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# Coordinate Systems MGE-05 WS2021

Exercise 1

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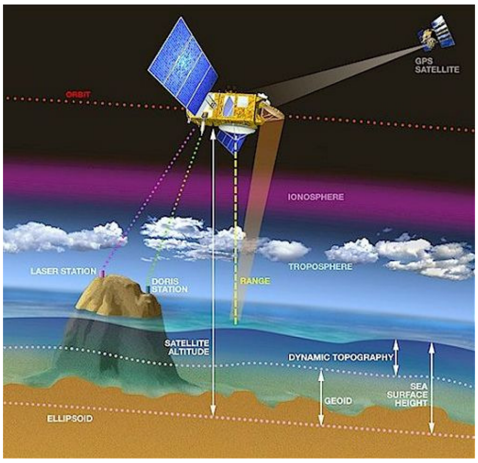
## Introduction

Radar altimeter satellites such as Jason-2, -3, Saral/Altika, Cryosat-2, Sentinel-3, etc., measure the range from the spacecraft to the sea surface. The ellipsoidal sea surface height (SSH) is then obtained as the satellite's height above the ellipsoid minus the measured range. Typical measurement accuracy is 2-3 cm, but by averaging many measurements one achieves mm-accuracy and better. The satellites' 3D positions are obtained from precise GNSS tracking. Altimetric SSH can be compared to SSH determined at tide gauges provided they are equipped with GNSS. In radar altimetry it is common to use the Topex/Poseidon (T/P) ellipsoid.




Radar altimetry principle (image credit: Aviso)

## Task 1.1

Given are the Cartesian coordinates in [m] of a satellite at time $t$,

$$X_{sat} = 4831342.4634 \quad Y_{sat} = 2833965.0779 \quad Z_{sat} = 5289590.6351$$

and the ellipsoidal longitude, latitude and height [m] of a nearby (few km) tide gauge station

$$\lambda_{tg} = 30.329000100° \quad \phi_{tg} = 43.592000088° \quad h_{tg} = 30.888 \ .$$

1. Compute ellipsoidal coordinates of the satellite w.r.t. the T/P ellipsoid.
2. What would be the (exact) difference in height if we were to use the GRS80 ellipsoid instead?
3. How big is the error expressed in [m], if we were accidentally to compute the spherical latitude instead of the ellipsoidal one?
4. How far, in [m], is the altimeter footprint (sub-satellite point on the ellipsoid) located from the tide gauge?

### Task 1.2

The satellite orbit has, at time $t$, an inclination of $i = 66.036006500°$, right ascension of ascending node (RAAN) of $\Omega = 335.188990200°$ and argument of perigee $\omega = 289.450123600°$. We assume here the orbit is perfectly circular, and the revolution time is $T = 110.0000$ min. Range measurements are provided every $\delta t = 1$ sec.

1. Compute the ellipsodial coordinates of the satellite footprint for one revolution, i.e. for times $t' = t + \delta t$. Hint: you will need to compute the true anomaly for times $t$ and $t'$.

### Task 1.3

In practice, different ellipsoids refer to different Cartesian systems that may be shifted (translated), rotated or scaled with respect to each other; i.e. one may have to apply a Helmert transformation to convert $X, Y, Z$ coordinates between them.

1. Show that, to first approximation, the change in ellipsoidal height $\delta h$ at a given location $\lambda, \phi$, when moving from one ellipsoid to another is found by

   $$\delta h = -\cos\phi \cos\lambda \, t_X - \cos\phi \sin\lambda \, t_Y - \sin\phi \, t_Z - a \, \delta m - \delta a + a \sin^2\phi \, \delta f$$

   where $t_X, t_Y, t_Z$ is the translation between the origin of the two systems, $\delta m$ a difference in scale, $\delta a$ the difference in the semi-major axes and $\delta f$ the difference in the flattening of the two ellipsoids (and rotations $\epsilon_x, \epsilon_y, \epsilon_z$ play no role).

   *Hint:* Make use of the Lecture Notes Prof. Kusche (Version 2018, p. 561).
2. In example 1, question 2 you have looked at the exact difference between T/P and GRS80 ellipsoidal heights, assuming the Cartesian systems are identical (no translation etc.). What difference does the approximate relation $\delta h = -\delta a + a \sin^2\phi \, \delta f$ provide? Is it sufficiently accurate?

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#### Formal Regulations

Your solution must include

- step-by-step explanation of the way of solving (what equations are used)
- all intermediate results
- all results must be provided with (the correct) units
- all results must be provided with the relevant number of digits
- all the above must be in a machine-readable format (i.e. not as scanned hand-written text)
- the codes that you used