysis to demonstrate the versability of the concept.

2 Concept for 2D-Shape Recovery

The proposed concept for 2D-shape recovery is based on a local and global analysis of the given shape using MDL, which may be iteratively applied, and is meant to lead to an efficient algorithm.

2.1 The MDL–Criterion for Structure Evaluation

The description length DL of a set $\hat{\mathcal{T}} = \{\hat{\mathcal{Q}}_\alpha\}$ of polygons approximating a set $\mathcal{P}$ of given points depends on both, the fit of the model to the data and the complexity of the model. The fit is measured by the weighed sum Ω of the residuals of a ML-estimation, whereas the complexity depends on the number of unknown parameters and the number n of given data, influencing the precision of the parameters. Let the non linear or linearized model be given by

$$E(\mathbf{y}) = \mathbf{g}(\boldsymbol{\beta}), \quad D(\mathbf{y}) = \boldsymbol{\Sigma}_{yy} \tag{1}$$

with u unknown parameters collected in $\boldsymbol{\beta}$, n observations collected in the vector $\mathbf{y}$, and their covariance matrix $\boldsymbol{\Sigma}_{yy}$. The description length is given by (cf. Rissanen 1987)

$$DL = \frac{\Omega}{2\,ln\,2} + \frac{u}{2} lb\, n \tag{2}$$

with the weighed sum of the squared residuals

$$\Omega = [\mathbf{y} - \mathbf{g}(\boldsymbol{\beta})]^T \boldsymbol{\Sigma}_{yy} [\mathbf{y} - \mathbf{g}(\boldsymbol{\beta})]$$

In case h constraints exist between the u' mutually dependent unknowns, we have $u = u' - h$ free unknown parameters. For a fixed number of u parameters the description length only depends on the fit Ω of model and data.

Equation (2) will be used to locally select hypothesis. Due to the equivalence of MDL and robust estimation (cf. section 3.2), also the selection of the global constraints is performed and evaluated based on (2).

2.2 The Strategy for Description Length Reduction

The procedure for recovering the 2D-shape consists of several steps, as a one step procedure does not seem to be feasible. The steps are the following:

1. **Preprocessing:** In the first step only local constraints, namely the noise model, is used. The given polygon set $\mathcal{S}$ is simplified by
   (a) *merging* collinear segments, if the given points $p_{j,\beta}$ refer to grid positions, only taking the rounding errors of the grid positions into account, and/or
   (b) *splitting* the resulting polygons according to classical recursive techniques (Douglas and Peucker 1973) with a narrow threshold T depending on the noise model, e. g. $T = 2\sigma_n$, where σ_n denotes the positional noise of the given points.

   We then obtain a first approximation $\hat{\mathcal{T}}^{(0)} = \{\hat{\mathcal{Q}}^{(0)}\}$ as starting point for iteration $\nu = 1$, which consists of $n_0 = \| \hat{\mathcal{T}}^{(0)} \| = \sum_\alpha \| \hat{\mathcal{Q}}^{(0)}_\alpha \| < m$ points. The named approaches for preprocessing are useful to reduce the number of points, even if the merging or splitting criterion is fixed in order to reduce the discretization noise only. Nevertheless, no model knowledge can be incorporated, which is possible using

