Module Coordinate Systems
Prof. Dr.techn. Michael Schindelegger
schindelegger@igg.uni-bonn.de

September 2020

Homework Assignment Earth Rotation

Transformation of station coordinates from a terrestrial to a celestial frame

Task description:

A VLBI (Very Long Baseline Interferometry) antenna at station Wettzell (Germany) takes part in a global VLBI experiment at **31 July 2020, 11:00 UTC**. As part of the subsequent analysis, the ITRF2014 coordinates of the station must be transformed to the celestial reference frame following the framework of the IAU 2000/2006 resolutions, considering all five Earth Orientation Parameters (precession/nutation, polar motion, and dUT1).

Input data and subroutines:

The ITRF2014 coordinates of the VLBI station Wettzell (code `7224`) are available for download from the IERS webpage (https://www.iers.org/IERS/EN/DataProducts/ITRF/itrf.html, SINEX format):

```
*CODE PT __DOMES__ T _STATION DESCRIPTION__ APPROX_LON_ APPROX_LAT_ _APP_H_
 7224  A 14201S004   WETTZELL Wettzell, Ger  12 52 38.8  49 08 42.0   669.1
   …
   169 STAX    7224  A     1 10:001:00000 m    2 0.407553967335915E+07 0.69526E-03
   170 STAY    7224  A     1 10:001:00000 m    2 0.931735482805480E+06 0.68664E-03
   171 STAZ    7224  A     1 10:001:00000 m    2 0.480162949550623E+07 0.67327E-03
```

The coordinates (second column from right, [m]) are valid at a specific reference epoch, i.e., 2010.0 in case of the ITRF2014. In high-accuracy applications, one would need to make use of the given velocities (last column) to perform linear extrapolation in time to the actual observation epoch.

Matlab (or Octave) subroutines needed to complete the task:

- `date2mjd.m`: converts a given UTC epoch to the corresponding modified Julian date
- `xys2000a.m`: evaluates the IAU2000 precession-nutation model for a given mjd input
- `Rx.m, Ry.m, Rz.m`: functions for the rotation matrices in the three coordinate directions

Some of the functions depend on other subroutines, which are provided too. The headers of the more comprehensive functions include a description of the input and output parameters and their units.

Specific tasks:

(1) *Nutation series*:

Familiarize yourself with the subroutine `xys2000a.m` by evaluating it for a time span of 25 years, from 1 January 1990 (0 UTC) to 31 December 2014 (0 UTC). The function input should be a vector of modified Julian dates, sampled from 1 January 1990 onwards in 5-day intervals. Use `date2mjd.m` to convert from calendar dates to modified Julian dates. Remove the linear trend of the resulting time series of nutation angles (Matlab command `detrend`) and plot both X and Y over time in units of meter. Make sure that the plot is intelligible (font size, legend, labels, etc.) and save it for inclusion in your report.

(2) *Complete transformation* $ITRS \longrightarrow ICRS$:

Transform the Wettzell station coordinates at epoch 31 July 2020 (11 UTC) to the celestial system/frame using the full IAU2000 formulation $\vec{x}_{ICRS} = Q\ R\ W\ \vec{x}_{ITRS}$ as summarized in the synopsis below. Consider the following:

- mjd_{UTC} is the UTC epoch given in the main task description but converted to a modified Julian date using `date2mjd.m`
- Values for (X, Y, s) are readily obtained from the subroutine `xys2000a.m`
- Values for dUT1 and polar motion at 31 July 2020 are published by international centers, e.g., http://hpiers.obspm.fr/eop-pc/index.php:
  $\mathrm{dUT1} = -0.2091724$ [s]
  $x_p = 0.199858$ [as], $y_p = 0.395993$ [as], $s' = 0$
- These values are valid for the entire day and must be converted to [rad] where needed
- Document the so-derived ICRS coordinates of station Wettzell

(3) *Approximate solution*:

Repeat the transformation while neglecting the effects of polar motion and nutation (matrices W and Q). Document the resulting ICRS coordinates and quantify the approximation error w.r.t. the solution from sub-task (2) as magnitude of the difference vector.

Synopsis of the transformation procedure (see also lecture slides):

Transformation $ITRS \longrightarrow ICRS$ of a station vector $\vec{x}$: $\vec{x}_{ICRS} = Q\ R\ W\ \vec{x}_{ITRS}$

(1) Matrix W accounting for the effect of polar motion: $W = R_z(-s')R_y(x_p)\ R_x(y_p)$
   - x_p, y_p: coordinates of the CIP in the ITRS
   - s': Position of the TIO (Terrestrial Intermediate Origin) on the CIP equator w.r.t. the ITRS meridian

(2) Matrix R considering the instantaneous phase angle of the Earth: $R = R_z(-ERA)$
   - ERA: Earth Rotation Angle = angle between the TIO and CIO (Celestial Intermediate Origin) on the CIP equator, directly proportional to dUT1

- $ERA = 2\pi(0.7790572732640 + 1.00273781191135448\, T_u)$
- $T_u = mjd_{UT1} - 51544.5 = (mjd_{UTC} + \mathrm{dUT1}) - 51544.5$, here T_u is the epoch of interest in mean days since J2000.0

(3) Precession/nutation matrix Q: $Q = R_z(-E)\, R_y(-d) R_z(E+s)$

- s: Position of the CIO on the CIP equator w.r.t. the ICRS prime meridian
- E, d: auxiliary quantities computed from the nutation angles X and Y, i.e., the coordinates of the CIP in the ICRS
  $X \; = \sin d \cos E$
  $Y \; = \sin d \sin E$

Remarks:

Your submission consists of a brief report as pdf and the Matlab (or Octave) script of your solution. Both files should be zipped and uploaded to eCampus, **with your first and last name indicated in the file name**. The report must be machine-readable (no scanned hand-written text) and includes:

- Explanations of the solution process in your own words, with references to the equations employed
- Final results of all sub-tasks (figure for sub-task 1) and important intermediate results
- Results provided with the correct units and a reasonably chosen number of digits