2. **Local MDL–Analysis:** A local analysis of all edges identifies the best simplification of the polygons $\hat{\mathcal{T}}^{(\nu-1)}$ according to the MDL principle. The end points (p_{i-1}, p_i) of an edge may be identified to show angles of either $\pm 90°$ or $180°$, in the last case leading to the elimination of a point. The evaluation of the 10 alternatives shown in Fig. 2 is discussed below. The simplification may reduce the number of polygon points by either
   (a) a fixed maximum number k, e. g. $k =$ number of polygons in $\mathcal{S}$,
   (b) a fixed maximum percentage, especially for large $n_\alpha = \| \mathcal{Q}_\alpha \|$, or
   (c) iterating until no change occur in the local configuration.
   Evaluating the reduction one may refer either to
   (a) the original data $p_{i.\beta}$ guaranteeing no loss in information or
   (b) to the actual data $q_{i.\alpha}^{(\nu-1)}$, possibly neglecting statistical dependencies between the actual data, thereby reducing algorithmic complexity.
   The result of this step is a set $\hat{\mathcal{T}}^{(\nu)}$, $\nu = 1, \ldots$ of possibly reduced polygons with modified coordinates. This step may be iterated until no reduction in the local DL is possible any more.
3. **Global Hypothesis Generation:** The edges $f_i(q_{i-1}^{(\nu)}, q_i^{(\nu)})$ of the the resulting polygons are now checked with respect to pairwise geometric relations. Orthogonality and parallelity are checked via the angle $\alpha(f_{i_1}, f_{i_2})$ using the test statistic $T_\alpha = \alpha/\sigma_\alpha$, where σ_α is determined via error propagation. Collinearity is checked by parallelity and orthogonal distance $d(p_{i_1}, f_{i_2})$ of one point to the other edge, again using error propagation. The result is a set $\mathcal{H} = \{h_k\}$ of hypothesis, which in general will not be independent.
4. **Constraint Robust Estimation:** The set of global hypothesis is used in a robust estimation (cf. Fuchs and Förstner 1995). Robust estimation is applied for three tasks, i. e. the elimination of globally inconsistent hypothesis, the determination of optimal coordinates of the points $q_{i.\alpha}^{(\nu)}$, and the determination of $\Omega^{(\nu)}$ used for the the global description length $DL^{(\nu)}$.
5. **Selection:** The global description length $DL^{(\nu)}$ according to (2) is used to decide on the quality of the final result. In case an iteration is performed in step 2, one may stop after this estimation. Otherwise, if $DL^{(\nu)} < DL^{(\nu-1)}$ the procedure continues with step 2 for further reduction. If $DL^{(\nu)} \geq DL^{(\nu-1)}$, we may either
   (a) stop, as there is no reduction in description length to be expected or
   (b) backtrack to step 2, e. g. using the second best selection of points.

The examples given later use the following sequence:

1. Preprocessing of $\mathcal{S}$ to eliminate collinear neighbouring edges.
2. Local MDL–analysis by eliminating a maximum number of points at a time referring to the actual data within this step, which is iterated until no local reduction of the description length is possible any more.
3. Global hypothesis generation, however restricting the generation of rectangular hypothesis to neighbouring edges.
4. Robust estimation.
5. Stop.

2.3 Computational Complexity

The algorithmic complexity of the different steps is given by $O(m)$ for merging and $O(m \log m)$ for splitting for preprocessing with $m = \| \mathcal{S} \|$, and $O(n^{(\nu)})$ with $n^{(\nu)} = \| \mathcal{T}^{(\nu)} \|$ for local MDL-analysis. If we iterate step 2 until no reduction is possible any more, the complexity is $O((r_0 n_0)^2)$ for constant maximum reduction and $O(r_0 n_0 \log(r_0 n_0))$ for a proportional reduction, where n_0 is the number of points after step 1 and $r_0 n_0$ is the number of points after the first reduction in step 2. The complexity for global hypothesis generation is $O((n^{(\nu)})^2)$, and $O((n^{(\nu)})^3)$ for the constraint estimation. If steps 2 to 4 are iterated, the complexity of these three steps is $O((r_0 n_0)^4)$ for constant maximum reduction and $O((r_0 n_0)^3 \log(r_0 n_0))$ for a proportional reduction. In the worst case we therefore obtain $O(c_1 m \log m + c_2 (q_0 n_0)^4)$, indicating, that the preprocessing (n_0) and the first MDL-based reduction (r_0) are decisive. Following this advice, we reach the lowest complexity, if step 2 is repeated until no reduction is possible any more and only once a global analysis is performed: $O(c_1 m \log m + c_2 n_0^2 + c_3 k^3)$, where k is the number of points in the final polygon set. Other strategies may be chosen.

3 Using the Shape Model

3.1 Local Constraints

First, we evaluate the hypotheses about neighbouring edges. This evaluation is based on the consideration of local configurations of four consecutive points. These configurations are set up by imposing orthogonality constraints on one or both angles at neighbouring points or by replacing them by a single point, possibly also introducing an orthogonality constraint in this point. Fig. 2 shows 10 alternatives ($a = 1, \ldots, 10$) which are used and evaluated using the description length given in (2).

The points 1 and 4 of the configurations are assumed to be fixed. Points 2 and 3 or the replaced point, dependent on the alternative, can move. The coordinates of these points are introduced as observations. Furthermore, the n_d orthogonal distances d_i of related points of the polygons $\mathcal{S}$ to an edge of $\mathcal{T}$ may be introduced as observations with $E(d_i) = 0$, relating the procedure to the original data. The area of the local configuration is assumed to be fixed and together with the orthogonalities introduced as weak constraints. A local adjustment yields the weighed sum of residuals Ω_a. Together with (2) the description length DL_a can be computed. Dependent on the description length, the local structure of the polygons $\mathcal{T}$ is adapted, replacing the local configurations by those alternatives which decrease the description lenght most.

3.2 Global Constraints

Global constraints establish geometric relations between any two polygon sides. Due to the transitivity of parallelity and collinearity relation, and similar relations including orthogonality relations, the found set of hypothesis will not be independent. On the other hand, sets of individually consistent hypothesis need not be jointly consitent. For both reasons, it is useful to introduce the relations as weak constraints, thus as observations, e. g. $E(\alpha - \frac{\pi}{2}) = \mathbf{g}(\boldsymbol{\beta})$ for the orthogonality constraint with a certain weight $w_i = 1/\sigma_i^2$, and apply robust estimation with a modified minimum function $\sum_i \rho(v_i)$, instead of $\sum v_i^2$ being non robust. The function ρ, which is of the form $\rho(x) = \min(\frac{x^2}{2}, \frac{k^2}{2})$, reflects two classes of hypothesis: accepted ones (x in $[-k, +k]$) and rejected ones (x outside $[-k, +k]$). The decision, whether a hypotheses is accepted using the MDL principle leads exactly to the same type of minimization function, however having the advantage that the threshold k results from the coding scheme (Förstner 1989). We used the robust estimation of our grouping technique (Fuchs and Förstner 1995).

4 Examples

The examples given in the following show results of our approach applied to range data and aerial images. Fig. 3 displays original range data[1], acquired by airborne laser scanning, and the result of a segmentation (cf. Weidner and Förstner 1994). The outlines of this segmentation are used as starting point for the vectorization. Fig. 4 shows results of the shape recovery for three building ground plans. From left to right, the polygons are displayed after preprocessing, local MDL–analysis, and global robust adjustment. For these data sets the local MDL–application leads to a reduction of points from 36 to 7, 134 to 29, and 98 to 36 resp. The hypothesis about geometric relations between edges of the polygons, which are introduced in the robust estimation, put constraints onto the edges, which results in the final polygons. These polygons are also displayed in Fig. 5 superimposed on the original range data. A qualitative evaluation shows little discrepancies, whereas the overall performance seems to be acceptable. The discrepancies are on one hand due to the suboptimal sequence as described in section 2, i. e. considering only the data in each iteration and not the originally observed polygons. On the other hand not all hypotheses passing through the robust estimation are actually correct.

Fig. 6 displays the results for range data acquired using a matching technique (Krzystek 1991), indicating the ability of our approach to deal with multiple polygons belonging to an object.

Fig. 7 shows the image data and the segmentation for the roof. The intermediate and the final polygons are again displayed from left to right. In this case, a representation of the different parts of the roof by a *network of polygons*, allowing points to belong to multiple sets $\mathcal{P}_\alpha$ would be appropriate.

5 Discussion

The results of the developed procedure applied to real data, though being convincing as a first step, show some deficiencies, which need to be analysed in

[1] The range data of Hannover was supplied by Dornier, Friedrichshafen.

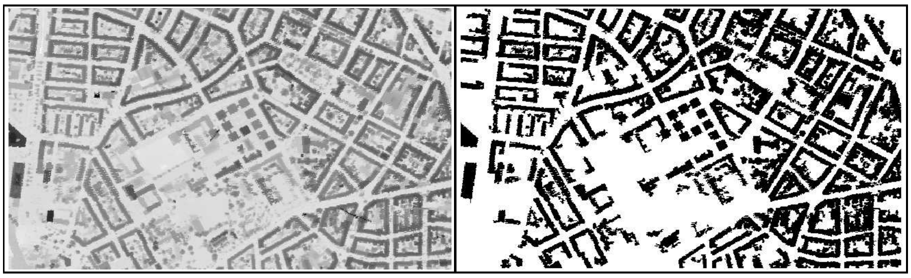

Fig. 3. Range data from airborne laser scanner and segmentation (Weidner and Förstner 1994)

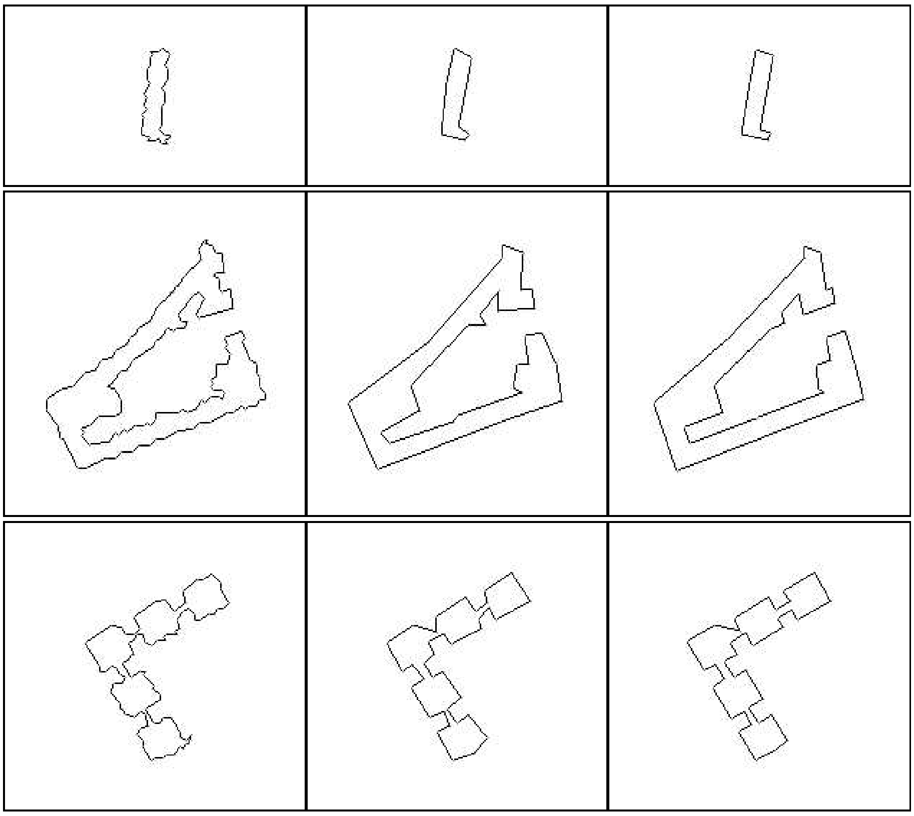

Fig. 4. Three examples for range data – left: original boundary from segmentation; middle: result of local MDL–analysis; right: recovered final shape

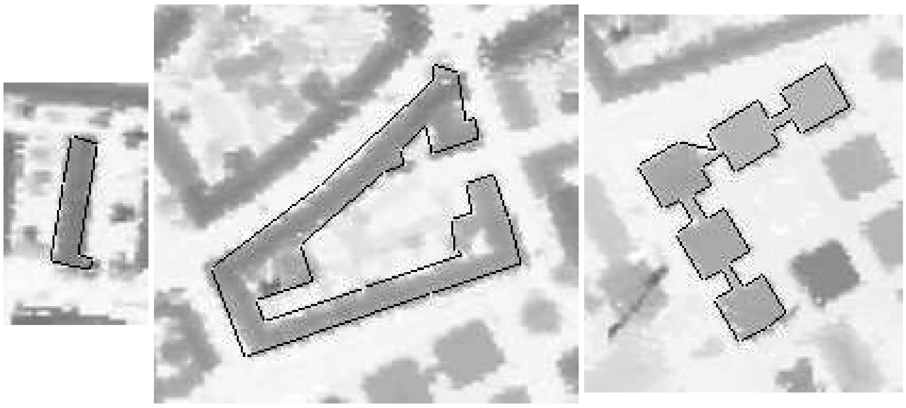

Fig. 5. Overlay of recovered final shapes on original data

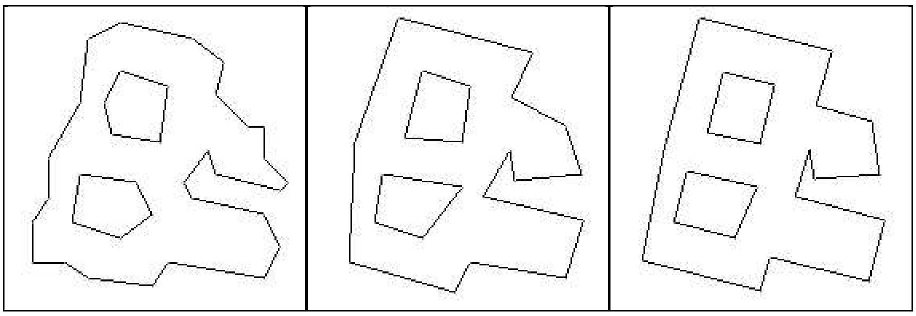

Fig. 6. Example for range data based on image matching and segmentation (cf. Fig. 4)

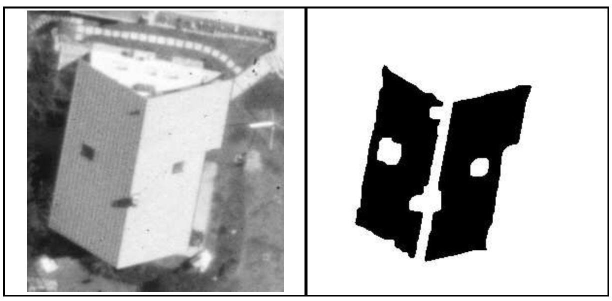

Fig. 7. Image data and blobs

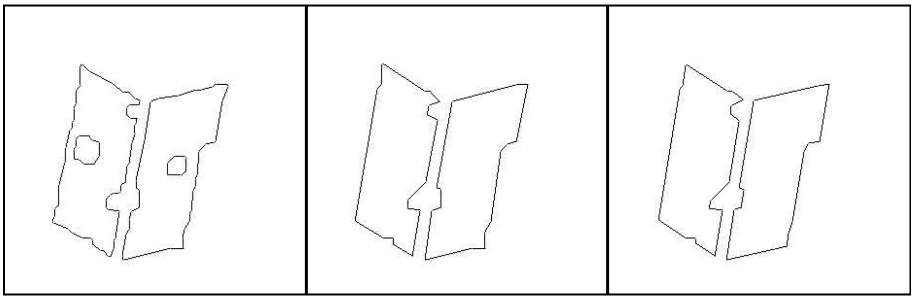

Fig. 8. Example for image data (cf. Fig. 4)

more detail, namely the high algorithmic complexity when not iterating the local MDL-analysis, the geometric deviations resulting from not referring to the original data during the iterations, and the evaluation of the hypothesis selected by the robust estimation. The approach may be generalized to more complex basic shape structures, such as circles, and to networks of polygons, necessary in simplifying image segmentation results.

References

Douglas, D.H.; Peucker, T.K. (1973): Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line or Its Caricature. *The Canadian Cartographer*, 10(2):112–122, 1973.

Fischler, M.A.; Wolf, H.C. (1994): Locating Perceptually Salient Points on Planar Curves. *IEEE T-PAMI*, 16(2):113–129, 1994.

Förstner, W. (1989): Image Analysis Techniques for Digital Photogrammetry. In: *Proceedings of 42nd Photogrammetric Week, Stuttgart*, pages 205–221, 1989.

Fua, P.; Hanson, A.J. (1987): Resegmentation Using Generic Shape Locating General Cultural Objects. *Pattern Recognition Letters*, 5:243–252, 1987.

Fuchs, C.; Förstner, W. (1995): Polymorphic Grouping for Image Segmentation. In: *Proceedings 5th International Conference on Computer Vision*, 1995.

Kass, M.; Witkin, A.; Terzopoulos, D. (1988): Snakes: Active Contour Models. *International Journal of Computer Vision*, pages 321–331, 1988.

Krzystek, P. (1991): Fully Automatic Measurement of Digital Elevation Models. In: *Proceedings of the 43rd Photogrammetric Week, Stuttgart*, pages 203–214, 1991.

Lin, C.; Huertas, A.; Nevatia, R. (1994): Detection of Buildings Using Perceptual Grouping and Shadows. In: *Proceedings Computer Vision and Pattern Recognition*, pages 62–69, 1994.

Rissanen, I. (1987): Minimum Description Length Principle. *Encyclopedia of Statistical Sciences*, 5:523–527, 1987.

Weidner, U.; Förstner, W. (1994): Towards Automatic Building Extraction from High Resolution Digital Elevation Models. *ISPRS Journal*, 1994. submitted paper.